Autonomous Robot Navigation Localization and Mapping Techniques for Mobile Robots

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Robots Intelligently Interacting With People 2006 ONASSIS LECTURE SERIES in COMPUTER SCIENCE

Autonomous Navigation Problem Statement

The ability of robots to navigate safely and reliably within their environments

- Operation in industrial environments
- Tour-guiding visitors in museums/exhibition sites
- Helping in household tasks
- Exploring unfriendly environments (volcanoes, sewer systems, underwater)
- Space applications
- (the list goes on)

Historical Walkthrough

... in the beginning: Robotics in ancient times - Talos



... and then: a multitude of robotic systems; Industrial robots





Existing, non-autonomous systems...



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... and non-existent, fully autonomous robots





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An Interesting Research Area

Theoretical interest...

 Mathematical and computational modeling of perception and action

... with important applications

- "Intelligent" robotic wheelchairs
- Robotic tour-guides in museums and exhibition sites
- Exploration of unknown and, possibly hostile, environments
- Routine tasks (surveillance, cleaning, etc)







The above are based to a great extent on the ability of <u>autonomous navigation</u>

Autonomous Navigation Research Directions

Given

- An environment representation - Map
- Knowledge of current position C
- Target position G
- A path has to be planned and tracked that will take the robot from C to G



Autonomous Navigation Research Directions

- During execution (runtime)
- Objects / Obstacles O may block the robot
- The planned path is nolonger valid
- The obstacle needs to be avoided and the path may need to be replanned





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Range Sensor Model

- Laser Rangefinder
- Model range and angle errors.

$$[x, y]^{T} = Exp(R(r, \phi)) = [r\cos(\phi), r\sin(\phi)]^{T}$$

$$\Sigma_{polar} = \begin{bmatrix} k_{\phi}\phi & 0\\ 0 & k_{\rho_0} + k_{\rho_1}r \end{bmatrix}$$

$$\Sigma_p = \nabla R \Sigma_{polar} \nabla R^T$$

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Markov Assumption

- State depends only on previous state and observations
- Static world assumption
- Hidden Markov Model (HMM)



Bayesian estimation: Attempt to construct the posterior distribution of the state given all measurements

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- •Most commonly Available:
 - •Initial State
 - Observations
 - •System (motion) Model

$$\mathbf{x}_{1} \leftrightarrow P(\mathbf{x}_{1})$$

$$\mathbf{y}_{1} \cdot \mathbf{y}_{T}$$

$$\mathbf{x}_{k} = f_{k}(\mathbf{x}_{k-1}) \quad \leftrightarrow \quad p(\mathbf{x}_{k} | \mathbf{x}_{k-1})$$

Measurement (observation) Model

$$\mathbf{y}_k = h_k(\mathbf{x}_k) \quad \leftrightarrow \quad p(\mathbf{y}_k \,|\, \mathbf{x}_k)$$

Inference - Learning

Localization (inference task)
 Compute the probability that the robot is at pose z at time t given all observations up to time t (forward recursions only)

$$P(x_t = z | y_1, y_2, ..., y_t)$$

 Map building (learning task) Determine the map m that maximizes the probability of the observation sequence.

$$m^* = \arg\max_{m} P(m|y_1, y_2, ..., y_T)$$

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Belief State

$$\underline{P(x_t \mid y_1, y_2, \dots, y_t)} = \frac{1}{c_t} P(y_t \mid x_t) \int_{Z} P(x_t \mid x_{t-1} = z) P(x_{t-1} = z \mid y_1, y_2, \dots, y_{t-1}) dz$$

How is the posterior distribution calculated?

How is the prior distribution represented?

- Discrete representation
 - Grid (Dynamic)
 - Samples

(Dynamic) Markov localization (Burgard98) Monte Carlo localization (Fox99)

- Continuous representation
 - Gaussian distributions
 Kalman filters (Kalman60)

Example: State Representations for Robot Localization

Continuous Representations

Discrete Representations

Grid Based approaches (Markov localization)

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Particle Filters (Monte Carlo localization)

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Kalman Filters - Equations

$$P(x_{t}|x_{t-1}) \approx N(Ax_{t-1}, \Gamma)$$

$$P(y_{t}|x_{t}) \approx N(Cx_{t}, \Sigma)$$
A: State transition matrix (n x n)

$$P(y_{t}|x_{t}) \approx N(Cx_{t}, \Sigma)$$
A: State transition matrix (n x n)
C: Measurement matrix (m x n)
w: Process noise ($\in \mathbb{R}^{n}$),
v: Measurement noise($\in \mathbb{R}^{m}$)
Process dynamics (motion model)

$$w_{t} \approx N(0, \Gamma)$$

$$w_{t} \approx N(0, \Sigma)$$
Where : $N(x; m, V) = \frac{1}{|2\pi V|^{1/2}} \exp\left(-\frac{1}{2}(x-m)^{T}V^{-1}(x-m)\right)$

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Kalman Filters - Update

 $\begin{array}{l} x_{t} = A_{t} x_{t-1} + w_{t} \\ y_{t} = C_{t} x_{t} + v_{t} \\ w_{t} \approx N(0, \Gamma) \\ v_{t} \approx N(0, \Sigma) \end{array}$

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Predict

Compute Gain

$$x_{t} = Ax_{t-1}$$
$$P_{t}^{-} = AP_{t-1}A^{T} + \Gamma$$

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$$K_t = P_t^{-}C^T \left(CP_t^{-}C^T + \Sigma\right)^{-1}$$

Compute Innovation

$$J_t = \hat{y}_t - C\hat{x}_t^-$$

Update

 $\hat{x}_t = \hat{x}_t - \mathbf{K}_t \mathbf{J}_t$ $P_t = (I - K_t C) P_k^{-1}$









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- Kalman Filter assumes that system and measurement processes are linear
- Extended Kalman Filter -> linearized Case

$$x_{t} = A_{t}x_{t-1} + w_{t}$$

$$y_{t} = C_{t}x_{t} + v_{t}$$

$$w_{t} \approx N(0,\Gamma)$$

$$v_{t} \approx N(0,\Sigma)$$

$$x_{t} = f(x_{t-1}) + w_{t}$$

$$y_{t} = g(x_{t}) + v_{t}$$

$$w_{t} \approx N(0,\Gamma)$$

$$v_{t} \approx N(0,\Sigma)$$



- Initialize State
 - Gaussian distribution centered according to prior knowledge
 large variance
- At each time step:

Example:

Localization – EKF

- Use previous state and motion model to predict new state (mean of Gaussian changes - variance grows)
- Compare observations with what you expected to see from the predicted state – Compute Kalman Innovation/Gain
- Use Kalman Gain to update prediction

Extended Kalman Filter

Project State estimates forward (prediction step)

Predict measurements

Compute Kalman Innovation

Compute Kalman Gain

Update Initial Prediction

$$\begin{split} \mu_{x_{t+1}^{-}} &= Exp(F(\mu_{x_{t}}, \alpha_{t})) \\ \Sigma_{x_{t+1}^{-}} &= \nabla F_{x} \Sigma_{x_{t}} \nabla F_{x}^{T} + \nabla F_{\alpha} \Sigma_{\alpha_{t}} \nabla F_{\alpha}^{T} \\ l_{t+1}^{-} &= H(\mu_{x_{t+1}^{-}}) \\ r_{t+1} &= l_{t+1} - l_{t+1}^{-} \\ \Sigma_{r_{t+1}} &= \nabla F_{x_{t+1}^{-}} \Sigma_{x_{t+1}^{-}} \nabla F_{x_{t+1}^{-}}^{T} + \Sigma_{l_{t+1}} \\ K_{t+1} &= \Sigma_{x_{t+1}^{-}} \nabla F_{x_{t+1}^{-}} \Sigma_{r_{t+1}^{-}}^{-1} \end{split}$$

$$\mu_{x_{t+1}} = \mu_{x_{t+1}} + K_{t+1}r_{t+1}$$
$$\Sigma_{x_{t+1}} = \Sigma_{x_{t+1}} - K_{t+1}\Sigma_{r_{t+1}}K_{t+1}^{T}$$

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EKF – Example motion model for mobile robot

- Synchro-drive robot
- Model range, drift and turn errors

$$\Sigma_{a_t} = \begin{bmatrix} k_r d_t & 0 \\ 0 & k_t f_t + k_d d_t \end{bmatrix}$$
$$\mu_{x_{t+1}} = Exp(F(\mu_{x_t}, a_t)) = \begin{bmatrix} x_t - d_t \sin(\theta_t) \\ y_t - d_t \cos(\theta_t) \\ \theta_t + d_t \end{bmatrix}$$

$$\Sigma_{x_{t+1}} = \nabla F_x \Sigma_{x_t} \nabla F_x^T + \nabla F_a \Sigma_{a_t} \nabla F_a^T$$





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Bayesian Methods Discrete Representation

Probabilistic localization – the case of global localization



Bayesian Methods Discrete Approaches

Grid-based representation of the state-space




Example: Localization – Grid Based

- Initialize Grid (Uniformly or according to prior knowledge)
- At each time step:
 - For each grid cell
 - Use observation model to compute P(y(k) | x(k))
 - Use motion model and probabilities to compute

 $\sum_{x(k-1)\in X} \left[P(x(k) \mid u(k-1), x(k-1)) P(x(k-1) \mid u(0:k-2), y(1:k-1)) \right]$

Normalize



Density plots of the robot state





... beginning

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small No of cycles

sufficient No of cycles



$$P(x_{t} | y_{1:t}) = \frac{1}{c_{t}} P(y_{t} | x_{t}) \int_{Z} P(x_{t} | x_{t-1} = z) P(x_{t-1} = z | y_{1:t-1}) dz$$

Motion model
Observation model
(=weight)

	Particle Filters SIS-R algorithm	
 Initialize particles randomly (Uniformly or according to prior knowledge) At each time step: 		
Sequential importance sampling	 For each particle: Use motion model to predict new pose (sample from transition priors) Use observation model to assign a weight to each particle (posterior/proposal) 	
Selection: Re-sampling Create A new set of equally weighted particles by sampling the distribution of the weighted particles produced in the previous step.		
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Particle Filters – Example 1



Use motion model to predict new pose (move each particle by sampling from the transition prior)

Particle Filters – Example 1



Particle Filters – Example 1















Discrete State Approaches

- Ability (to some degree) to localize the robot even when its initial pose is unknown.
- Ability to deal with noisy measurements, such as from ultrasonic sensors.
- Ability to represent ambiguities.
- Computational time scales heavily with the number of possible states (dimensionality of the grid, size of the cells, size of the map).
- Localization accuracy is limited by the size of the grid cells.

Continuous State Approaches

- Perform very accurately if the inputs are precise (performance is optimal in the linear case).
- Computational efficiency.
- Requirement that the initial state of the robot is known.
- Inability to recover from catastrophic failures caused by erroneous matches or incorrect error models.
- Inability to track Multiple Hypotheses about the location of the robot.

Hybrid Approaches

- Combination of characteristics from both methods
- Hybrid methods very popular in many scientific areas
 - Control theory
 - Economics

Proposed Model switching state-space model (SSSM)

The Switching State-Space model

• M continuous State Vectors

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• One discrete "switch" variable





Example Belief State

H. Baltzakis and P. Trahanias, Autonomous Robots 2003

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Switching state-space Model

Combines both models

Continuous Model

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- Accurate performance
- Computational efficiency
- Initial state must be known
- Inability to recover from catastrophic failures
- Inability to track Multiple
 Hypotheses

Inherits strengths Eliminates weaknesses

Discrete Model

- Perform even when initial pose is unknown
- Deal with noisy measurements
- Represent ambiguities
- Computational time scales heavily
- Lecalization accuracy limited

Localization

Belief state is intractable

- Mixture of M^T Gaussians
- Grows exponentially with time



Solution

- Selection (eg. Cox94, Jensfelt99, Roumeliotis00, Duckett01) Only keep the most probable paths in model histories (Multiple Hypothesis Tracking)
- <u>Collapsing (eg. Murphy98)</u> *Approximate the mixture of M^T Gaussians with a mixture of M^r Gaussians (r: small number, eg. 1,2,3)*

Localization – Discrete Model Corner Point Visibility



Localization – Discrete Model Corner Point Visibility



Localization - Discrete Model (Observation – Transition)



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Localization – Continuous Model (EKF)

Project State estimates forward (prediction step)

Predict already mapped features to the predicted state

Compute Kalman Innovation

Compute Kalman Gain

Update Initial Prediction

$$\mu_{x_{t+1}^{-}} = Exp(F(\mu_{x_{t}}, \alpha_{t}))$$

$$\Sigma_{x_{t+1}^{-}} = \nabla F_{x} \Sigma_{x_{t}} \nabla F_{x}^{T} + \nabla F_{\alpha} \Sigma_{\alpha_{t}} \nabla F_{\alpha}^{T}$$

$$l_{t+1}^{-} = H(\mu_{x_{t+1}^{-}})$$

$$r_{t+1} = l_{t+1} - l_{t+1}^{-}$$

$$\Sigma_{r_{t+1}} = \nabla F_{x_{t+1}^{-}} \Sigma_{x_{t+1}^{-}} \nabla F_{x_{t+1}^{-}}^{T} + \Sigma_{l_{t+1}}$$

$$K_{t+1} = \sum_{x_{t+1}} \nabla F_{x_{t+1}} \sum_{r_{t+1}}^{-1} \sum_{r_{t+1}}^{-1} \sum_{r_{t+1}}^{-1} \nabla F_{r_{t+1}} \sum_{r_{t+1}}^{-1} \sum_{r_$$

$$\mu_{x_{t+1}} = \mu_{x_{t+1}^-} + K_{t+1}r_{t+1}$$
$$\Sigma_{x_{t+1}} = \Sigma_{x_{t+1}^-} - K_{t+1}\Sigma_{r_{t+1}}K_{t+1}^T$$

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Localization - Results (Simulated)



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Localization - Results (Real world – FORTH 1st floor)



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Localization - Results (Real world – Outside our lab)





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Mapping Problem Statement



Mapping – Kalman Tracker

- Simultaneously estimate the robot position as well as the positions of landmarks (stochastic mapping)
 - Augment state vector to also include landmark positions

$$x = \begin{bmatrix} x_r & y_r & x_{l1} & y_{l1} & x_{l2} & y_{l2} & \cdots & x_{\ln_l} & y_{\ln_l} \end{bmatrix}^T$$

Mapping – Kalman Tracker

$$x(k+1) = x(k) + \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \\ \vdots & \vdots \\ 0 & 0 \end{pmatrix} \begin{pmatrix} u_x(k) \\ u_y(k) \end{pmatrix} + \begin{pmatrix} v_{rx}(k) \\ v_{ry}(k) \\ 0 \\ \vdots \\ 0 \end{pmatrix}$$

$$x(k+1) = Fx(k) + Gu(k) + v(k)$$

Mapping – Discrete Bayesian Approach

 Recursive Bayesian filtering for estimating the robot positions along with a map of the environment

$$P(x(1:k), m | u(0:k-1), y(1:k))$$

= $aP(y(k) | x(k), m) \int A \cdot B \cdot dx(1:k-1)$

$$A = (P(x(k) | u(k-1), x(k-1)))$$

$$B = P(x(1:k-1), m | u(0:k-2), y(1:k-1)))$$

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Mapping – Discrete Bayesian Approach

- Estimating the full posterior is not tracktable
- Incremental scan matching
- Let at time k-1 the localization and map estimates:

 $\hat{x}(k-1)$ $\hat{m}(\hat{x}(1:k-1), y(1:k-1))$

 At time k – after moving and getting a new measurement y(k)

 $\hat{x}(k) = \arg \max_{x(k)} \left\{ P(y(k) \mid x(k), \hat{m}(\hat{x}(1:k-1), y(1:k-1))) P(x(k) \mid u(k-1), \hat{x}(k-1)) \right\}$

Mapping – Discrete Bayesian Approach

Estimating the full posterior is not tracktable
FastSLAM

$$P(x(1:k), m | u(0:k-1), y(1:k))$$

= $P(m | u(0:k-1), y(1:k)) \bullet P(x(1:k) | u(0:k-1), y(1:k))$

Usually implemented via particle filters

Mapping Challenge: Loops in the robot's path

- As the robot moves and maps features, errors in both the state and the mapped features tend to increase with time
- When already mapped areas are visited (a loop is detected), the mapping algorithm should be able to correct its state and eliminate the accumulated errors
- Complicated robot paths, nested loops or loops that close simultaneously are difficult cases.

Our approach

Off-line Feature-mapping algorithm:

- Loop detection is accomplished via a hybrid localizer with global localization capabilities (SSSM) that creates hypotheses whenever known areas (corner points) are visited
- All hypotheses created by the localizer, whenever loops are detected, are tracked individually within their own copy of the map.
- The best path through hypotheses histories is selected, a Kalman smoother redistributes errors and an iterative procedure corrects the map

H. Baltzakis and P. Trahanias, ICRA 2006

Features


Algorithm overview 1

- Mapping starts with one hypothesis (dominant)
- Existing line segments used for localization while the map is created.
- Non existing segments are inserted in the map





- Detected corner points result in creation of new hypotheses
- Hypotheses eventually vanish if observation sequences do not confirm their validity





Algorithm overview 4

- Upon entering previously mapped areas (corners detected), new hypotheses are created at the correct robot poses
- Correct hypotheses will eventually become more probable since observations confirm their validity



Algorithm overview 5

All hypotheses are tracked within their own copy of the map.

Multi-hypothesis Mapping

Haris Baltzakis & Panos Trahanias

Example - Artificial PHASE A

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Map Rectification - Iterative Algorithm

Treat map features as parameters of the dynamical system according to which the robot's state evolves

 <u>E-STEP</u>: Localize the robot using all available measurements.
(obtain max a-posteriori estimates of robot states)

 M-STEP: Recalculate map features





Results (simulated running example)

Initial map

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Example - Artificial PHASE B

Results (simulated running example)





Multi-hypothesis Mapping Haris Baltzakis & Panos Trahanias

Example - Castello di Belgioioso

PHASE A

Belgioioso dataset available from university of Freiburg

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Multi-hypothesis Mapping Haris Baltzakis & Panos Trahanias

Example - Castello di Belgioioso

PHASE B

Belgioioso dataset available from university of Freiburg

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Results (Radish - cmu_nsh_level_a)

Multi-hypothesis Mapping Haris Baltzakis & Panos Trahanias

Example - Radish - cmu_nsh_level_a PHASE A

Multi-hypothesis Mapping Haris Baltzakis & Panos Trahanias

Example - Radish - cmu_nsh_level_a PHASE B

Radish cmu_nsh_level_a data set submitted by Nick Roy

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Mapping - Results (Real world – FORTH 1st floor)



Mapping - Results (Real world – FORTH 1st FLOOR)



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Visual Information Processing

- Laser range finders provide fast and accurate depth information for 2D slices of the environment
- Various objects are invisible to the laser range finder.
- Vision can provide extra information for crucial tasks such as obstacle avoidance.





Visual Information Processing



Visual Information Processing (simulated example)



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Visual Information Processing (Efficiency considerations)

 Depth computations take place only where inconsistencies are detected

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- For collision avoidance depth computations can be further eliminated.
 - Criterion 1. Visual range defers significantly to laser range data
 - Criterion 2. Visual range is shorter that corresponding laser suggests
 - Criterion 3. Visual range is neither too far nor to close to the robot.

Crit(Combination of Criteria max low



Visual Information Processing (Real world example – outside our lab)









H. Baltzakis, A. Argyros and P. Trahanias, MVA 2003

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Real Application (Robotic Tour-guide in exhibition site)

The TOURBOT & WebFAIR Projects:

- Autonomous mobile robots in populated environments (serving realvisitors)
- Also operating over the web (serving web-visitirs)



P. Trahanias et al, IEEE RAM 2005

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