# On Humanoid Control 

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## Humanoids can move the environment for humans

- Implies that
$\leftrightarrow$ Walking on a flat floor and rough terrain
$\Rightarrow$ Going up and down stairs and ladders
$\Rightarrow$ Lying down, crawling and getting up
$\Leftrightarrow$ Falling down safely and getting up
$\Leftrightarrow$ Opening and closing doors
- Humanoids move on two, three or four feet.


## Humanoid as a Controlled Plant

A humanoid robot is a multilink structure that is not fixed to the environment and moves in the environment and/or moves the environment by the contact force between the robot and the environment in the gravity field.

## Humanoid Control Problem

When the initial and final configurations of a humanoid robot is given, find motions of the robot that can transfer it from the initial configuration to the final configuration through a sequence of the contact states.


## Control Algorithms

- Inverted Pendulum Scheme

1. Plan motions of the robot
2. Change the position of the next footprint to keep the planned configuration of the robot
ZMP (Zero Moment Point) based Scheme
3. Plan a sequence of footprints.
4. Change the configuration of the robot to keep the planned sequence of the footprints

## Motions vs. Contact Force

$$
\begin{aligned}
& M\left(\mathbf{g}-\ddot{\mathbf{p}}_{G}\right)=\mathbf{f}_{C} \\
& \mathbf{p}_{G} \times M\left(\mathbf{g}-\ddot{\mathbf{p}}_{G}\right)-\dot{\mathbf{L}}=\boldsymbol{\tau}_{C}
\end{aligned}
$$

$\mathbf{p}_{G}$ : Position of the center of the gravity
$\mathbf{L}$ : Angular momentum about the COG
$\mathbf{f}_{C}$ : Contact force
$\boldsymbol{\tau}_{C}:$ Contact torque


## Inverted Pendulum Scheme

[Gubina, Hemami and McGee 1974]


## Footprints may have a constraint



## ZMP based Scheme

[Vukobratovic and Stepanenko 1972]


And more....

## What is ZMP (Zero Moment Point)?



Fig. 1. Zero-moment point (ZMP).


Center of Pressure
-ZMP NEVER leaves the support polygon!
-ZMP can be measured by force sensors in feet.

## How ZMP is used?

- When the ZMP is inside the support polygon, the contact between the feet and the floor should be kept.
- When the contact is kept, the posture of the robot should
 be kept without falling down.


## From the ZMP to the COG

Target ZMP pattern


Trajectory of the center of mass


## Motions vs. Contact Force

$$
\begin{aligned}
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$\boldsymbol{\tau}_{C}: \quad$ Contact torque


## Cart-Table Model



- A running cart on a mass-less table
- The table has a small support area


## The ZMP of the Cart-Table Model



$$
\begin{aligned}
\tau_{Z M P} & =M g\left(x-x_{0}\right)-M \ddot{x} z_{h} \\
= & 0 \\
&
\end{aligned}
$$

ZMP equation

$$
x_{0}=x-\frac{Z_{h}}{g} \ddot{x}
$$

## Input and Output



Walking pattern generation

Find the cart trajectory to realize the given ZMP pattern

## Servo tracking control of the ZMP



Proper dynamics of Cart-Table model

## From ZMP reference to Cart motion




Walking pattern


The cart must move before ZMP changes !
$\rightarrow$ Servo controller must use FUTURE information

## The Preview Control

On a winding road, we steer a car by watching ahead, by previewing the future reference.


- Concept and naming
[Sheridan 1966]
- LQ optimal controller
[Tomizuka and Rosenthal 1979]
[Katayama et.al 1985]

$$
\begin{aligned}
& \text { Control law } \\
& u_{k}=-K_{I} \sum_{i=0}^{k}\left(x_{0 k}-x_{0 k}^{r e f}\right)-K_{x} x_{k}-\left[f_{1}, f_{2}, \cdots f_{N}\right]\left[\begin{array}{c}
x_{0(k+1)}^{r e f} \\
\vdots \\
x_{0(k+N)}^{r e f}
\end{array}\right]
\end{aligned}
$$

Accumulate Servo error

State feedback Preview gain
Target ZMP of N -step future

## Preview gain



$$
\begin{aligned}
& f_{i}=\left(R+B^{T} P B\right)^{-1} B^{T}(A-B K)^{T *(i-1)} C^{T} Q \\
& K \equiv\left(R+B^{T} P B\right)^{-1} B^{T} P A
\end{aligned}
$$

LQ optimal feedback gain


The preview gain is approximately zero after 1.6 s future. Controller does not need the future ZMP farther than 1.6s.

## ZMP Tracking Example




## Walking Pattern Generator


[Kajita et al.]

## Experiment of HRP-2



## Configuration of the Feedback Controller



## Feedback Controller is essential



Without stabilizer


With stabilizer

## Feedback Control of the Table Orientation



Cart on a table

Equation of motion:

$$
\left(x^{2}+z_{c}^{2}\right) \ddot{\theta}+\ddot{x} z_{c}-g\left(z_{c} \sin \theta+x \cos \theta\right)+2 x \dot{x} \dot{\theta}=0
$$

$$
\text { Linearize at } \theta \approx 0
$$

$$
\left(x^{2}+z_{c}^{2}\right) \ddot{\theta}=g x-g z_{c} \theta-z_{c} \ddot{x}
$$



Table inclination

Cart acceleration

## Feedback Control of the Cart Position

[Nagasaka, Inaba and Inoue, 1999]


ZMP equation with sensor delay $T$

$$
x_{0}=\frac{1}{1+s T}\left(x-\frac{z_{h}}{g} \ddot{x}\right)
$$

System representation

$$
\frac{d}{d t}\left[\begin{array}{c}
x_{0} \\
x \\
\dot{x}
\end{array}\right]=\left[\begin{array}{ccc}
-1 / T & 1 / T & 0 \\
0 & 0 & 1 \\
0 & 0 & 0
\end{array}\right]\left[\begin{array}{c}
x_{0} \\
x \\
x
\end{array}\right]+\left[\begin{array}{c}
-Z_{c} /(g T) \\
0 \\
1
\end{array}\right] \ddot{x}
$$

Stabilization by state feedback
Used in the walking controller of H 7 . Good for robots with hard feet.

$$
\ddot{x}=-k_{1} x_{0}-k_{2} x-k_{3} \dot{x}
$$



ZMP error

## Experiment on a Slope



## HRP-2 walks on a Rough Terrain



Gap $< \pm 20$ mm Slope $<5 \%$

## J apanese Traditional Dance


[Nakaoka et al. 2005]

## The First Running Humanoid Sony Qrio [Dec.,2003]



Qrio runs at $0.84 \mathrm{~km} / \mathrm{h}$.

## Running Biped [AIST Apr.,2004]



Speed 0.58km/hour


Slow motions

## Walking on a floor with low friction



$$
\mu: 0.5 \rightarrow 0.1
$$

$$
\mu: 0.5 \rightarrow 0.05
$$

$\mu=0.15$ between a tire of a car and a wet snow surface

## Reduction of the Slip by a Tuning of Walking Pattern

-Rotation about Yaw-axis may occur at $\mu=0$. 3 due to the change of the acceleration when the supporting leg is exchanged.
-The pattern generator is tuned to reduce the peak of the jerk.


## Walk on a Floor with a Low Friction



## Humanoids can move the environment for humans

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$\Rightarrow$ Lying down, crawling and getting up
$\Leftrightarrow$ Falling down safely and getting up
$\Leftrightarrow$ Opening and closing doors
- Humanoids move on two, three or four feet.


## Contact States Graph



## Balance Control for the Transition



The position of the torso link is under a compliance control.


## Dynamic Simulation



## Lying down and Getting up



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- Humanoids move on two, three or four feet.


## Preliminary Experiment for Falling



With the knee extended


With the knee bended

## Falling Motion of a Leg Robot



## Impact Test



## Falling Motion of Humanoid Robot



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$\Rightarrow$ Falling down safely and getting up
$\Rightarrow$ Opening and closing doors
$\Rightarrow$ Arms and legs coordination
- Humanoids move on two, three or four feet.


## Several Configurations of Arm/Leg Coordination



Walking and Touching


## Leaning on



Balancing


Pulling


Pushing

## A Generalized ZMP [Harada et al. 2004]



2D Convex Hull
3D Convex Hull

## A Generalized ZMP [Harada et al. 2004]



# A Generalized ZMP [Harada et al. 2004] 

Small Acceleration



## A Generalized ZMP [Harada et al. 2004]



## Projection of the ZMP



## Numerical Example



## Area of the Generalized ZMP



## Push a heavy object


25.9 kg
[Harada et al. 2004]

## Pushing a button with the support of a hand


[Harada et al. 2004]

## An Open Question

- Can we plan motions of humanoid robots based on the unified criterion?
- What the ZMP criterion can judge?
$\Rightarrow$ The ZMP can judge if the contact should be kept without solving the equations of motions when the robot moves on a flat plane under the sufficient friction assumption.


## Our Goal are

- to create a new criterion that can judge the contact stability of humanoids which may touch an arbitrary terrain with two, three or four feet, and
- to prove that the criterion is equivalent to ZMP in a specific case and more universal, and to claim to say "Adios ZMP".


## Related Works

- Legged Robots
\& ZMP [Vukobratovic 1972]
$\Leftrightarrow$ Locomotion with hand contact [Yoneda 1996]
$\Rightarrow$ FRI [Goswani 1999]
$\Rightarrow$ FSW [Saida 2003]
$\Leftrightarrow$ Generalized ZMP [Harada 2004]
- Mechanical Assembly
$\Rightarrow$ Strong and Weak Stability [Trinkle 1997]


## Formulation



Gravity and Inertia Wrench

$$
\begin{aligned}
\mathbf{f}_{G} & =M\left(\mathbf{g}-\ddot{\mathbf{p}}_{G}\right) \\
\boldsymbol{\tau}_{G} & =\mathbf{p}_{G} \times M\left(\mathbf{g}-\ddot{\mathbf{p}}_{G}\right)-\dot{\mathcal{L}}
\end{aligned}
$$

$\mathbf{p}_{G}$ : Center of the mass
$\mathcal{L}$ : Angular momentum around COG
Contact Wrench

$$
\begin{aligned}
& \mathbf{f}_{C}=\sum_{k=1}^{K} \sum_{l=1}^{L} \varepsilon_{k}^{l}\left(\mathbf{n}_{k}+\mu_{k} \mathbf{k}_{k}^{l}\right) \\
& \boldsymbol{\tau}_{C}=\sum_{k=1}^{K} \sum_{l=1}^{L} \varepsilon_{k}^{l} \mathbf{p}_{k} \times\left(\mathbf{n}_{k}+\mu_{k} \mathbf{t}_{k}^{l}\right) \\
& \Rightarrow \text { Polyhedral Convex Cone }
\end{aligned}
$$

## Strong Stability Criterion

- The contact state must be stable if (-fg,-T g) is an internal element of the contact wrench cone under the sufficient friction assumption.
(proof)
The work done by ( $\mathrm{fG}, \mathrm{T} \mathrm{G}$ ) is negative for any motion;

$$
\forall\left(\delta x_{G}, \Omega_{G}\right) \neq 0,\left(-f_{G,}-\tau_{G}\right) \in \operatorname{int}(C W C) ;\left(\delta x_{G}, \Omega_{G}\right) \llbracket\left(f_{G}, \tau_{G}\right)<0,
$$ where the CWC is given by

$$
\mathbf{f}_{C}=\sum_{k=1}^{K} \sum_{l=1}^{L}\left(\varepsilon_{k}^{l} \mathbf{n}_{k}+\varepsilon_{k}^{l} \mathbf{t}_{k}^{\prime}\right) \quad \boldsymbol{\tau}_{C}=\sum_{k=1}^{K} \sum_{l=1}^{L}\left(\varepsilon_{k}^{l} \mathbf{p}_{k} \times \mathbf{n}_{k}+\varepsilon_{k}^{l} \mathbf{p}_{k} \times \mathbf{t}_{k}^{l}\right)
$$

## Example 1. Walking on a horizontal plane with sufficient friction



## Strong Stability Determination by ZMP



$$
\begin{aligned}
& \frac{M\left(\ddot{z}_{G}+g\right) x_{G}-M \ddot{x}_{G} z_{G}-\dot{\mathcal{L}}_{y}}{M\left(\ddot{z}_{G}+g\right)}=\sum_{k=1}^{K} \lambda_{k} x_{k} \\
& \frac{M\left(\ddot{z}_{G}+g\right) y_{G}-M \ddot{y}_{G} z_{G}+\dot{\mathcal{L}}_{x}}{M\left(\ddot{z}_{G}+g\right)}=\sum_{k=1}^{K} \lambda_{k} y_{k} \\
& \sum_{k=1}^{K} \lambda_{k}=1, \lambda_{k} \geq 0
\end{aligned}
$$

## Equivalence between the ZMP and the CWC

ZMP

$$
\begin{gathered}
\frac{M\left(\ddot{z}_{G}+g\right) x_{G}-M \ddot{x}_{G} z_{G}-\dot{\mathcal{L}}_{y}}{M\left(\ddot{z}_{G}+g\right)}=\sum_{k=1}^{K} \lambda_{k} x_{k} \\
\frac{M\left(\ddot{z}_{G}+g\right) y_{G}-M \ddot{y}_{G} z_{G}+\dot{\mathcal{L}}_{x}}{M\left(\ddot{z}_{G}+g\right)}=\sum_{k=1}^{K} \lambda_{k} y_{k} \\
\sum_{k=1}^{K} \lambda_{k}=1, \lambda_{k} \geq 0
\end{gathered}
$$

CWC

$$
\begin{gathered}
M\left(\ddot{z}_{G}+g\right) y_{G}-M \ddot{y}_{G} z_{G}+\dot{\mathcal{L}}_{x}=\sum_{k=1}^{K} \varepsilon_{k}^{0} y_{k} \\
-M\left(\ddot{z}_{G}+g\right) x_{G}+M \ddot{x}_{G} z_{G}+\dot{\mathcal{L}}_{y}=-\sum_{k=1}^{K} \varepsilon_{k}^{0} x_{k}
\end{gathered}
$$

Dividing the equations by $M\left(\ddot{z}_{G}+g\right)=\sum_{k=1}^{K} \varepsilon_{k}^{0}$

## Equivalence between the ZMP and the CWC

ZMP

$$
\begin{aligned}
& \frac{M\left(\ddot{z}_{G}+g\right) x_{G}-M \ddot{x}_{G} z_{G}-\dot{\mathcal{L}}_{y}}{M\left(\ddot{z}_{G}+g\right)}=\sum_{k=1}^{K} \lambda_{k} x_{k} \\
& \frac{M\left(\ddot{z}_{G}+g\right) y_{G}-M \ddot{M}_{G} z_{G}+\dot{\mathcal{L}}_{x}}{M\left(\ddot{z}_{G}+g\right)}=\sum_{k=1}^{K} \lambda_{k} y_{k} \\
& \sum_{k=1}^{K} \lambda_{k}=1, \lambda_{k} \geq 0 \\
& \frac{M\left(\ddot{z}_{G}+g\right) y_{G}-M \ddot{y}_{G} z_{G}+\dot{\mathcal{L}}_{x}}{M\left(\ddot{z}_{G}+g\right)}=\sum_{k=1}^{K} \frac{\varepsilon_{k}^{0}}{\varepsilon} y_{k} \\
& \frac{-M\left(\ddot{z}_{G}+g\right) x_{G}+M \ddot{x}_{G} z_{G}+\dot{\mathcal{L}}_{y}}{M\left(\ddot{z}_{G}+g\right)}=-\sum_{k=1}^{K} \frac{\varepsilon_{k}^{0}}{\varepsilon} x_{k} \\
& \sum_{k=1}^{K} \frac{\varepsilon_{k}^{k}}{\varepsilon}=1, \frac{\varepsilon_{k}^{0}}{\varepsilon} \geq 0
\end{aligned}
$$

## The CWC for a 2D-Robot on a Line



## A Desired Trajectory in the CWC



The CWC is the direct product of a 2D polyhedral cone and 1D linear subspace, which is identical for the single and double support phases.

## Example 2. Robot on a Stair (1/2)



$$
\begin{array}{r}
M\left(\ddot{z}_{G}+g\right) y_{G}-M \ddot{y}_{G} z_{G}+\dot{L}_{x} \\
=\sum_{k=1}^{K} \varepsilon_{k}^{0} y_{k}-\lambda_{1}^{y} z_{F 1}-\lambda_{2}^{y} z_{F 2} \\
-M\left(\ddot{z}_{G}+g\right) x_{G}+M \ddot{x}_{G} z_{G}+\dot{L}_{y} \\
=-\sum_{k=1}^{K} \varepsilon_{k}^{0} x_{k}+\lambda_{1}^{x} z_{F 1}+\lambda_{2}^{x} z_{F 2}
\end{array}
$$

## Example 2. Robot on a Stair (2/2)

$$
\begin{aligned}
& M\left(\ddot{z}_{G}+g\right) y_{G}-M \ddot{y}_{G} z_{G}+\dot{L}_{x} \\
& =\sum_{k=1}^{K} \varepsilon_{k}^{0} y_{k}-\lambda_{1}^{y} z_{F 1}-\lambda_{2}^{y} z_{F 2}
\end{aligned}
$$

$$
=\sum_{k=1}^{K} \varepsilon_{k}^{0} y_{k}-M \ddot{y}_{G}\left(\left(\frac{\lambda_{1}^{y}}{\lambda_{1}^{y}+\lambda_{2}^{y}}\right) z_{F 1}+\left(\frac{\lambda_{2}^{y}}{\lambda_{1}^{y}+\lambda_{2}^{y}}\right) z_{F 2}\right)
$$

where

$$
\begin{aligned}
& M \ddot{y}_{G}=\lambda_{1}^{y}+\lambda_{2}^{y} \\
& M\left(\ddot{z}_{G}+g\right) y_{G}-M \ddot{y}_{G}\left(z_{G}-z_{F}\right)+\dot{L}_{x}=\sum_{k=1}^{K} \varepsilon_{k}^{0} y_{k} \\
& \quad z_{F}=\left(\frac{\lambda_{1}^{y}}{\lambda_{1}^{y}+\lambda_{2}^{y}}\right) z_{F 1}+\left(\frac{\lambda_{2}^{y}}{\lambda_{1}^{y}+\lambda_{2}^{y}}\right) z_{F 2}
\end{aligned}
$$

## Pattern Generation of the COG

$$
\begin{aligned}
& \frac{d}{d t}\left(\begin{array}{l}
y_{G} \\
\dot{y}_{G} \\
\dot{y}_{G}
\end{array}\right)=\left(\begin{array}{lll}
0 & 1 & 0 \\
0 & 0 & 1 \\
0 & 0 & 0
\end{array}\right)\left(\begin{array}{l}
y_{G} \\
\dot{y}_{G} \\
\dot{y}_{G}
\end{array}\right)+\left(\begin{array}{l}
0 \\
0 \\
1
\end{array}\right) u_{y} \\
& \xi_{x}=\left(\begin{array}{lll}
-M g & 0 & M\left(z_{G}-z_{F}\right)
\end{array}\right)\left(\begin{array}{l}
y_{G} \\
\dot{y}_{G} \\
\dot{y}_{G}
\end{array}\right) \\
& u_{y}=\ddot{y}_{G} \\
& \xi_{\alpha}=\tau_{y}^{\text {ref }}-\dot{x}_{y}^{\text {ref }}
\end{aligned}
$$

## The CWC for a 2D-Robot on a Stair



## A Desired Trajectory in the CWC



The CWC is the direct product of a 2D polyhedral cone and 1D linear subspace, which is not identical for the single support phases of the lower and the higher feet, and is the product of the 2D cone and 2D linear subspace for a double support phase.

## Vertical Trajectory of the ZMP



Pseudo Plane on which the ZMP trajectory is defined [Honda]


Equivalent trajectory of the ZMP based on the proposed criterion

## Horizontal Contact Force while Climbing Stairs




Black curve is generated from a continuous trajectory in the CWC Red curve is generated from a discontinuous one in the CWC

## ZMP vs.CWC

|  | ZMP | CWC |
| :--- | :---: | :---: |
| Flat plane <br> Foot contact <br> Sufficient friction | Strong Stability | Strong Stability |
| Arbitrary terrain <br> Hand/Foot contact <br> Sufficient friction | N/A | Strong Stability |

## Summary of the CWC

- The proposed criterion is equivalent to ZMP in the specific case and can judge the strong stability in generic cases.
- Therefore we claim to say "Adios ZMP", and the voice can be louder when we can plan motions in a variety of cases based on the proposed criterion.


## Open Problems in the Control

- Robust walking
$\Leftrightarrow$ Biped walking is still not robust enough for a large disturbance.
- Walking on a natural rough terrain
$\Rightarrow$ Walking must be more generalized with the recognition of the working environment.
- Falling motion control
$\Rightarrow$ The human-size humanoid just crashes when it falls down without a proper control.


## Humanoids in Real Environment



## Research Platforms



HRP-2 (Kawada)
154cm, 0.5M Euro
Available at LAAS, France


HOAP (Fujitsu)
60cm, 50K Euro

## Free Research Platform



OpenHRP: Open Architecture Humanoid Robotics Platform http://www.aist.go.jp/is/humanoid/openhrp/

## Implementation Features of OpenHRP

## Distributed Object System based on CORBA

- Concurrent development using an arbitrary operating system and language
$\Rightarrow$ OpenHRP is written in J ava, C++ and runs on Linux and Windows


## CORBA Objects of OpenHRP



## ISE: Integrated Simulation Environment



## Our Current Challenge

- A Famous Project of Takeo Kanade
$\Rightarrow$ EyeVision at the Superball
$\Leftrightarrow$ Let's watch NBA in the court.
- Our Challenge
$\Rightarrow$ Let's go to the cafeteria with a humanoid.
- Robust biped walking
- Going up and down stairs
- Opening and closing doors
- 3D SLAM


## Powered Suits



HAL [U of Tsukuba]


Bleex [UC Berkeley]

## Autonomous Walking-Aid



## A Measure of the Ability of a Robot

- Artificial Intelligence
$\Leftrightarrow$ This robot has the intelligence that is compatible to three years old child.
- Mobility of a Humanoid
$\Rightarrow$ This robot has the mobility that is compatible to eighty years old person.


## Evxapı $\quad$ т $\omega$



May, my dog on a summer vacation

