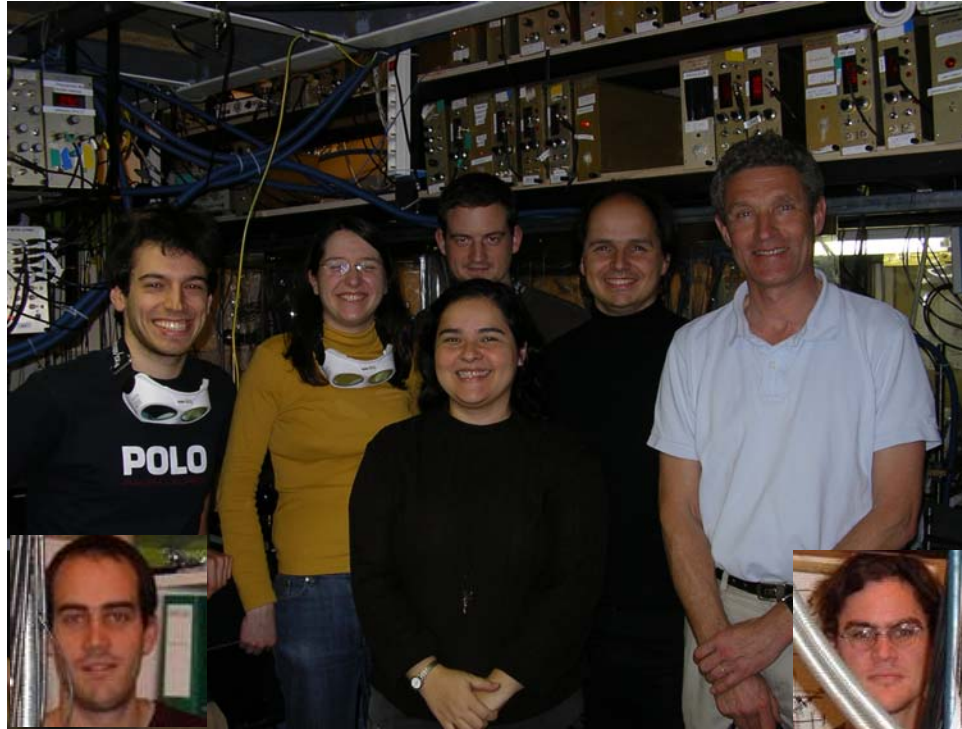


Tunable Fermi gases experiments in the BEC-BCS crossover



Collège de France



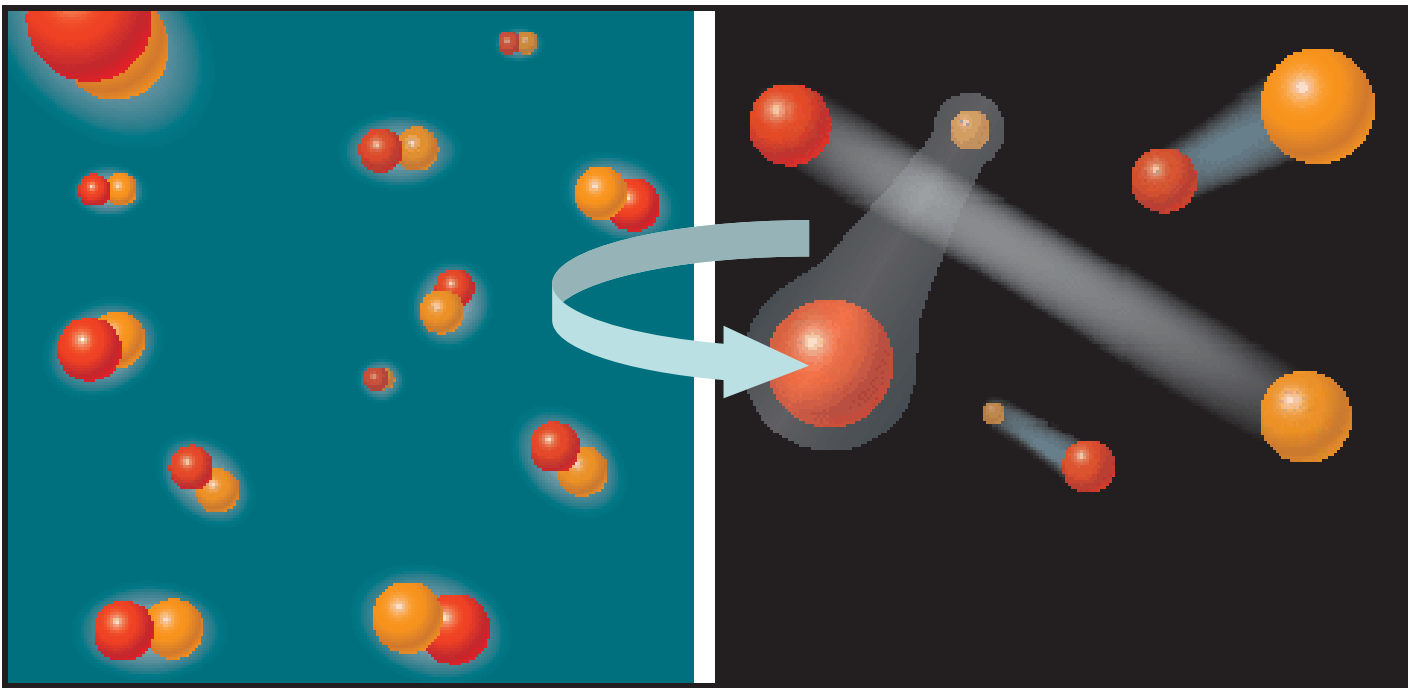
L. Tarruell, M. Teichmann, G. Duffy, S. Nascimbène, N. Navon,
J. McKeever, K. Magalhães, F. Chevy, C. Salomon

Laboratoire Kastler Brossel, ENS, Paris

D. Petrov, G. Shlyapnikov, R. Combescot, Y. Castin,

Fermi superfluid and Bose-Einstein condensate of Molecules

Fermions with two spin states with attractive interaction



BEC of molecules ← Interaction strength → BCS fermionic superfluid
Bound state No bound state

Leggett, Eagles, Nozières, Schmidt-Rink,... '80

Dilute gases: Feshbach resonance

Outline

General methods for ultracold Fermi gas manipulation

Tuning the interaction in the gas

Molecule formation and Bose-Einstein condensation of fermion dimers

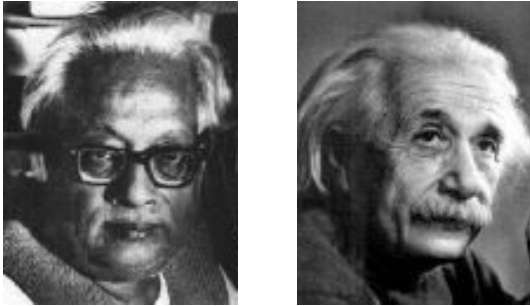
Crossover experiments and superfluidity

Superfluidity with spin population imbalance

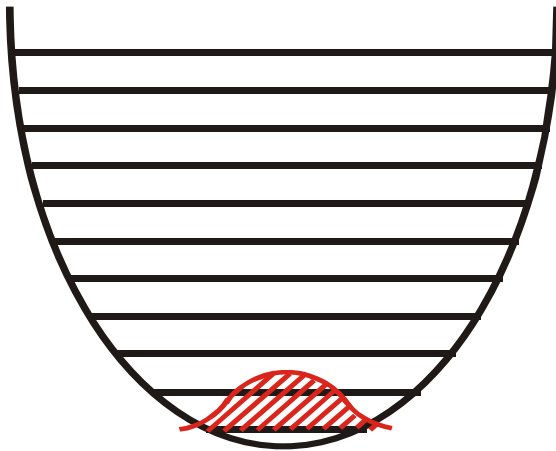
Prospects

Quantum statistics in harmonic traps

- Bose-Einstein statistics (1924)



Bose-Einstein condensate



Bose enhancement

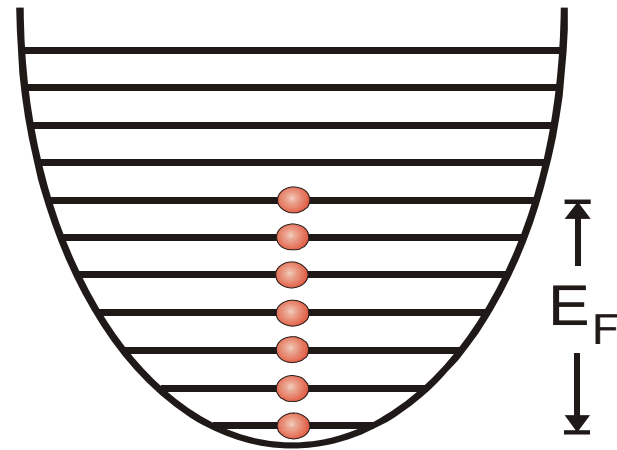
$$T_C = \frac{\hbar\omega}{k_B} (0.83 N)^{1/3}$$

Dilute gases: 1995, JILA, MIT

- Fermi-Dirac statistics (1926)



Fermi sea

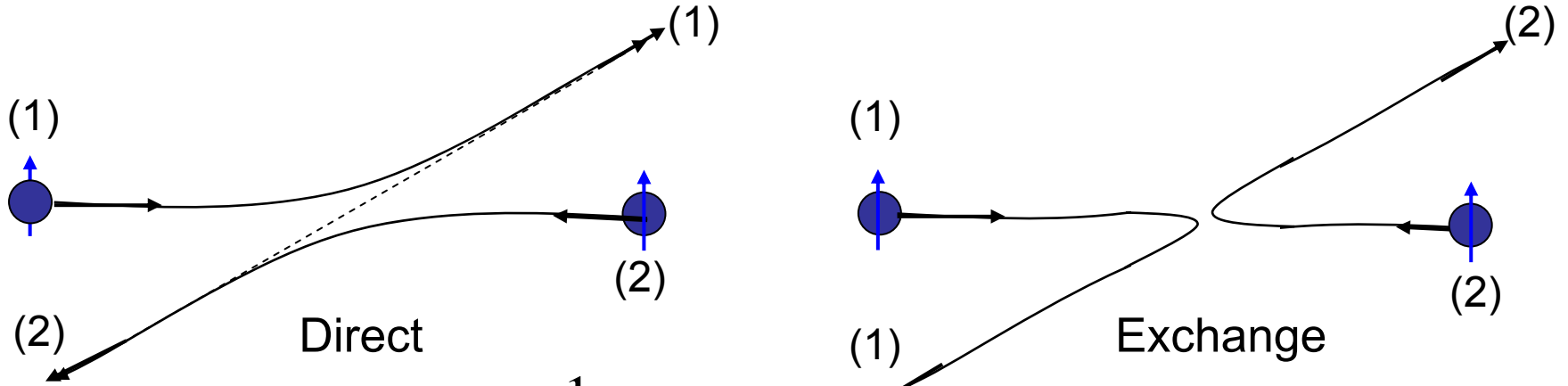


Pauli Exclusion

$$T \ll T_F = \frac{\hbar\omega}{k_B} (6 N)^{1/3}$$

Dilute gases: 1999, JILA

Collisions between identical particles and quantum statistics



$$|\psi_f\rangle = \frac{1}{\sqrt{2}}(1 + \varepsilon P_{21}) |1: k e_z, 2: -k e_z\rangle$$

Scattering amplitude interfere with + sign for bosons and – for fermions

At low temperature, s-wave only

Bosons

$$\sigma = 8\pi a^2$$

Fermions

$$\sigma = 0$$

Good for clocks: no interaction shift

But evaporation is more difficult

Sympathetic cooling

Cooling Methods

Solution 1: sympathetic cooling (with bosons)

^6Li - ^7Li , ^6Li - ^{23}Na , ^6Li - ^{87}Rb , ^{40}K - ^{87}Rb ,.....

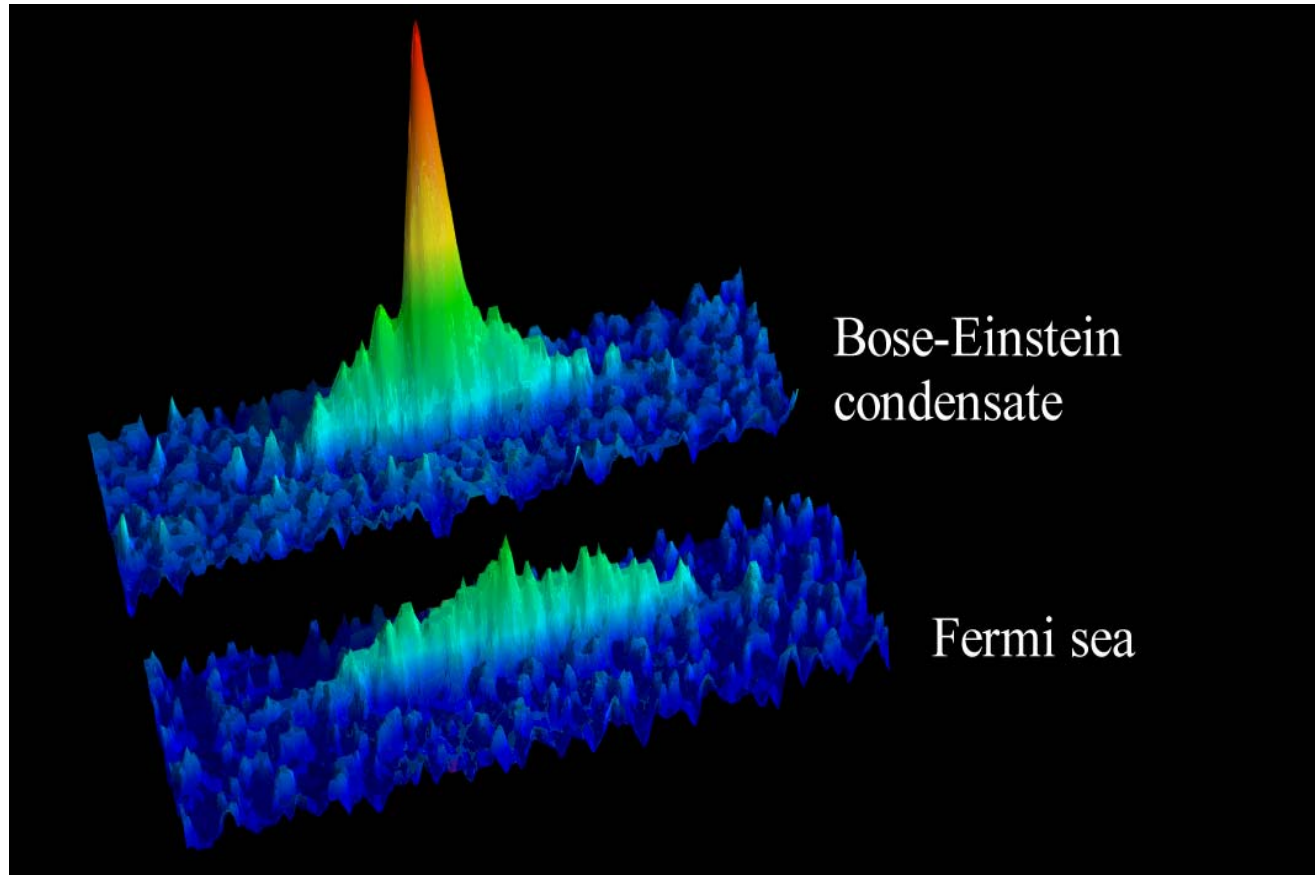
Solution 2: mixture of spin states in magnetic trap

Solution 3: mixture of spin states in optical trap + Feshbach resonance

| | | | |
|-------------------|----------------|------|--|
| ^{40}K | JILA Boulder | 1999 | magnetic trap, spin mixture |
| | LENS Florence | 2002 | magn. trap & sympathetic cooling Rb |
| | ETH Zurich | 2004 | magn. trap & sympath. cooling Rb |
| | Univ. Hamburg | 2005 | magn. trap & sympath. cooling Rb |
| | Univ Toronto | 2005 | chip magn. trap & sympath. cooling Rb |
| ^6Li | Rice univ. | 2001 | magn. trap, sympathetic cooling ^7Li |
| | ENS Paris | 2001 | magn. trap, sympathetic cooling ^7Li |
| | Duke Univ. | 2001 | optical dipole trap, mixt.of spin states |
| | MIT Boston | 2002 | magn. trap, sympathetic cooling ^{23}Na |
| | Uni. Innsbruck | 2003 | optical dipole trap, mixt.of spin states |
| | Univ. Tübingen | 2005 | chip magn. trap, sympathetic cooling Rb |
| | Uni. Swinburne | 2007 | optical dipole trap, mixt.of spin states |
| ^{171}Yb | Univ. Kyoto | 2006 | optical dipole trap, mixt.of spin states |

Bose-Einstein condensate and Fermi sea

2001
ENS



10^4 Li 7 atoms, in thermal equilibrium with
 10^4 Li 6 atoms in a Fermi sea.

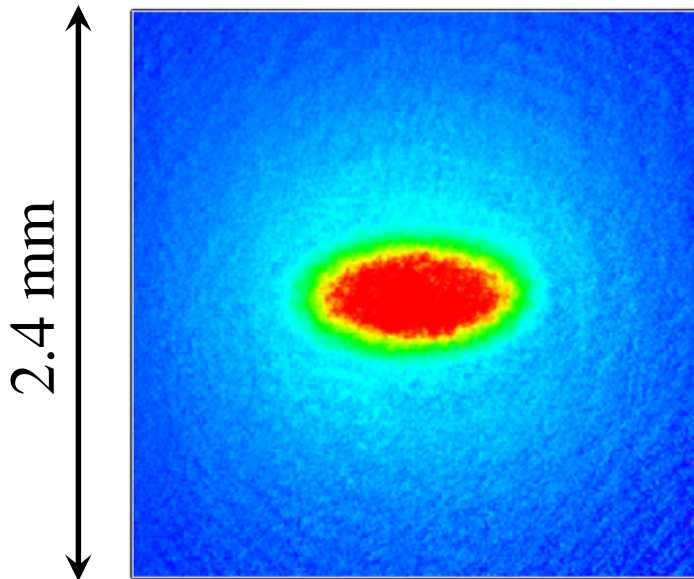
Quantum degeneracy: $T = 0.28 \mu\text{K} = 0.2(1) T_C = 0.2 T_F$

Lithium-Sodium mixture (MIT)

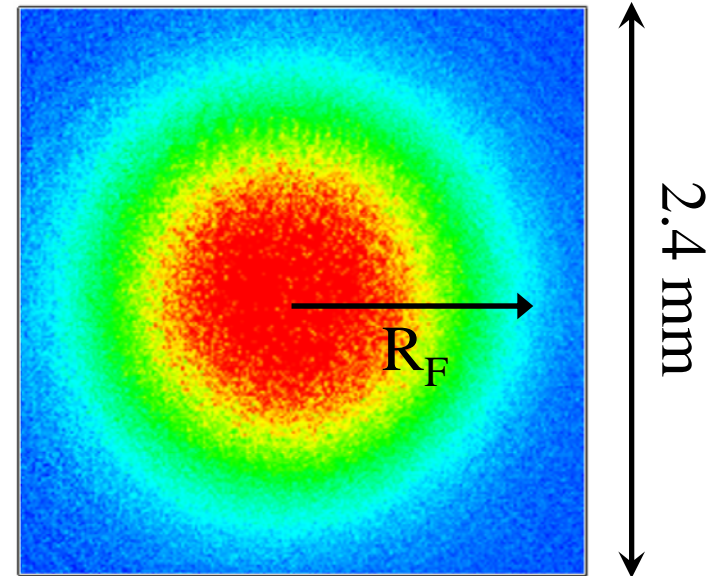
Use Sodium $F = 2$ as refrigerator
to cool Lithium in $F = 3/2$ (state $|6\rangle$) in a magnetic trap
20s forced evaporation on Na results in typically

10^7 atoms in BEC (w/o Li)

50 10^6 Li atoms at $\frac{T}{T_F} < 0.3$



50 ms time of flight
 $\omega = 2\pi \times (72,72,18)$ Hz



12 ms time of flight
 $\omega = 2\pi \times (142,142,36)$ Hz

All-optical method

all-optical approach by John Thomas group at Duke Univ., Durham, NC, USA

CO₂ laser trap

65 W
 $\lambda = 10.6 \mu\text{m}$



trap depth $690 \mu\text{K}$

loading of a few 10^6 atoms *directly from the MOT*

after plain evaporation for 5s:

1.3×10^6 atoms @ $5 \mu\text{K}$, p.s.d. 8×10^{-3} ($T/T_F = 2.8$) Now 2×10^5 atoms at $T/T_F = 0.1$

Also Innsbruck Univ.

ultrastable CO₂ trapping of lithium fermions
O'Hara et al., PRL 82, 4204 (1999)

Optical Traps

Dipole force: far-off resonance laser : very low photon scattering rate
Flexible geometry, 1 or several beams, adjustable aspect ratio

Decouples trapping function and magnetic tuning for Feshbach Resonances

Can be switched on and off very fast

Easy modulation of trap depth or position using acousto-optic modulators
Excitation of collective modes, rotating trap,.....

3D, 2D, 1D optical lattices by interference of several laser beams

See e.g. Proceedings of 2006 Varenna School on Cold Fermi Gases
on cond-mat, and book to appear in 2007
And R. Grimm, Y. Ovchinnikov '00

BCS theory: Cooper pairs

Bardeen, Cooper, Schrieffer, 1957

Superconductivity of metals at low temperature

Homogeneous Fermi gas, k_F , E_F at zero temperature

Ad two fermions, 1 and 2 with different spin states, with attractive interaction: $a_{\uparrow\downarrow} < 0$

$$V(\vec{r}_1 - \vec{r}_2) = V \delta(\vec{r}_1 - \vec{r}_2) \quad \text{with } V < 0$$

The state with correlated pairs of fermions has energy lower than E_F .

$$\vec{k}, -\vec{k} \quad \text{Pairs at Fermi surface: } |k| \geq k_F$$

If T is low enough, a superfluid phase is produced

Critical temperature:

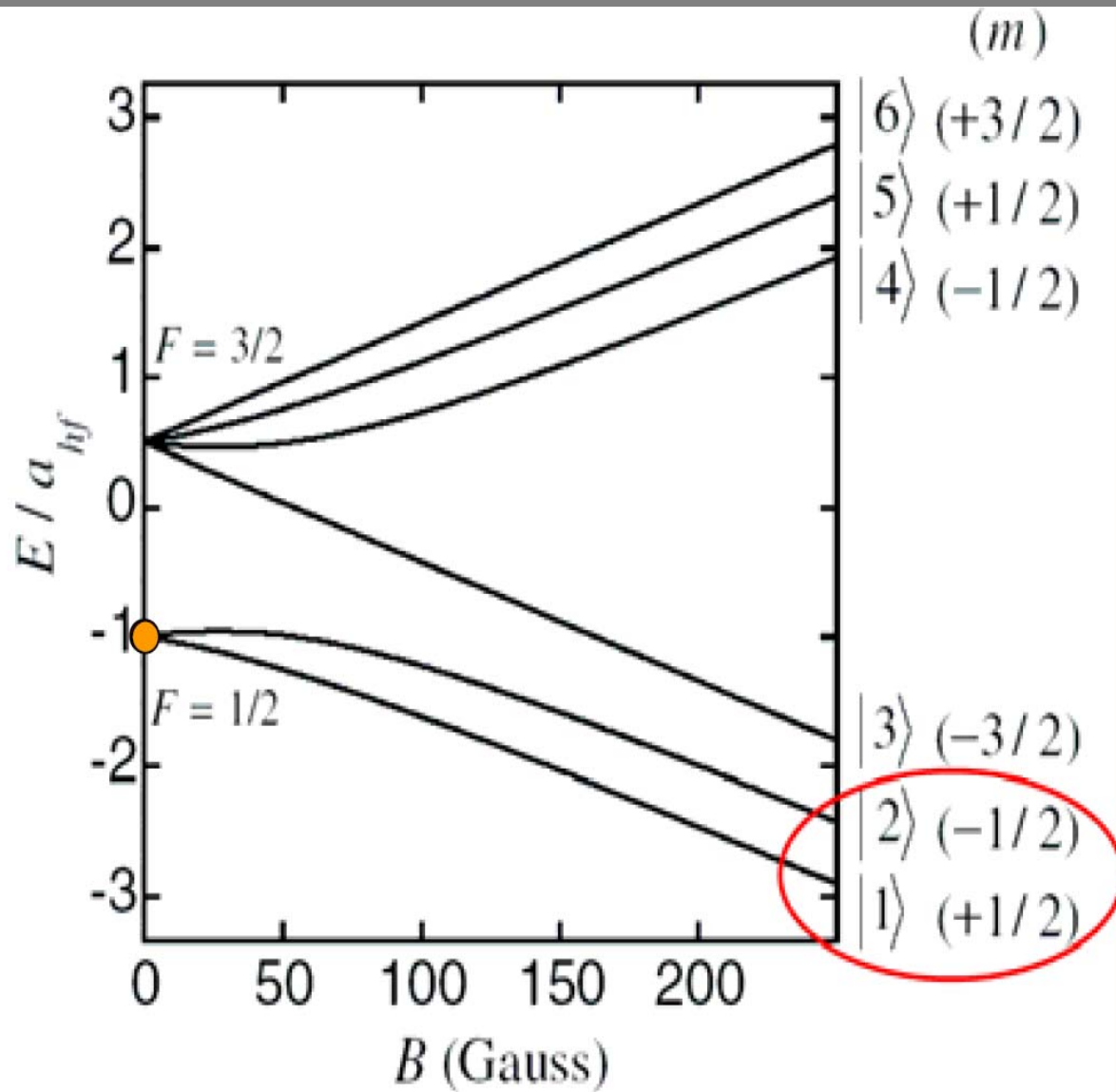
$$T_{BCS} \sim 0.28 T_F e^{-\frac{\pi}{2 k_F |a|_{\uparrow\downarrow}}}$$

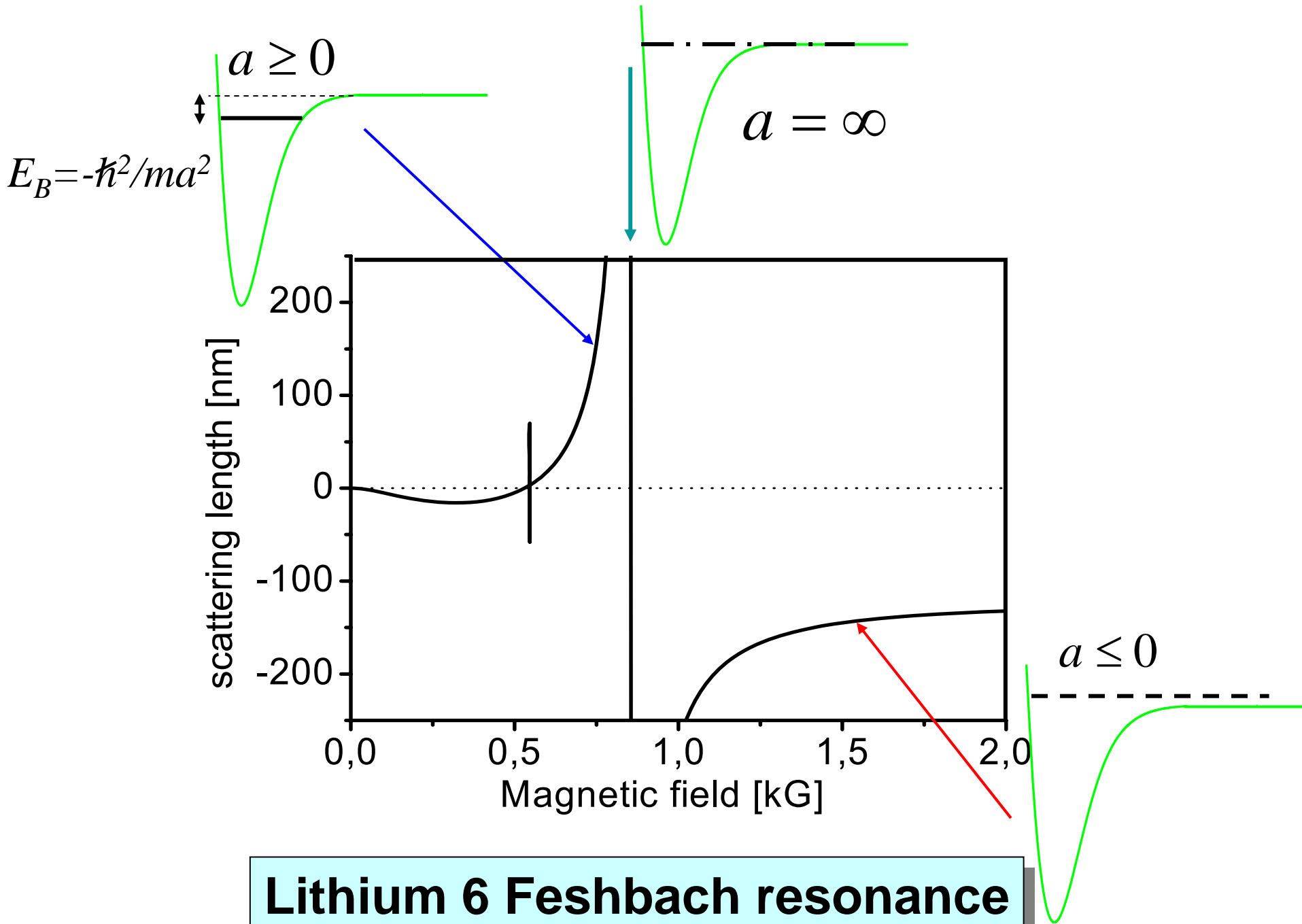
$$\text{Validity: } k_F a \ll 1$$

Example: $k_F a = -0.2$, $T_{BCS} = 10^{-4} T_F$ is very small !!

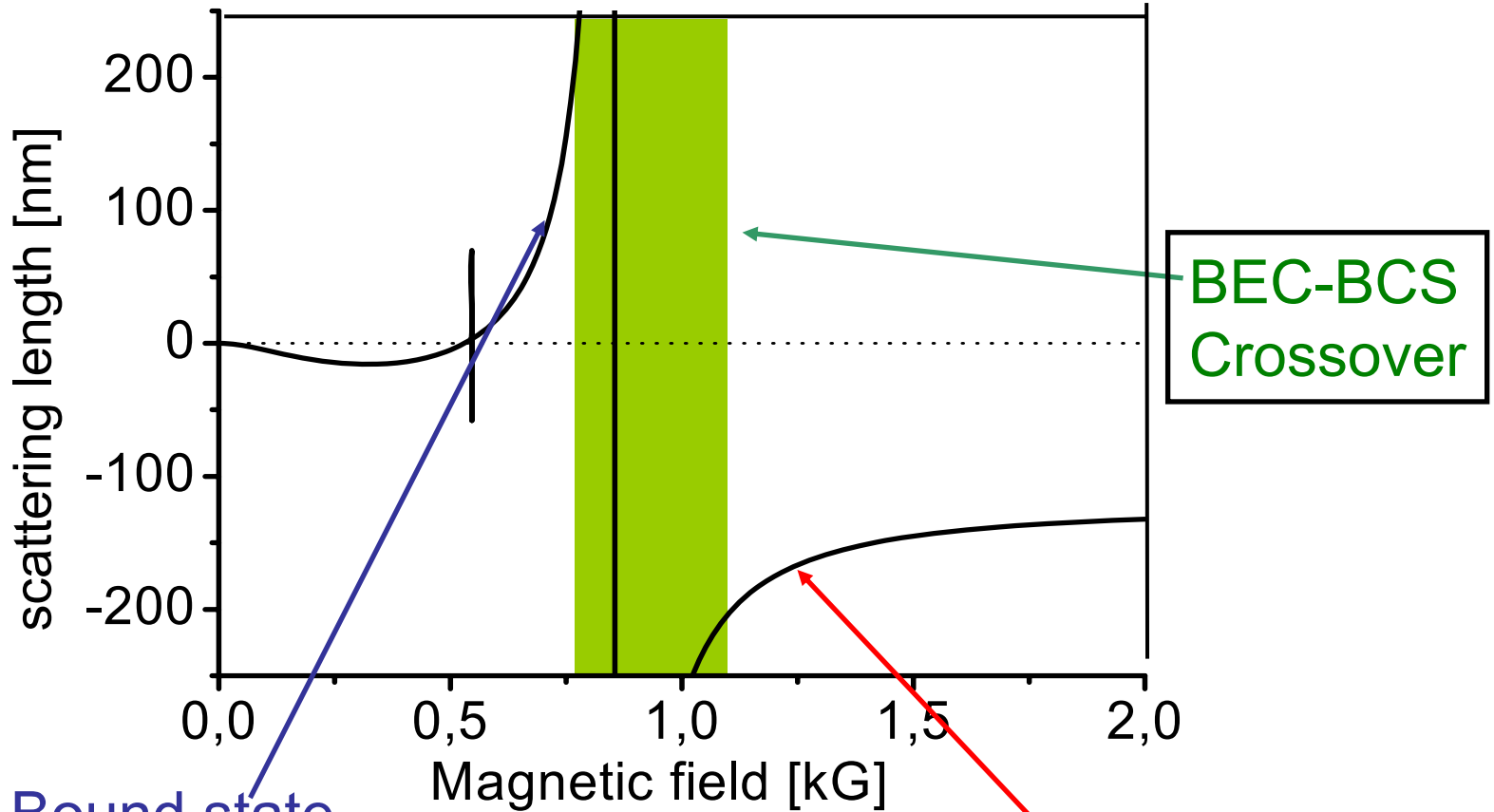
Tuning atom-atom interactions

${}^6\text{Li}$ Ground state in magnetic field





interacting fermions



Bound state

$$Eb = -\frac{\hbar^2}{ma^2}$$

condensate of molecules

No bound state

BCS phase

Experimental approach

Cooling of ${}^7\text{Li}$ and ${}^6\text{Li}$

1000 K: oven



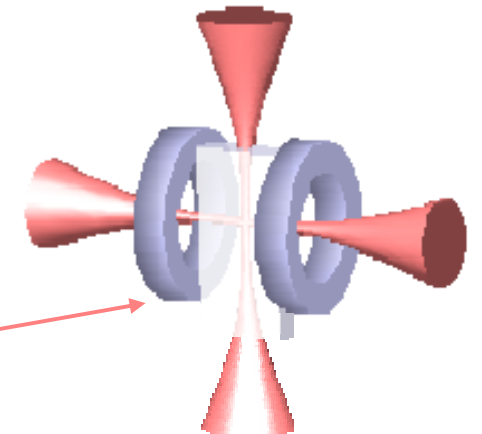
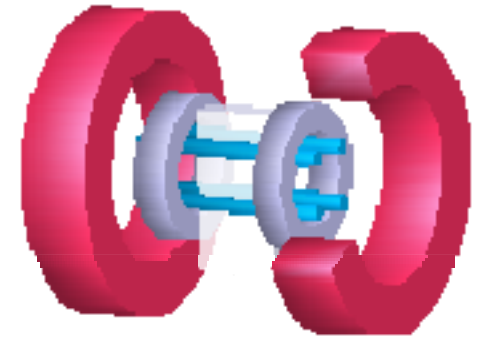
1 mK: laser cooling



10 μK : evaporative cooling
in magnetic trap

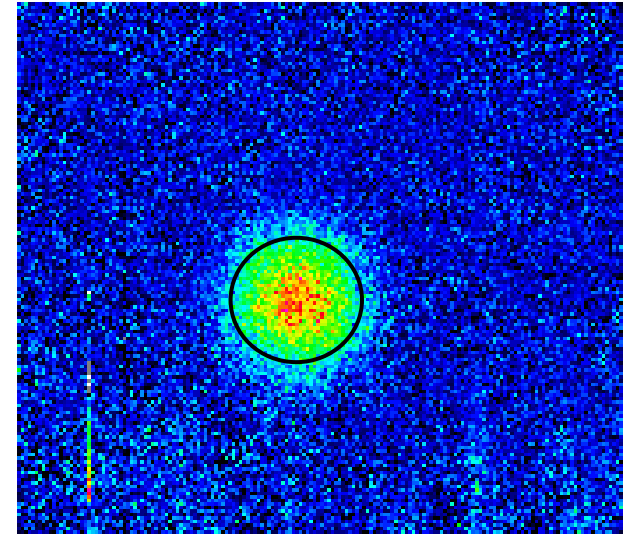
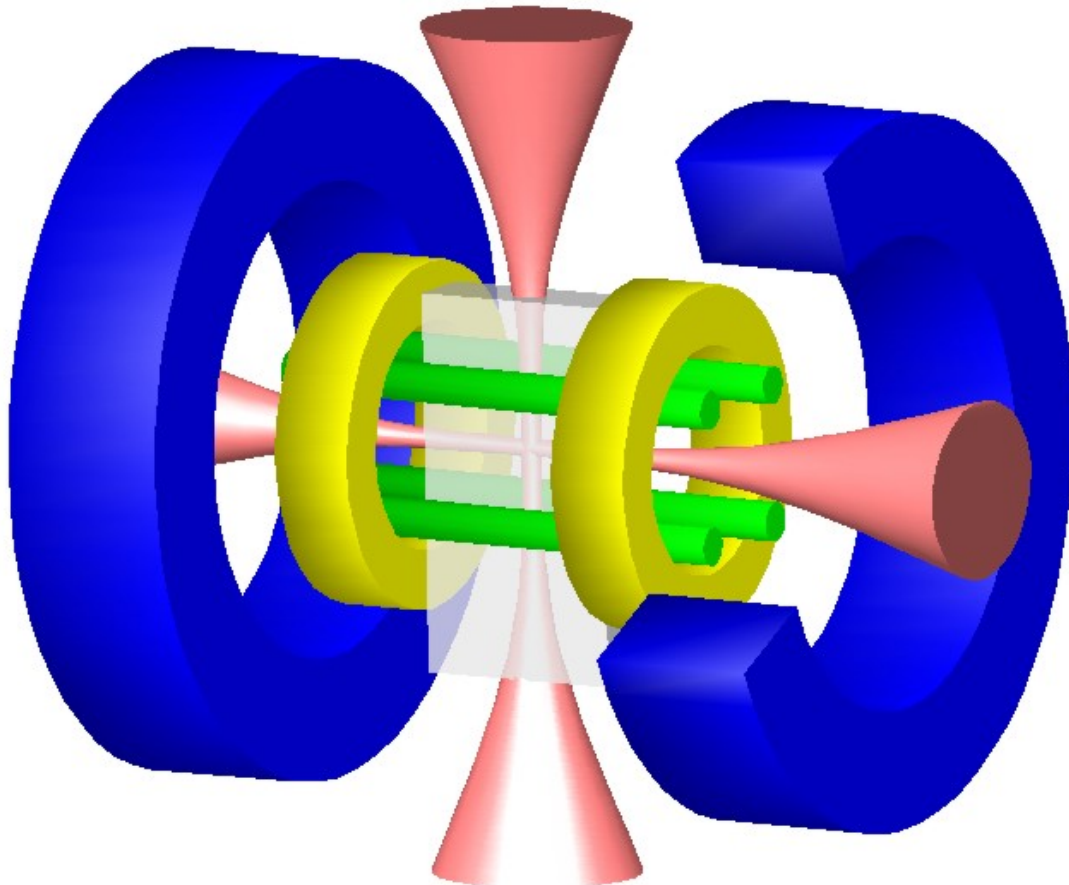
$$E = -\vec{\mu} \cdot \vec{B} = +|\vec{\mu}||\vec{B}|$$

Tuning the interactions in optical trap
Final evaporation in optical trap



Optical trap

Evaporation of ${}^6\text{Li}$ gas in an optical trap



$$T_F = 5 \mu\text{K}$$

$$T/T_F = 0.2$$

$$N_{\text{total}} = 1 \cdot 10^5$$

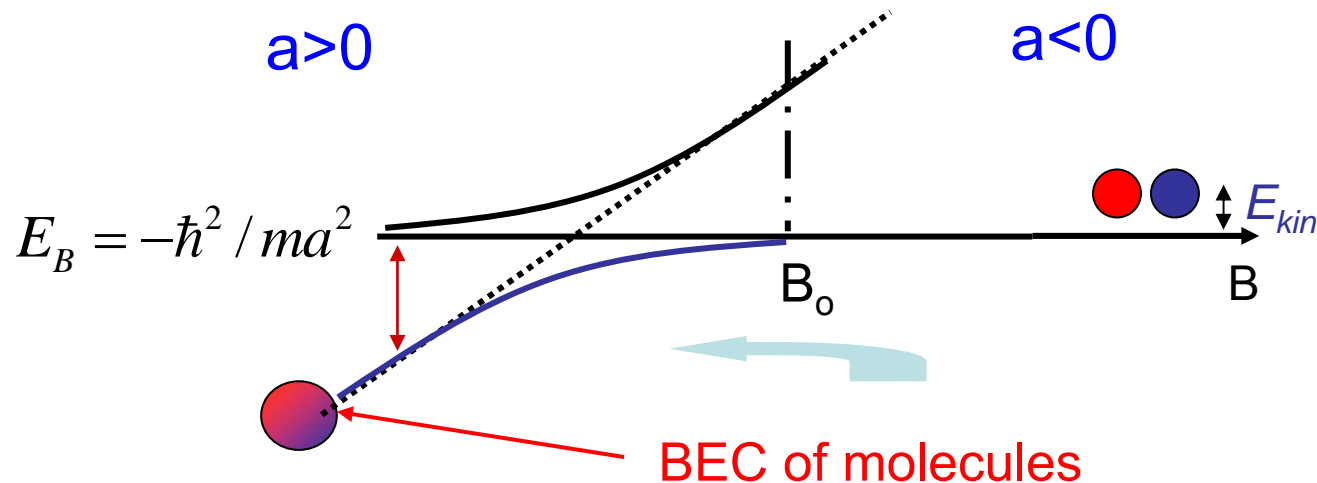
Two YAG beams with 2.5 W and waist of $38 \mu\text{m}$

Temperature is measured in the weakly interacting regime ($B < 200 \text{ G}$)
by fit to the finite T Fermi distribution

Difficult to get T in the crossover region (except in imbalanced case, MIT)
Thermal fraction on molec BEC side, or universal thermodynamics at unitarity

Molecule production Method 1

JILA 03
ENS 03



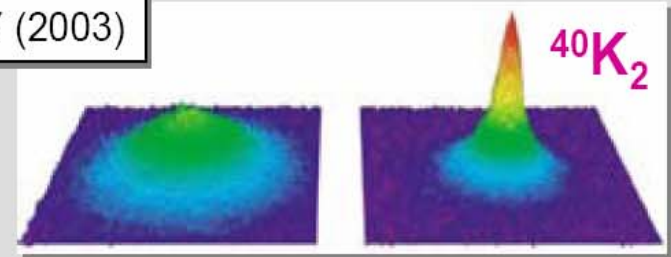
Recipe: in region $a < 0$, cool a gas of fermions below T_F
Slowly scan across resonance towards $a > 0$
Typically : 1000 G to 770 G in 200 ms
This produces molecules with up to 90% efficiency !
Reversible process ! Entropy is conserved.
If $T < 0.2 T_F$, BEC of molecules

Condensates of molecules

in situ

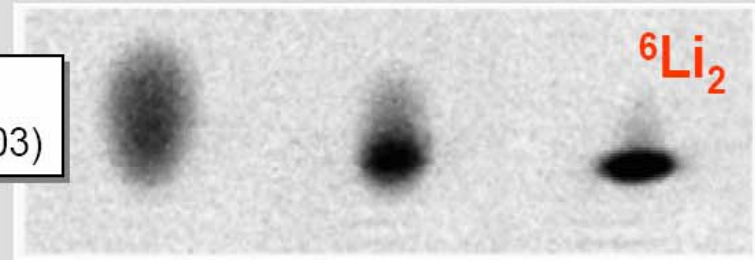
expansion

JILA Greiner et al.,
Nature 426, 537 (2003)

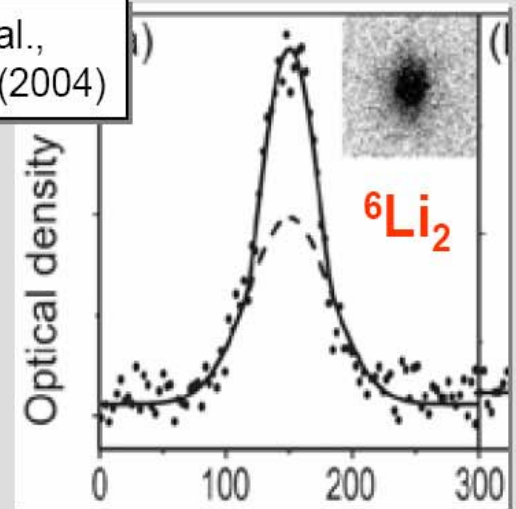


Innsbruck Bartenstein et al.,
PRL 92, 120401 (2004)

MIT Zwierlein et al.,
PRL 91, 250401 (2003)

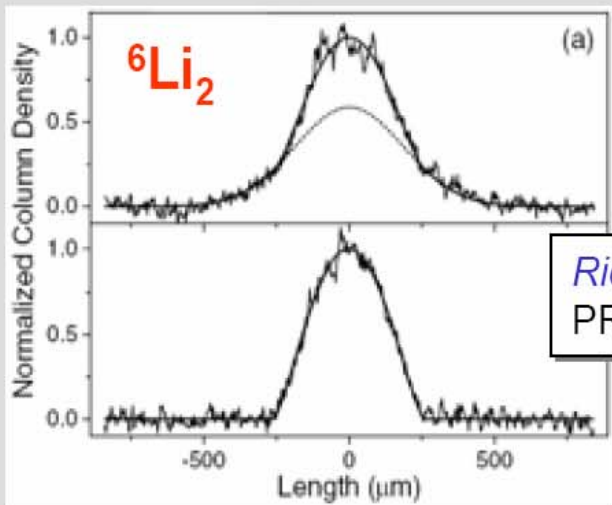
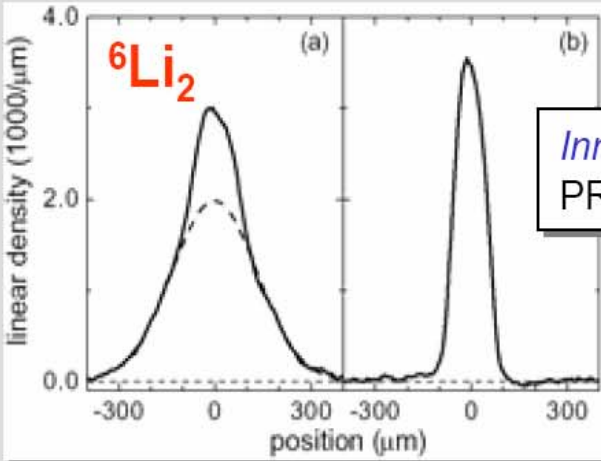


ENS Bourdel et al.,
PRL 93, 050401 (2004)



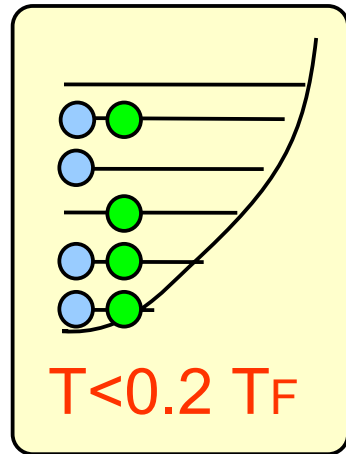
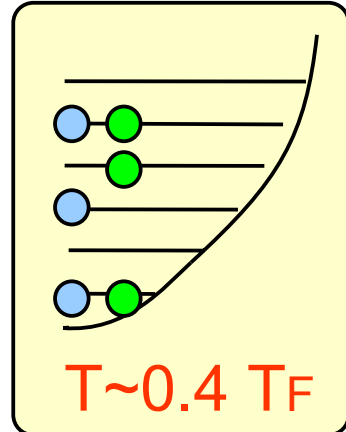
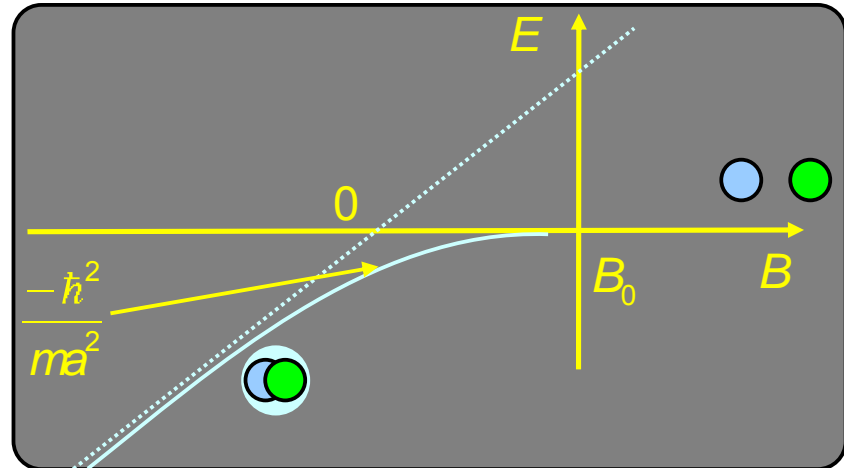
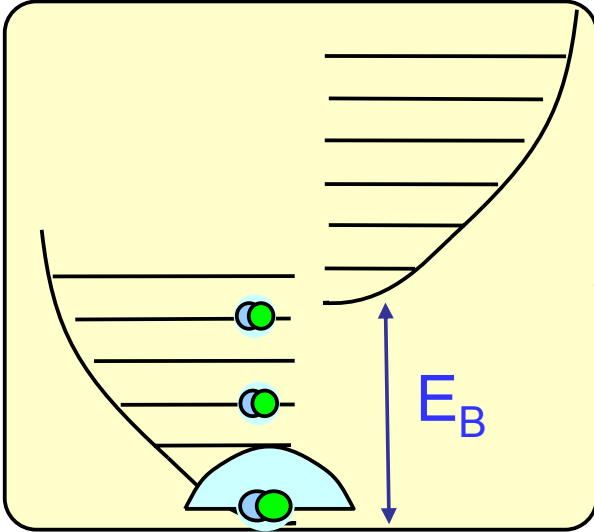
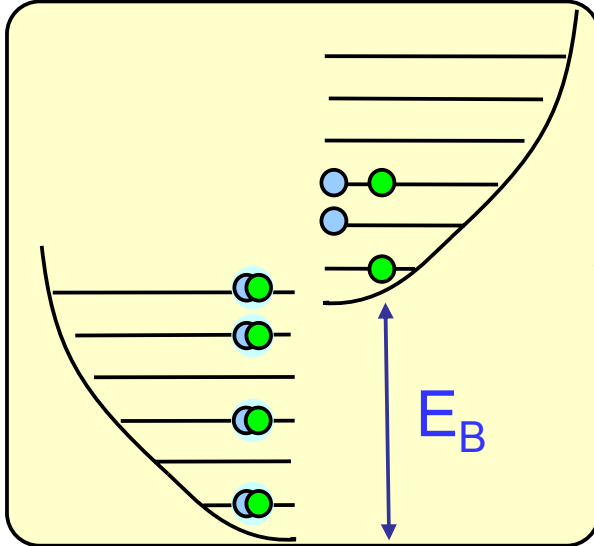
Rice Partridge et al.,
PRL 93, 020404 (2005)

2007: Swinburne Univ



A simple thermodynamic model

conservation of entropy



See also T. Koehler lecture

Questions on BEC-BCS crossover

BEC of molecules: excellent starting point for exploring the crossover

Q1: Lifetime of molecules ?

Q2: interaction between molecules ?

Q3: What happens in strongly correlated regime: unitarity: $k_F a \gg 1$?

Q4: Can we measure the excitation gap ?

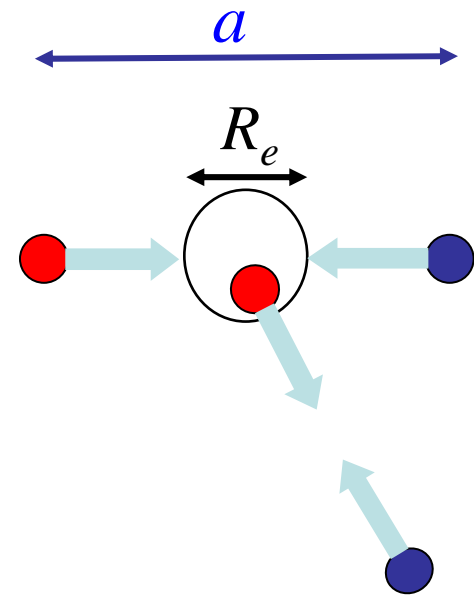
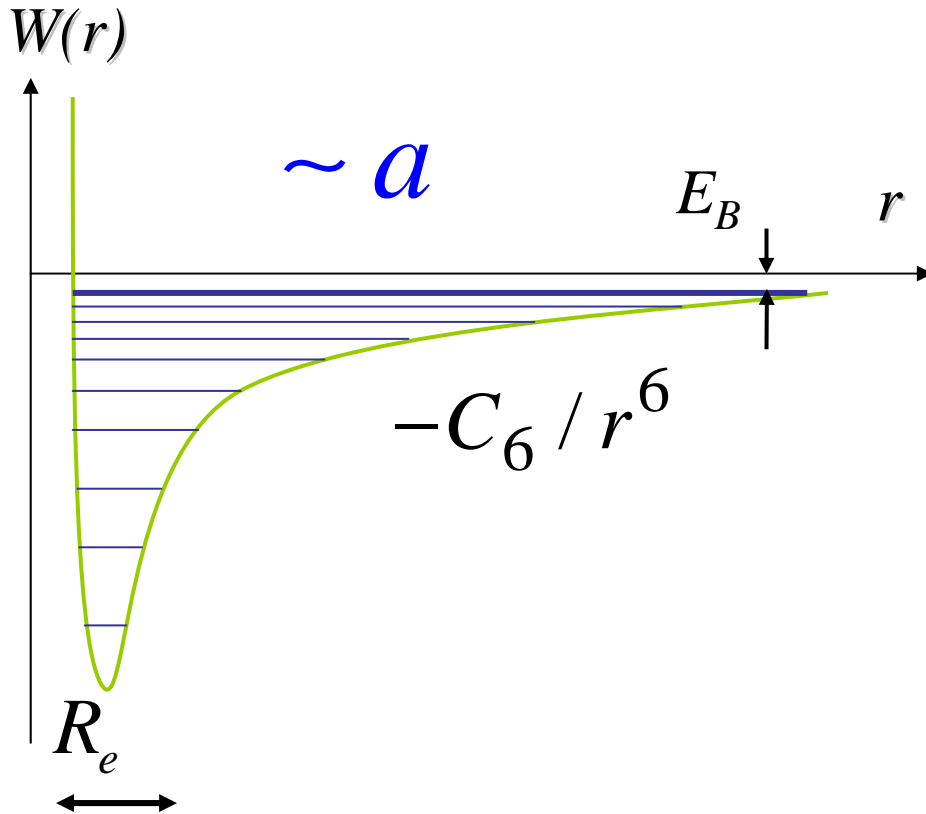
Q5: How to probe superfluidity in crossover regime ?

Q6: what is the momentum distribution of particles ?

Q7: superfluidity with imbalanced spin populations ?

Remarkable stability of weakly bound molecules

Suppression of vibrational relaxation for fermion dimers



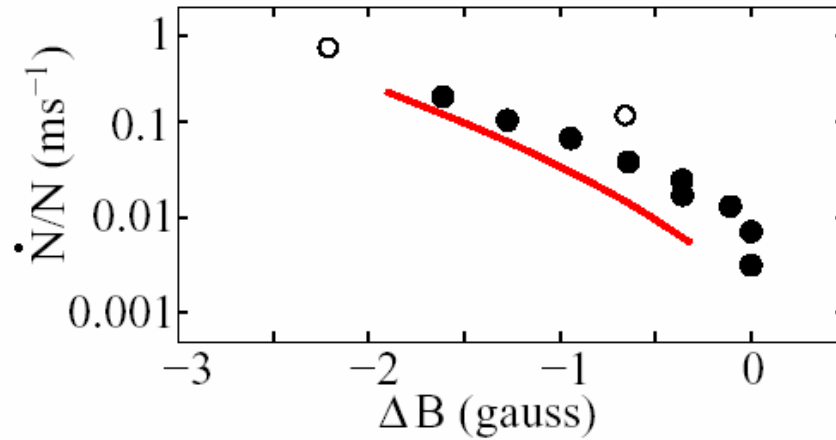
Pauli exclusion principle
 Inhibition by factor $(a/R_e)^2 \gg 1$

Binding energy: $E_B = \hbar^2 / ma^2$
 Momentum of each atom: \hbar/a

$G \sim 1/a^s$ with $s = 2.55$ for dimer-dimer coll.
 3.33 for dimer-atom coll.

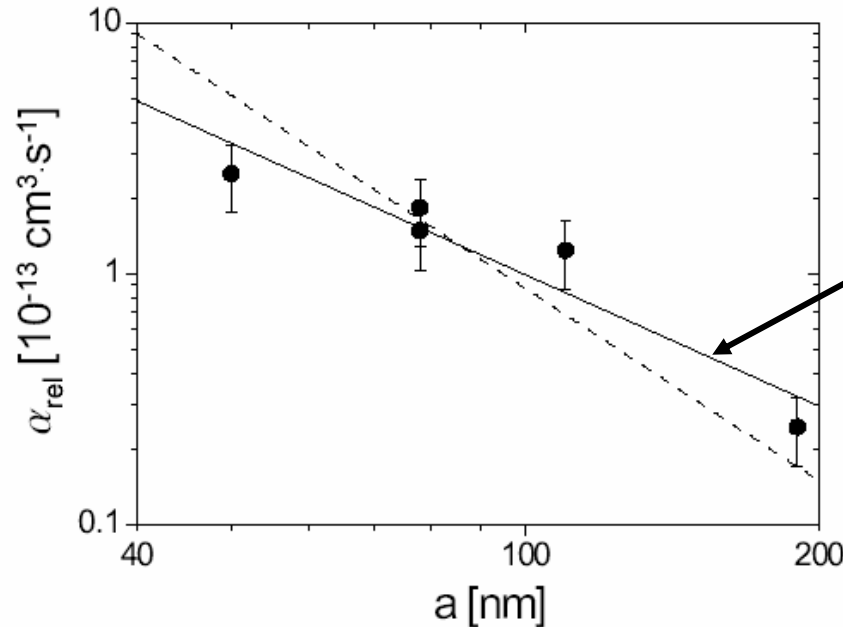
Comparison with experiments

$^{40}\text{K}_2$, JILA
Regal 04



$$\beta_{\text{exp}} \sim a^{-2.3 \pm 0.4}$$

$^6\text{Li}_2$
ENS 04



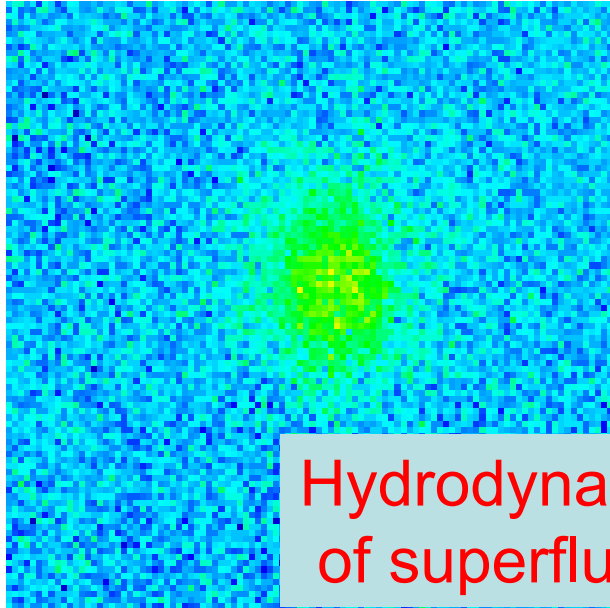
$$\beta_{\text{exp}} \sim a^{-1.9 \pm 0.8}$$

$$\beta_{\text{th}} \sim a^{-2.55}$$

On resonance, lifetime of strongly interacting gas exceeds 30 s !

Interaction between molecules measurement of a_{mm}

nearly pure condensate $\lambda=0.1$



Hydrodynamic expansion is signature
of superfluidity on BEC side

$$T \leq 0.9 \mu K = T_c^0 / 3$$

In trap TF radius:

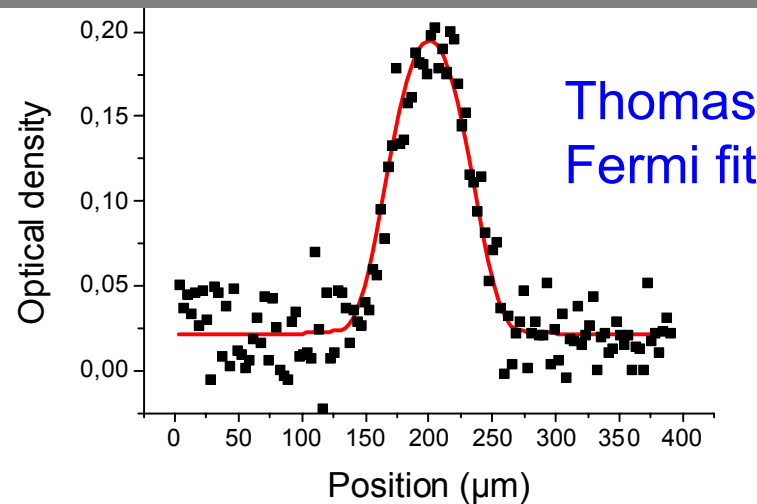
$$R_x = 26 \mu m, R_y = 2.75 \mu m$$

Good agreement with theory:

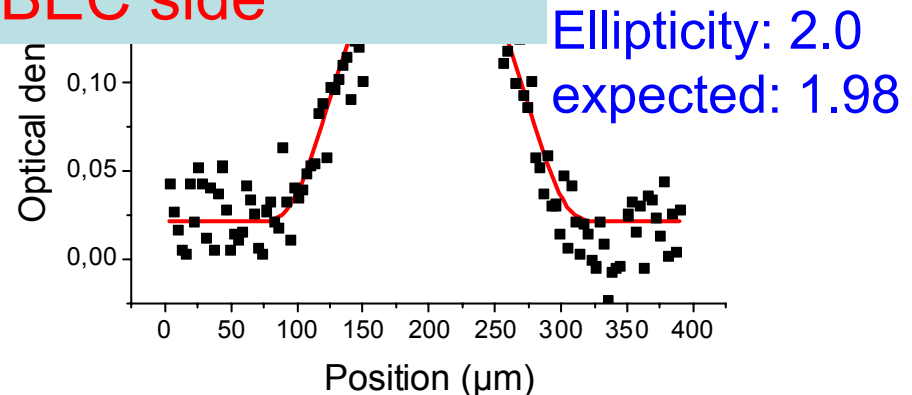
$$a_{mm} = 0.6 a = 0.6 \times 306 = 183 \text{ nm}$$

D. Petrov, G. Shlyapnikov, C.S.

Excludes: $a_{mm} = 2a$



Thomas
Fermi fit



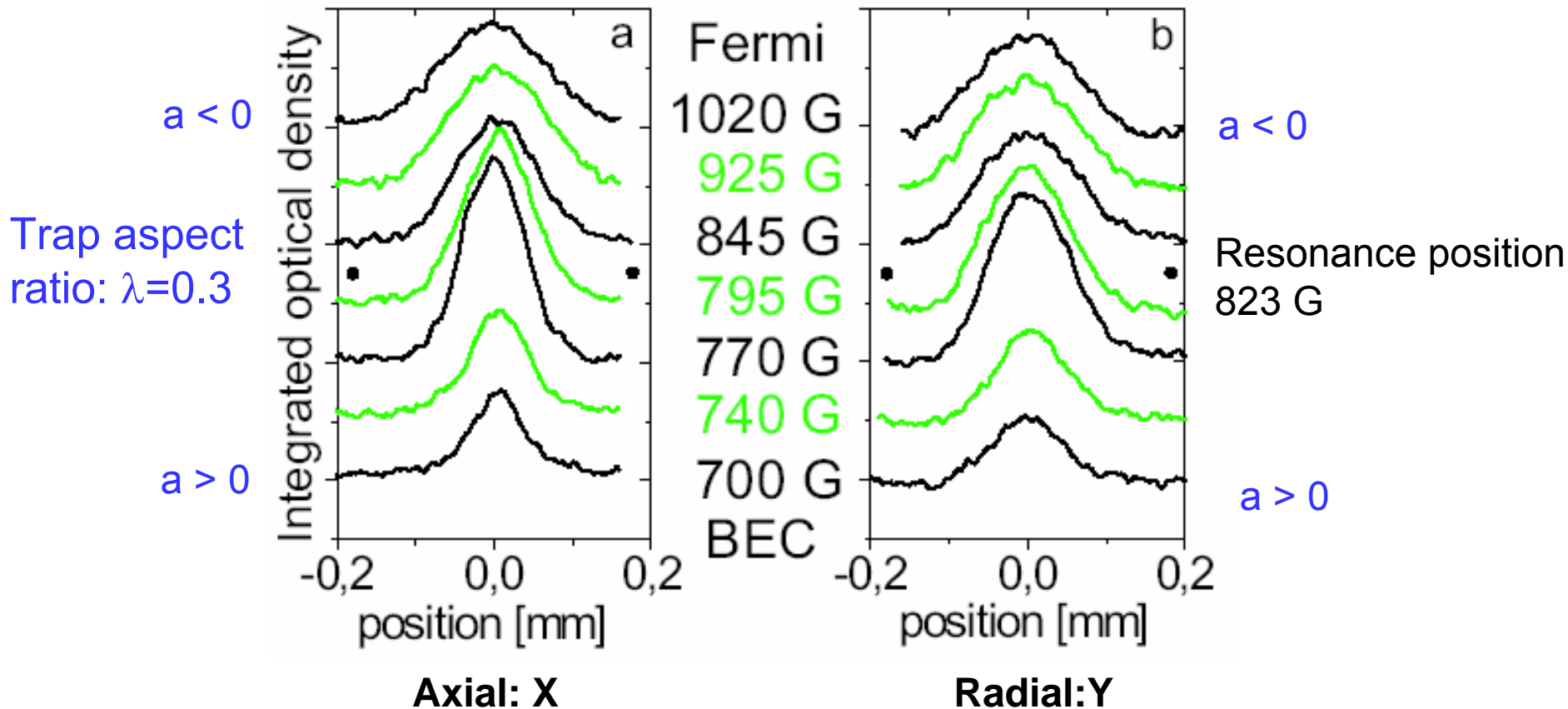
Ellipticity: 2.0
expected: 1.98

From hydrodynamic expansion

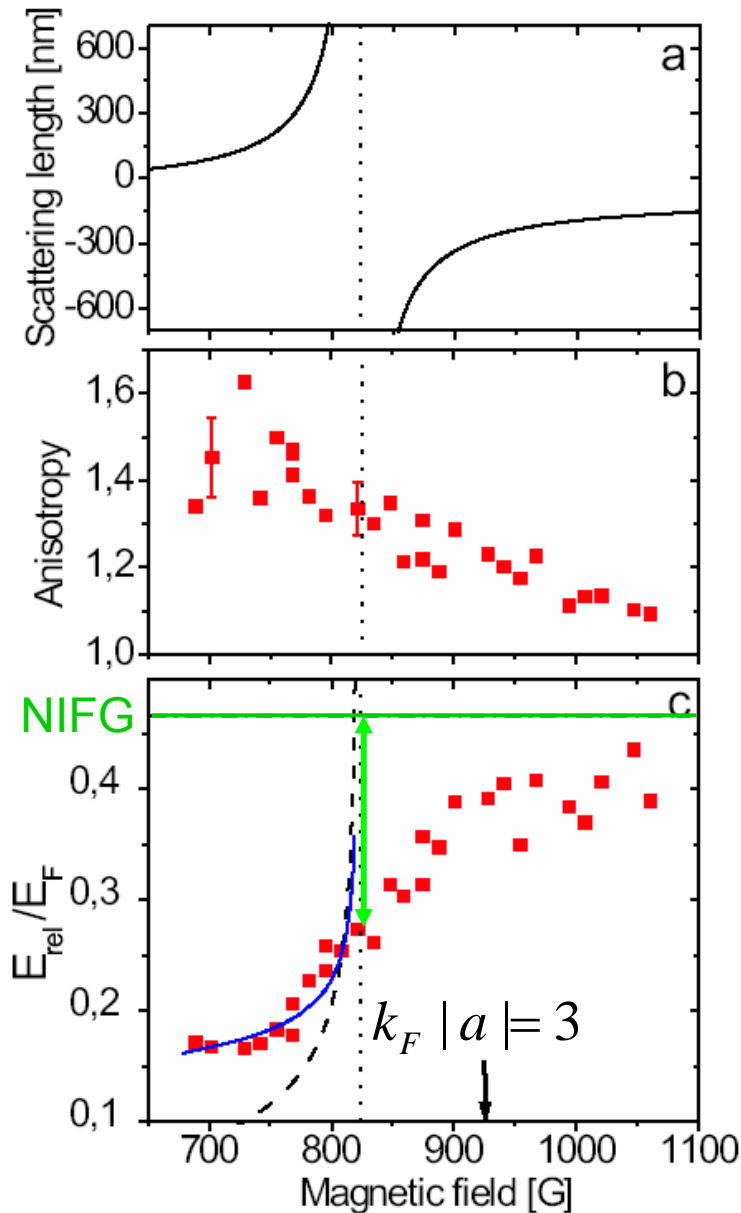
$$\text{At 770 G: } a_{mm} = 170_{-60}^{+100} \text{ nm}$$

BEC – BCS crossover expansion images

Prepare nearly pure condensate at 770G: $4 \cdot 10^4$ mol., $N_0/N \geq 70\%$
Change magnetic field slowly across FR: rate: 1-2 G/ms
Take 1.4 ms TOF image



BEC – BCS crossover: on resonance



On resonance $k_F a \gg 1$, behavior should no longer depend on a .
Equation of state should have same density dependence as ideal Fermi gas

$$\mu = \frac{\hbar^2}{2m} (6\pi^2)^{2/3} (1 + \beta) n^{2/3}$$

$\beta = 0$: ideal Fermi gas

$\beta \neq 0$ at unitarity

On resonance: $E_R = \sqrt{1 + \beta} E_R^0$

Where E_R^0 is the release energy of non interacting Fermi gas in harm. trap

We find: $\beta = -0.58(15)$

A fundamental quantity in many-body theories
Good agreement with QMC method (Carlson 02
Giorgini 04, UMASS-ETH coll. 05)

Universal equation of state of Fermi gas with equal spin populations

balanced Fermi gas ($\mu_{\uparrow} = \mu_{\downarrow}$)

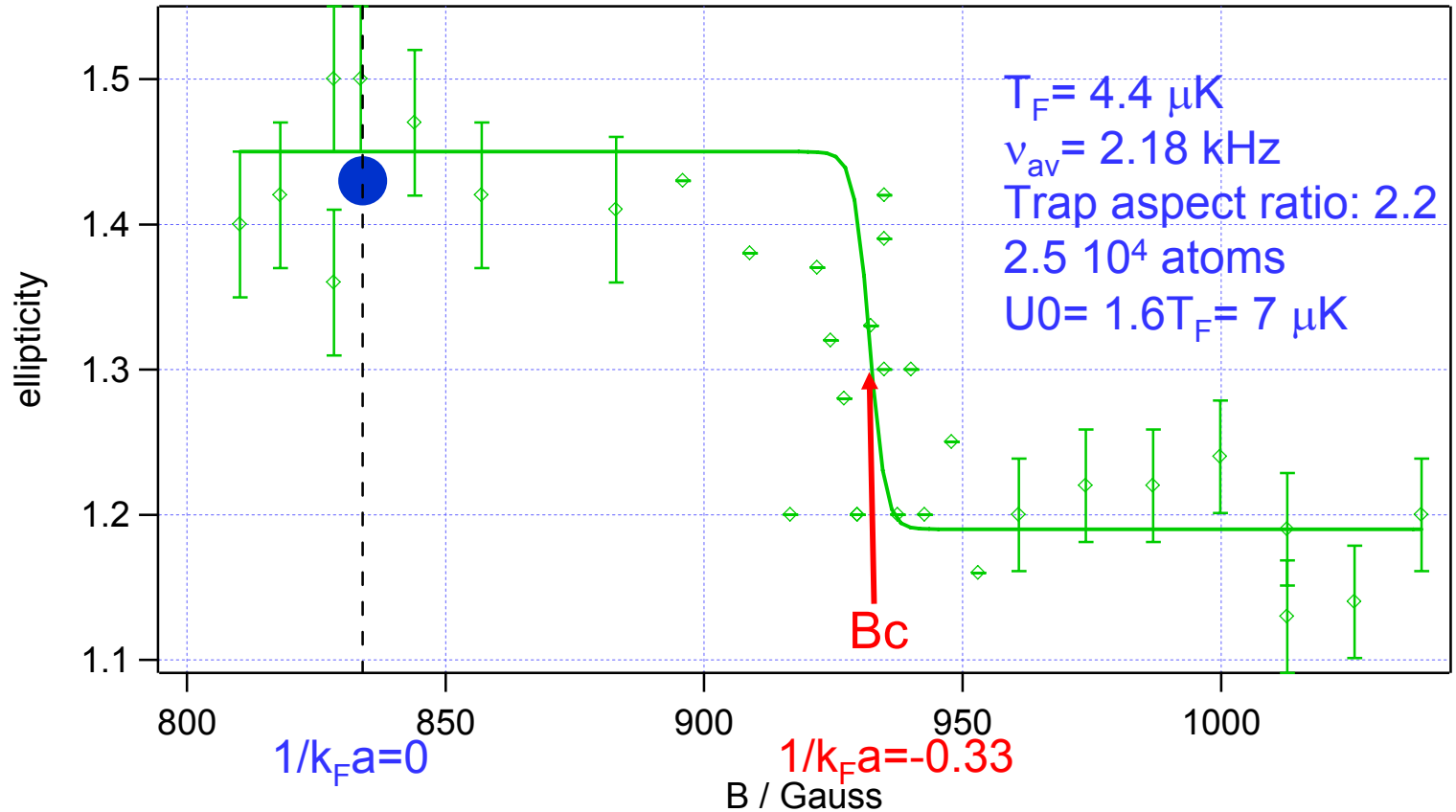
$$n = \frac{1}{6\pi^2} \left(\frac{2m\mu_{\uparrow}}{\hbar^2} \right)^{3/2} \quad \text{x numerical factor}$$

$$\mu_{\uparrow} = \xi \frac{\hbar^2}{2m} (6\pi^2 n)^{2/3} = \xi E_F$$

Determination of ξ

| | | | | | |
|-------------------|-----------------------------------|----------|---------------|----------------------|---------|
| Experiment | <i>ENS (⁶Li)</i> | 0.42(15) | Theory | <i>BCS</i> | 0.59 |
| | <i>Rice (⁶Li)</i> | 0.46(5) | | <i>Astrakharchik</i> | 0.42(1) |
| | <i>JILA(⁴⁰K)</i> | 0.46(10) | | <i>Perali</i> | 0.455 |
| | <i>Innsbruck (⁶Li)</i> | 0.27(10) | | <i>Carlson</i> | 0.42(1) |
| | <i>Duke (⁶Li)</i> | 0.51(4) | | <i>Hausmann</i> | 0.36 |

Aspect ratio at low temperature



Abrupt decrease near 930 G !

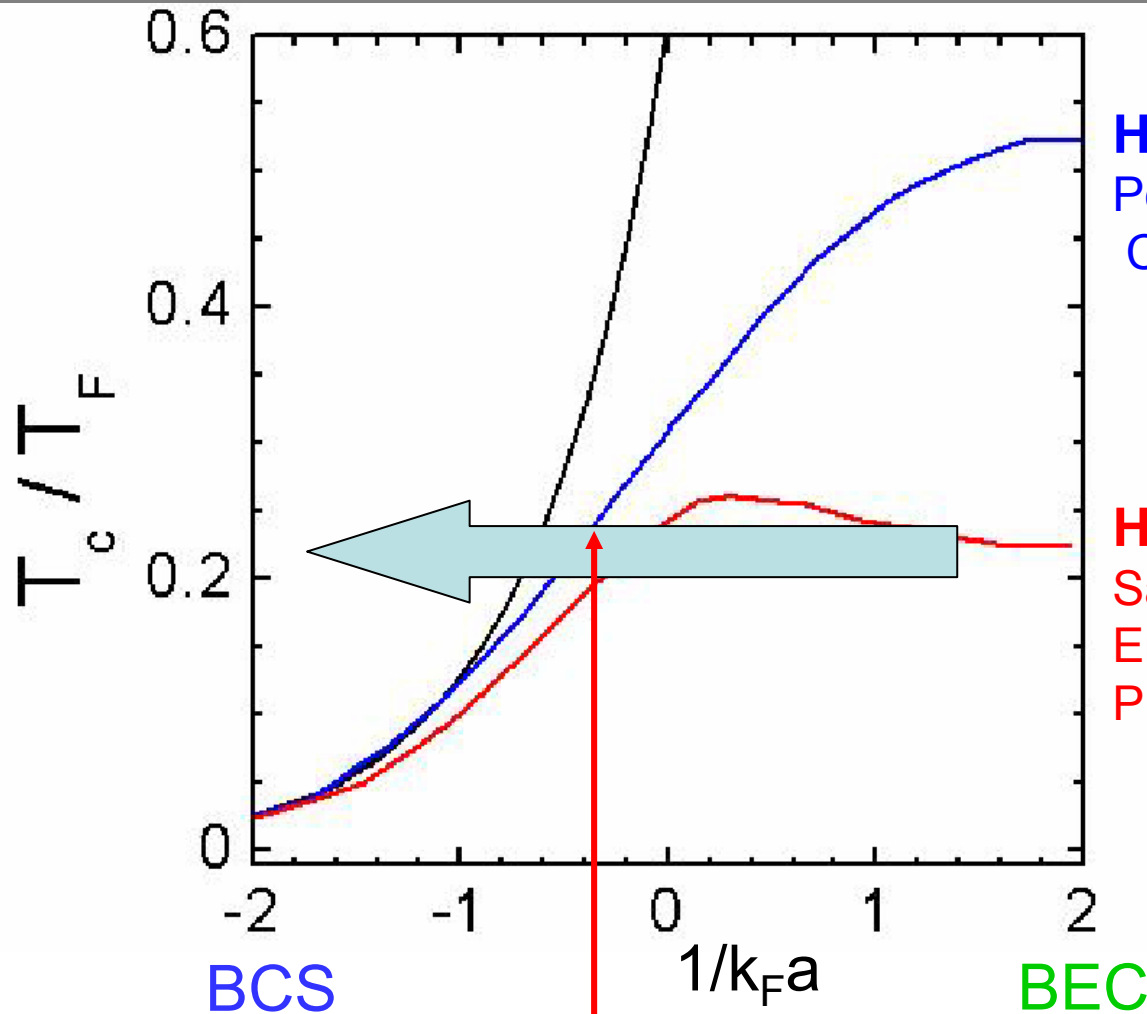
On resonance: agreement with hydrodynamic prediction

At B_c : crossing of the critical temperature near 930 Gauss.

For $T > T_c$, generalized Cooper pairs are broken, hence loss of superfluidity.

At higher T , the step smoothes and shifts towards smaller $1/k_F|a|$

Critical temperature in BEC-BCS crossover



Harmonic trap

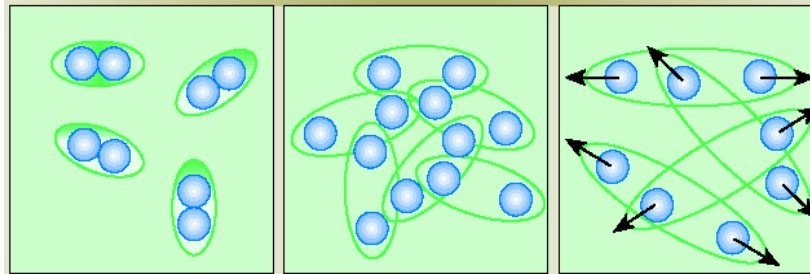
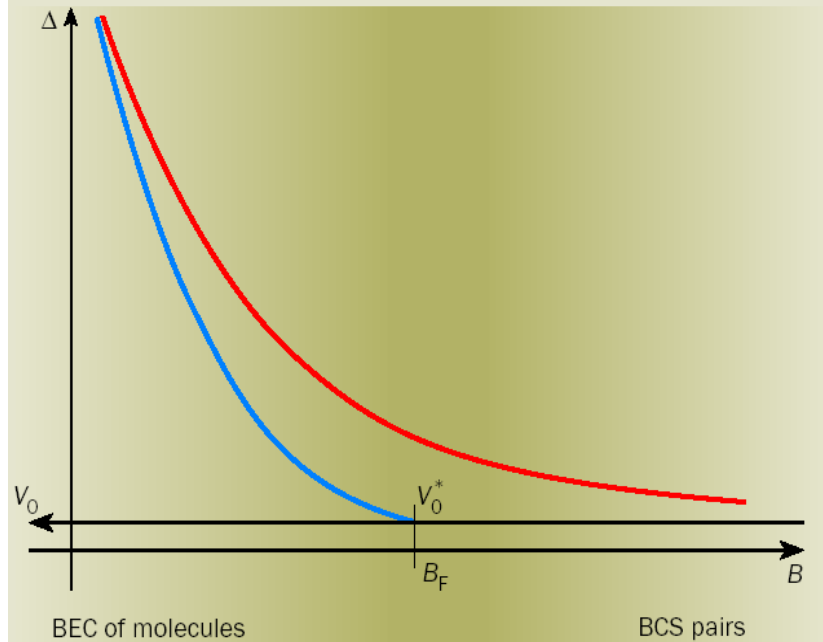
Perali et al.,
Cond-mat 0311309

Homogeneous case

Sa de Melo, Randéria,
Engelbrecht,
PRL 71, (1993)

At B_c , $1/k_F a = -0.33$ in trap

Phase diagram at T=0



Molecular BEC
 Strongly bound
 Size: $a \ll n^{-1/3}$
 $n^{-1/3}$: average dist.
 between particles

On resonance
 $na^3 > 1$ or $k_F a \geq 1$
 Pairs stabilized by Fermi sea

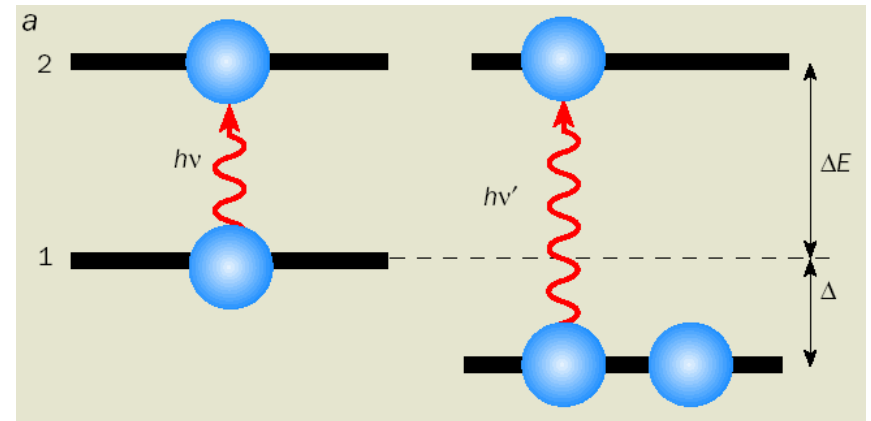
F. Chevy
 C.S.
 Physics World
 March 05

BCS regime:
 $k_F |a| \ll 1$
 Cooper pairs $\mathbf{k}, -\mathbf{k}$
 Well localized in
 Momentum: $k \sim k_F$
 Delocalized in
 position

Observation of pairing gap

Innsbruck
C. Chin et al., Science 04

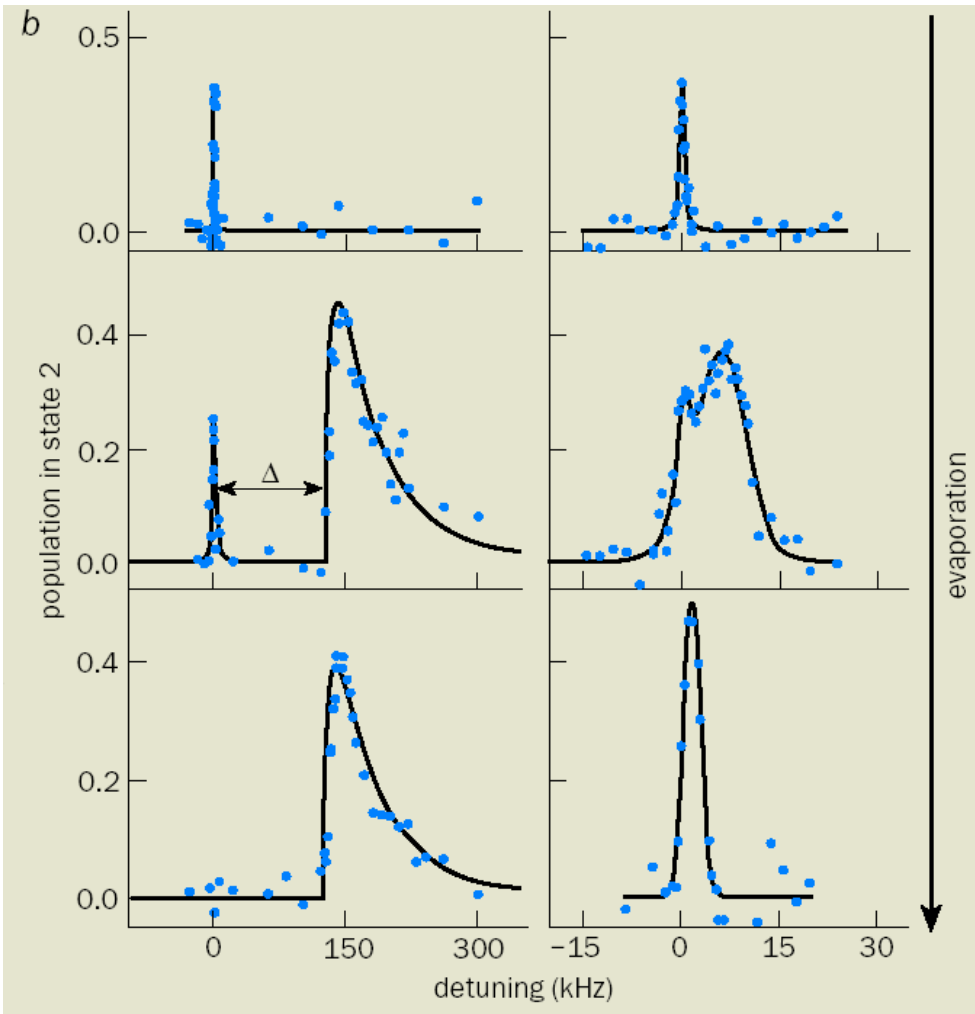
$T = 5 T_F$



RF dissociation spectroscopy

$T < 0.2 T_F$
 $T_F = 1.2 \mu\text{K}$

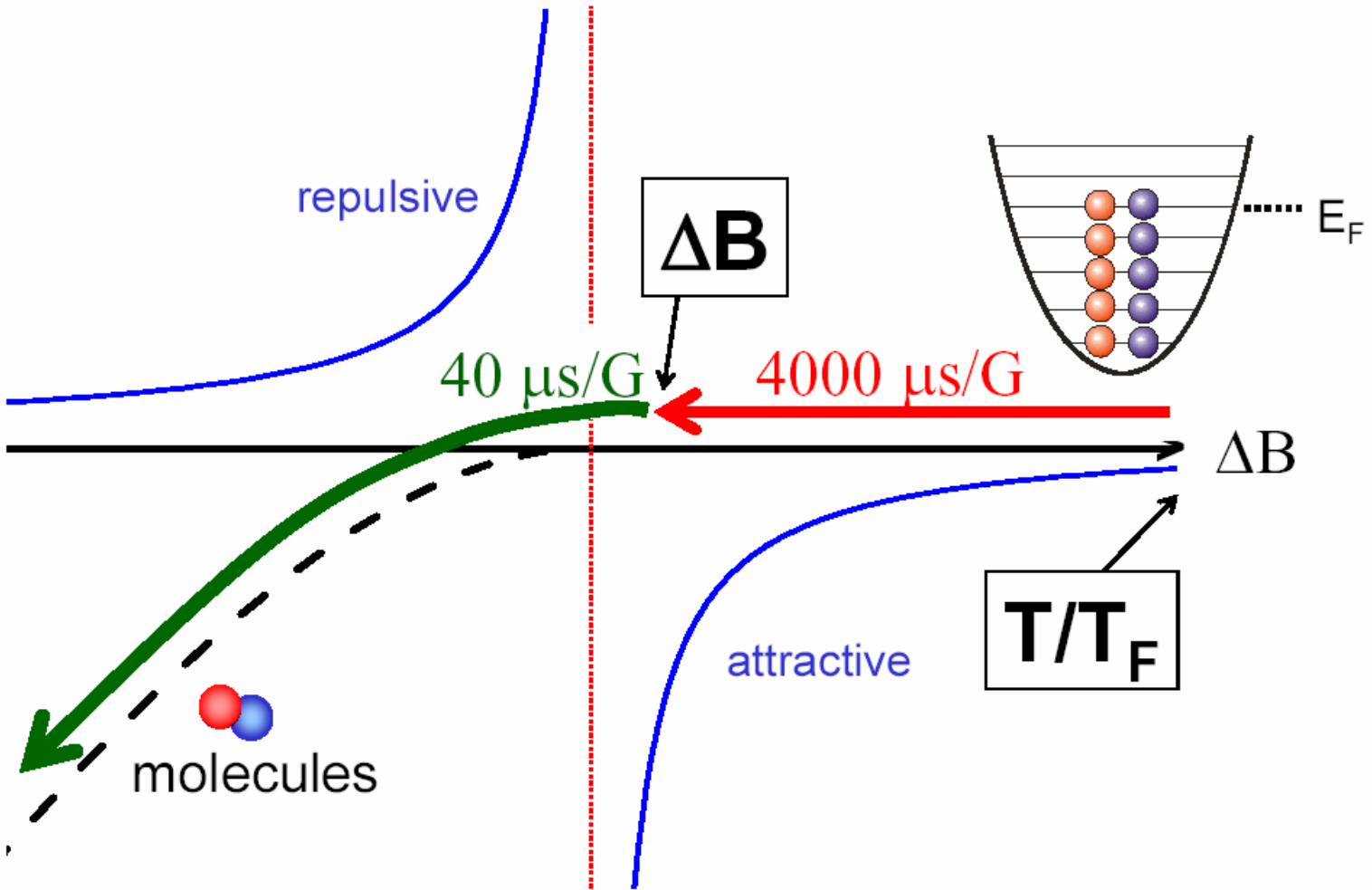
On resonance: $h\Delta \sim 0.2 E_F$



BEC side

BCS side

Are fermion pairs condensed?



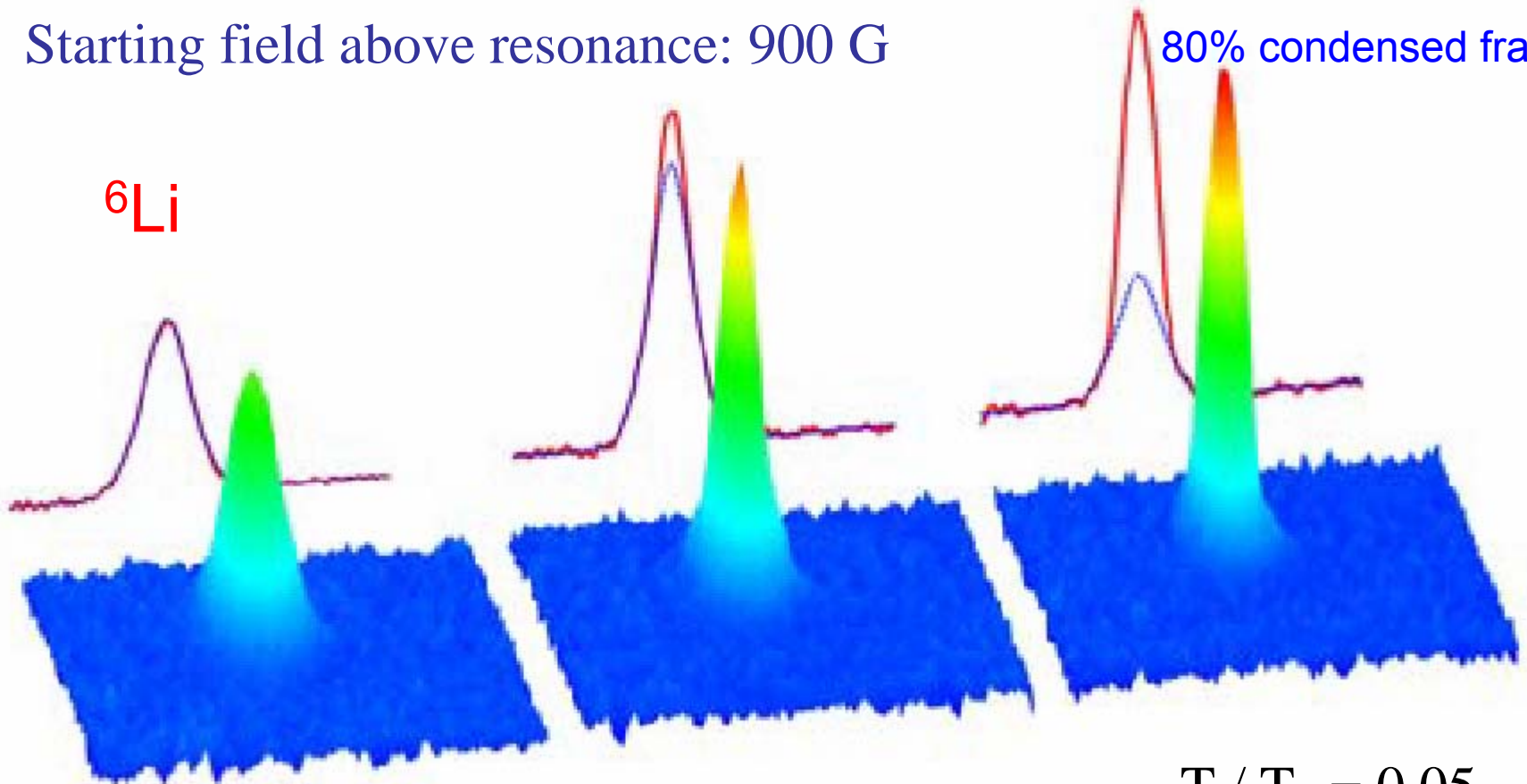
Condensation of fermionic pairs: JILA, MIT

C. Regal, PRL 04
M. Zwierlein et al., 04

Starting field above resonance: 900 G

80% condensed fraction

${}^6\text{Li}$



Initial

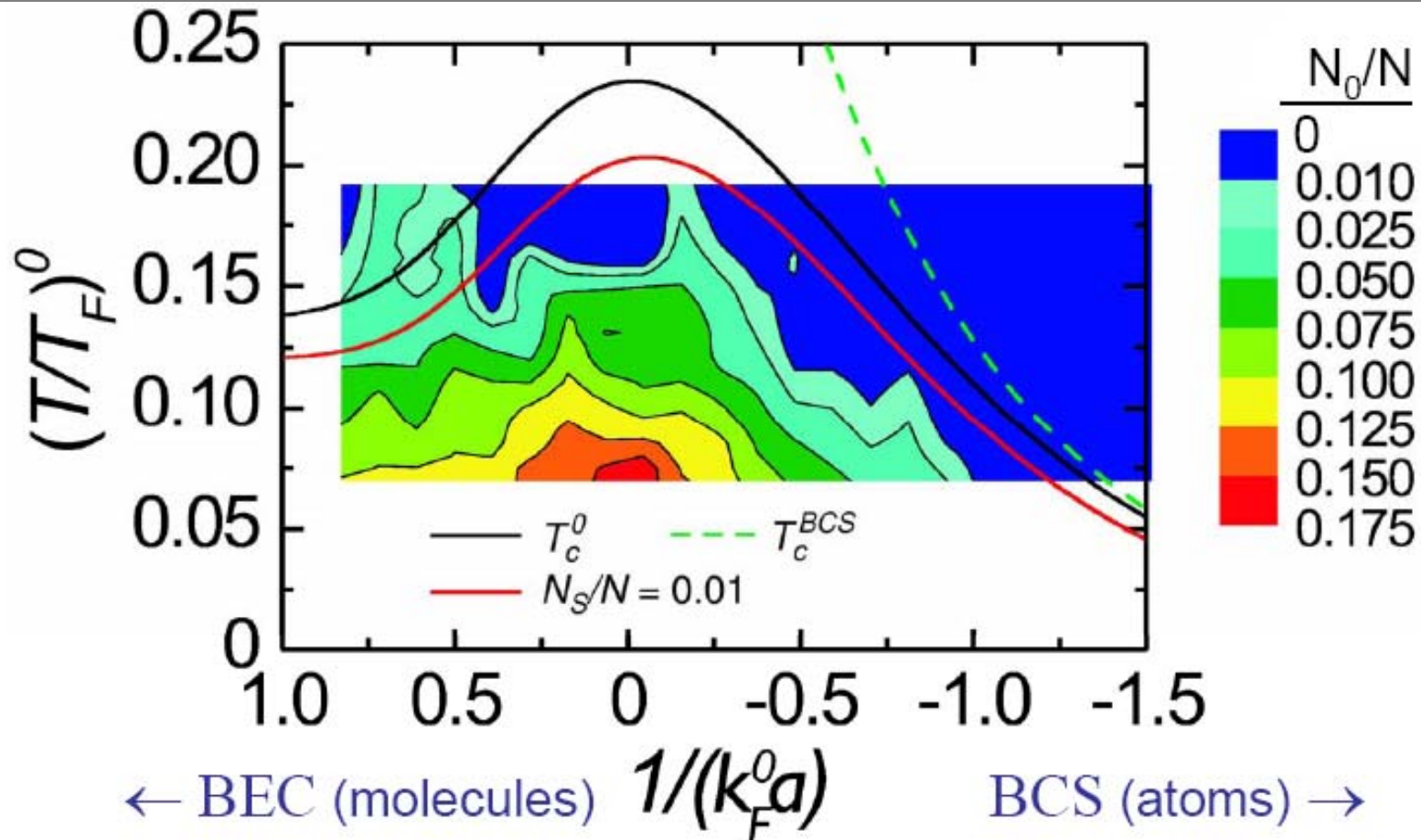
temperature: $T / T_F = 0.2$

$T / T_F = 0.1$

$T / T_F = 0.05$

High condensate fraction indicates the presence of $k, -k$ pairs on resonance side where no molecular bound state exists

Pair condensation transition temperature: 40K



Expt: C.A. Regal, M. Greiner, and D. S. Jin, PRL **92**, 040403 (2004)

Theory: Q. Chen *et al.*, PRA **73**, 041601 (2006)

Direct proof of superfluidity

So far:

Anisotropic expansion

Collective modes

Pairing gap

Condensate fractions

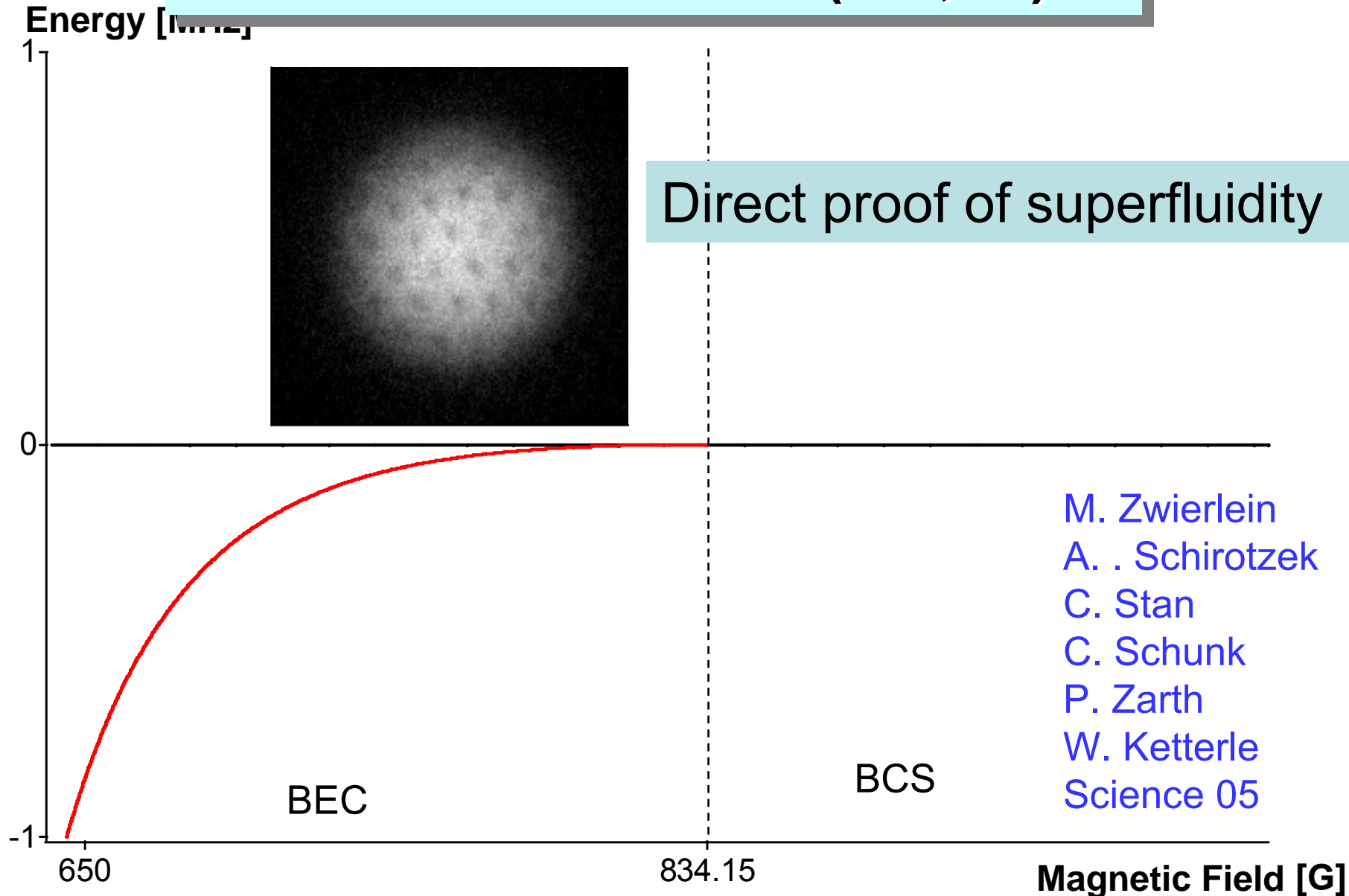
are evidence for superfluid behavior

Direct proof of superfluidity in the system ?

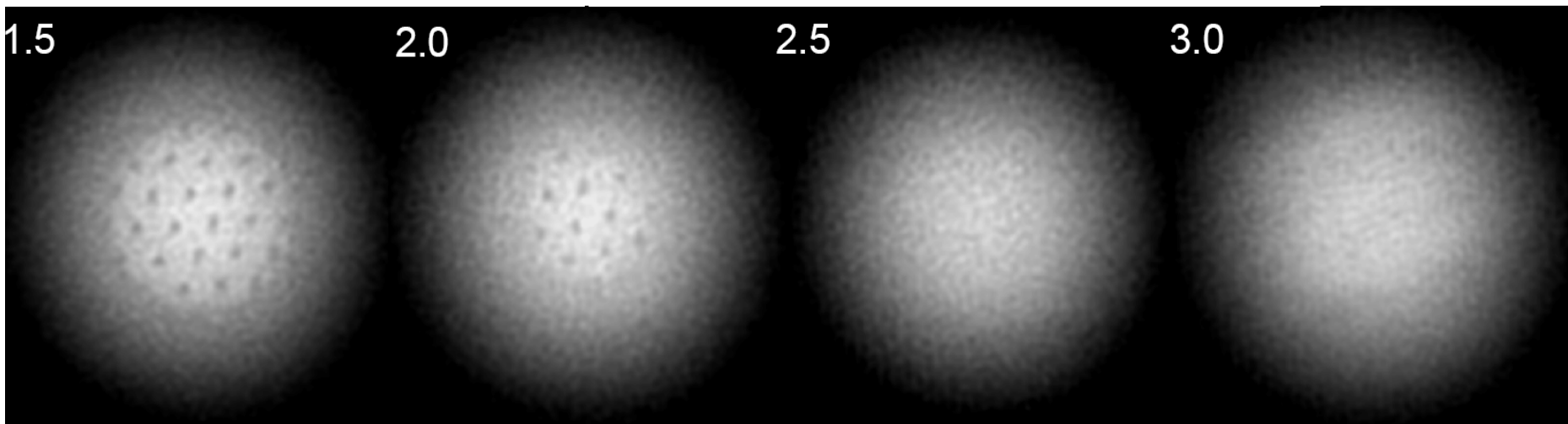
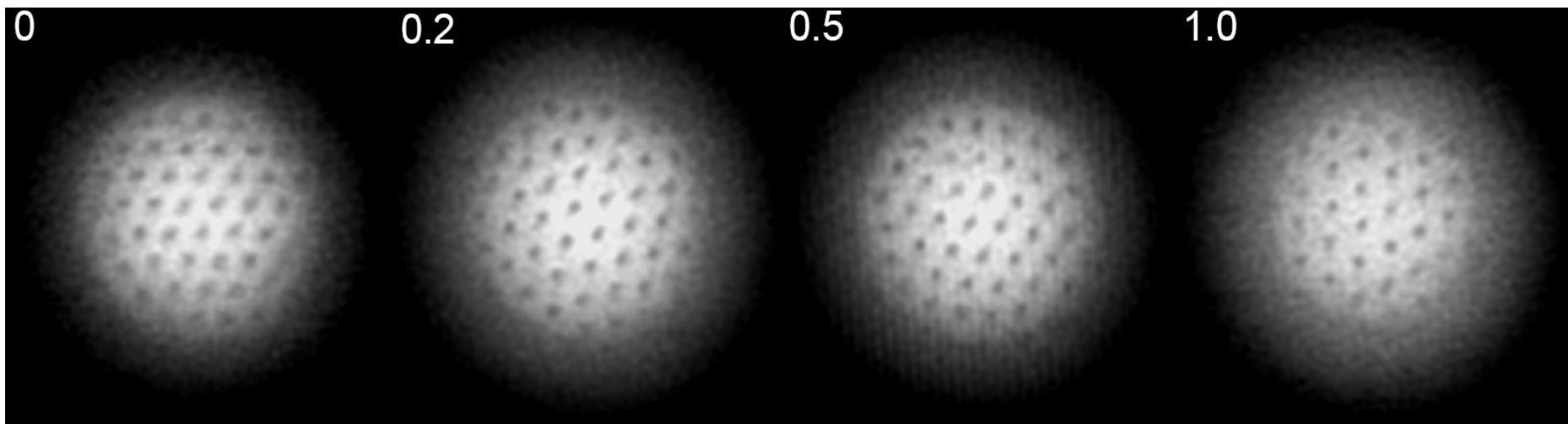
Put the gas in rotation

In contrast to classical gas, the superfluid Fermi gas should exhibit quantized vortices, $(\hbar/2m)$ (Sandro Stringari's lecture)

Observation of vortex lattices in the BEC-BCS crossover (MIT, 05)



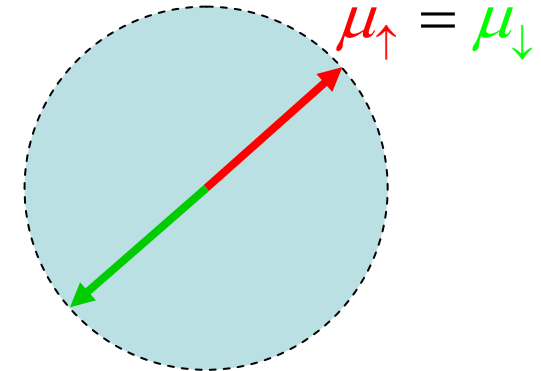
Pair breaking in TOF [ms] 930 G



Superfluidity
with imbalanced
spin populations

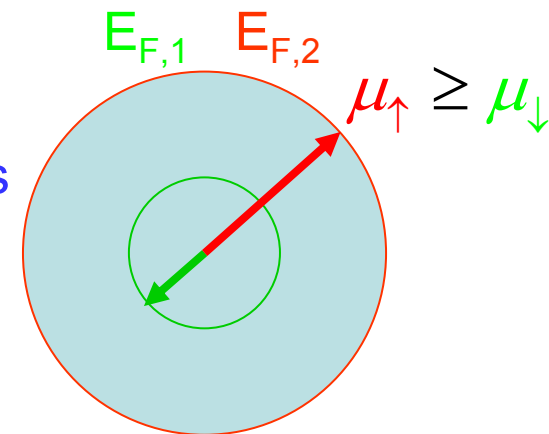
Imbalanced Fermi gas: motivation

Attractive Fermi gas with equal spin population
⇒ BCS theory, pairing at edge of Fermi surface



What is the nature and existence of superfluidity
when spin population is imbalanced ?

Mismatched density and/or pairing with different masses



Ex:

Superconductors in magnetic field or
quark matter

Cold gases: MIT and Rice expt

$$E_{F,i} = \frac{\hbar^2 k_{F,i}^2}{2m_i} = \frac{\hbar^2}{2m_i} \left(6\pi^2 n_i \right)^{2/3}$$

Overview of Theoretical scenarios

Chandrasekhar and Clogston: stability of the paired state : $\mu_{\uparrow} > \mu_{\downarrow}$

Conversion of a particle: $\downarrow \rightarrow \uparrow$

Decrease the grand potential $H - \mu_{\uparrow} N_{\uparrow} - \mu_{\downarrow} N_{\downarrow} : \mu_{\uparrow} - \mu_{\downarrow}$

Cost of pair breaking: Δ

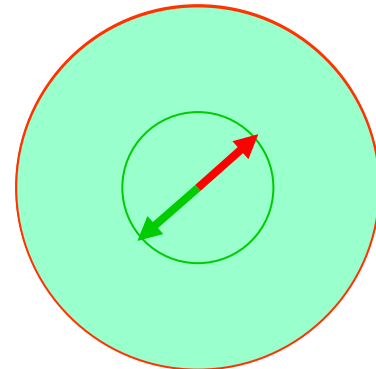
\Rightarrow Paired state stable for $\mu_{\uparrow} - \mu_{\downarrow} < \Delta$

And beyond?

Polarized phase : One spin species (Carlson, PRL **95**, 060401 (2005))

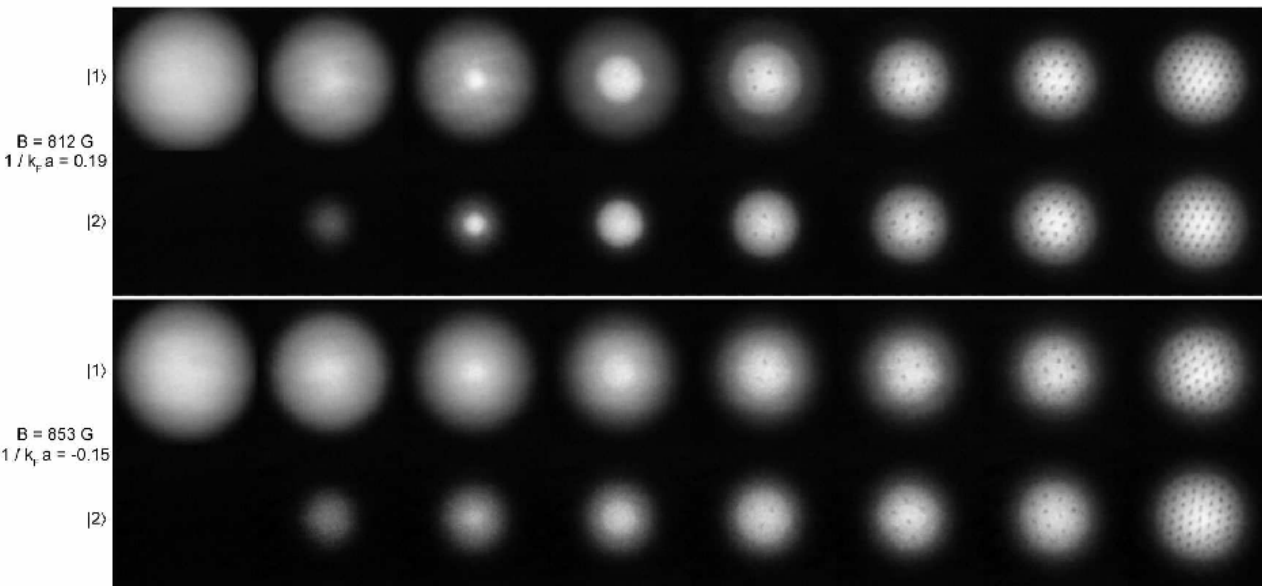
FFLO Phase (Fulde Ferrell Larkin Ovchinnikov) : pairing in $\mathbf{k}_{\uparrow} - \mathbf{k}_{\downarrow} \neq 0$
(C. Mora et R. Combescot, PRB **71**, 214504 (2005))

Sarma phase (internal gap) : pairing in $\mathbf{k}_{\uparrow} - \mathbf{k}_{\downarrow} = 0$
 \Downarrow opening of a gap in the Fermi sea of majority species. (Liu, PRL **90**, 047002 (2003))

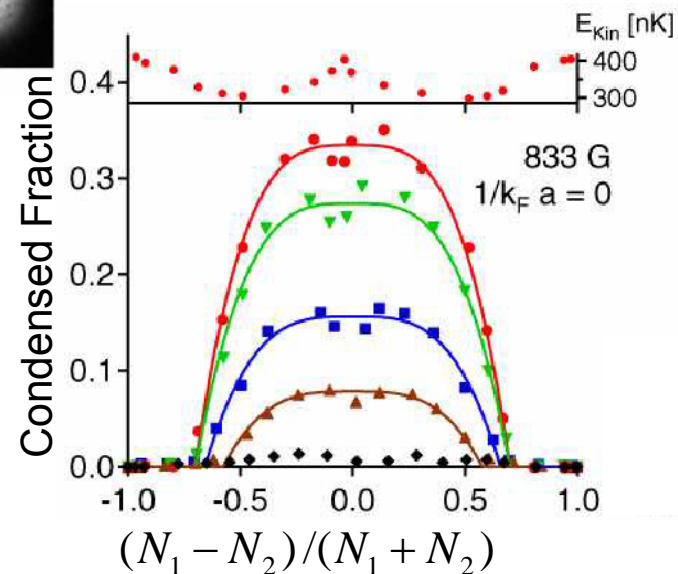


MIT experiment

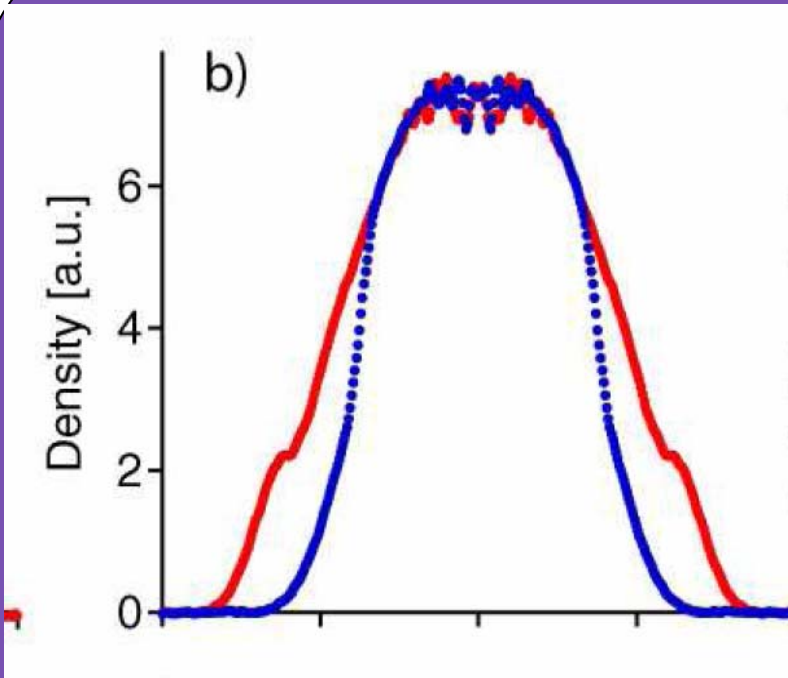
(Science Express, December 22, 2005)



Superfluidity observed in Time of flight
Loss of superfluidity for large
Spin population imbalance



Experimental results



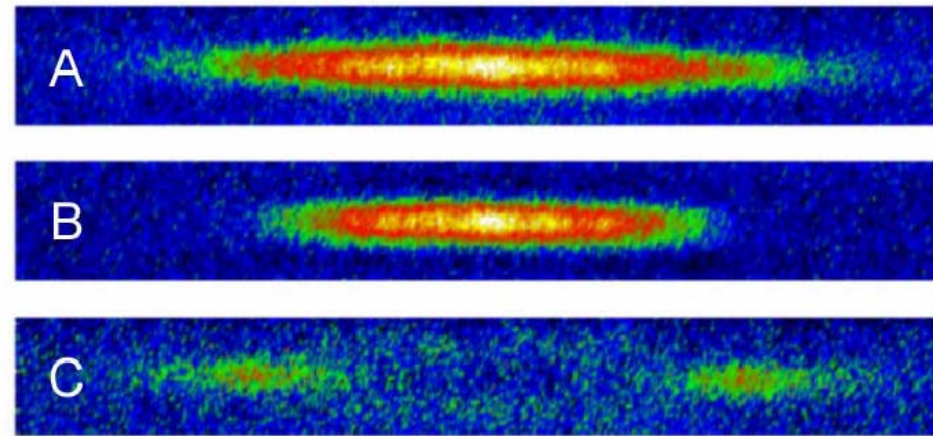
MIT: 3 phases

- Fully paired superfluid core
- Intermediate mixture
- Fully polarized rim

M.W. Zwierlein, *et al.*, Science, **311**
(2006) 492.

Rice: 2 phases

- Fully paired superfluid core
- Fully polarized rim



G. Partridge, W. Li, R.I. Kamar, Y.-A. Liao,
R.G. Hulet, Science, **311** (2006)
503.

G. Partridge *et al.*, Cond-mat 0608455

Rice Univ: phase separation at unitarity

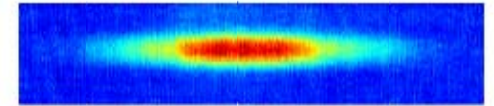
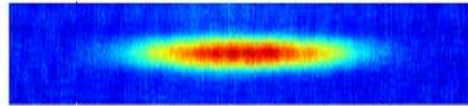
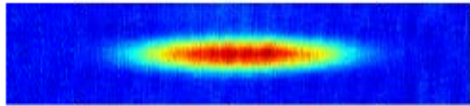
$$P = (N_1 - N_2) / (N_1 + N_2)$$

$P = 0$

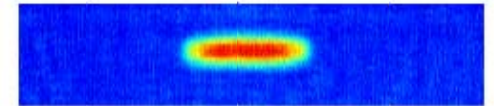
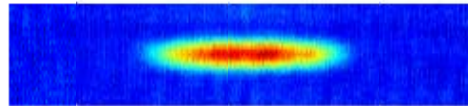
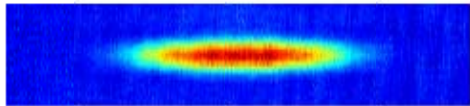
$P = 0.18$

$P = 0.37$

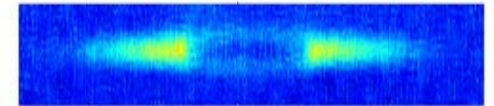
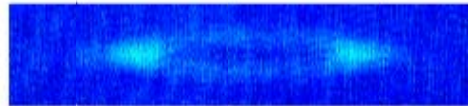
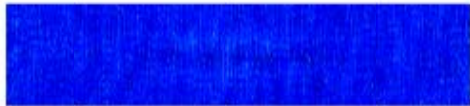
$|1\rangle$



$|2\rangle$



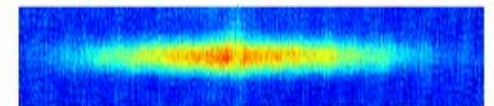
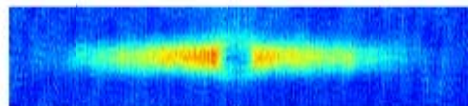
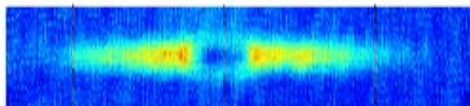
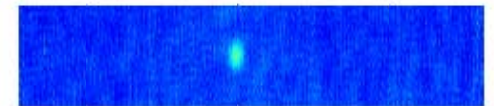
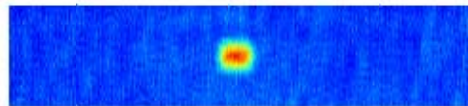
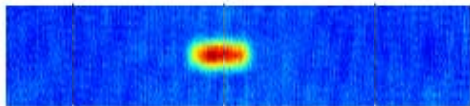
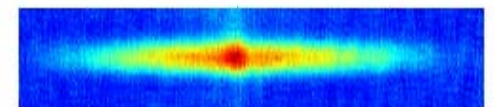
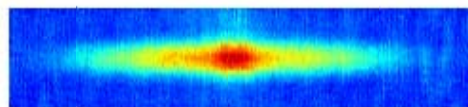
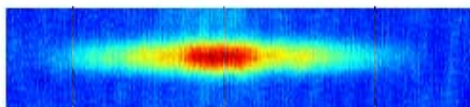
$|1\rangle - |2\rangle$



$P = 0.6$

$P = 0.79$

$P = 0.95$



Avalanche of recent publications !

P. Pieri and G.C. Strinati cond-mat/0512354 : diagrammatic method

Extrapolation from BEC regime

W. Yi and L.-M. Duan, cond-mat/0601006 : BCS at finite temperature

M. Haque and H.T.C. Stoof, cond-mat/0601321 : BCS at T=0

T.N. de Silva and E.J. Mueller, cond-mat/0601314 : BCS at T=0

D. Sheehy, L. Radzihovsky, PRL 06

A. Bulgac, M. McNeil Forbes '06

K. Levin et al., 06

M. Parish, Nature Physics 3 '07

.....

F. Chevy approach:

Assumptions:

1) Unitarity: universal parameter $\mu = (1 + \beta) E_F = \xi E_F$ known

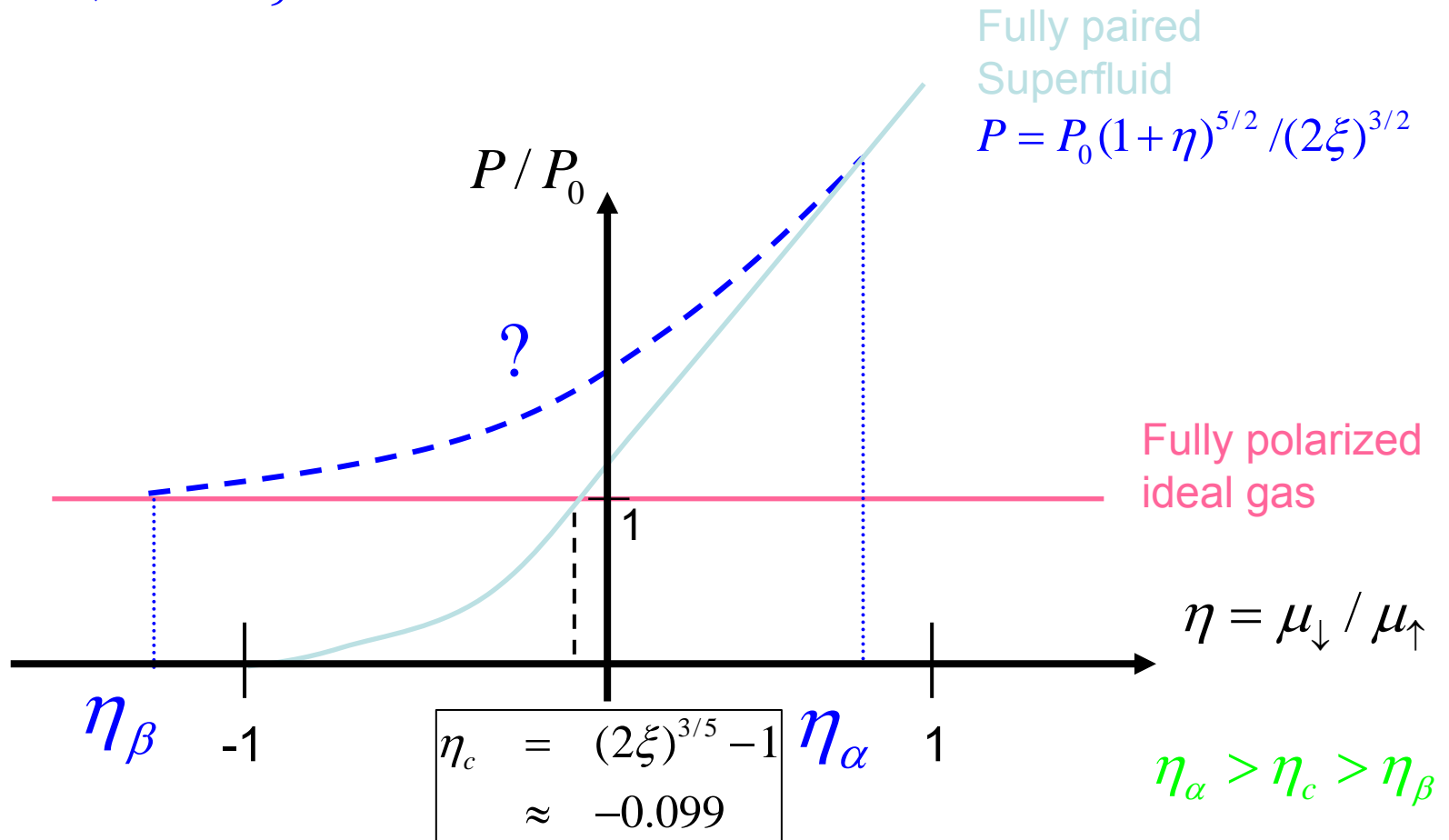
2) Grand canonical description, Local density approx,

3) T=0 approach

Universal phase diagram of the homogeneous unitary system (2)

F. Chevy, PRA 06

$$\left. \begin{aligned} \Omega &= -PV \\ dP &= \sum_{\sigma=\uparrow\downarrow} n_{\sigma} d\mu_{\sigma} \end{aligned} \right\} \Rightarrow \text{Just need to know } n(\mu)$$



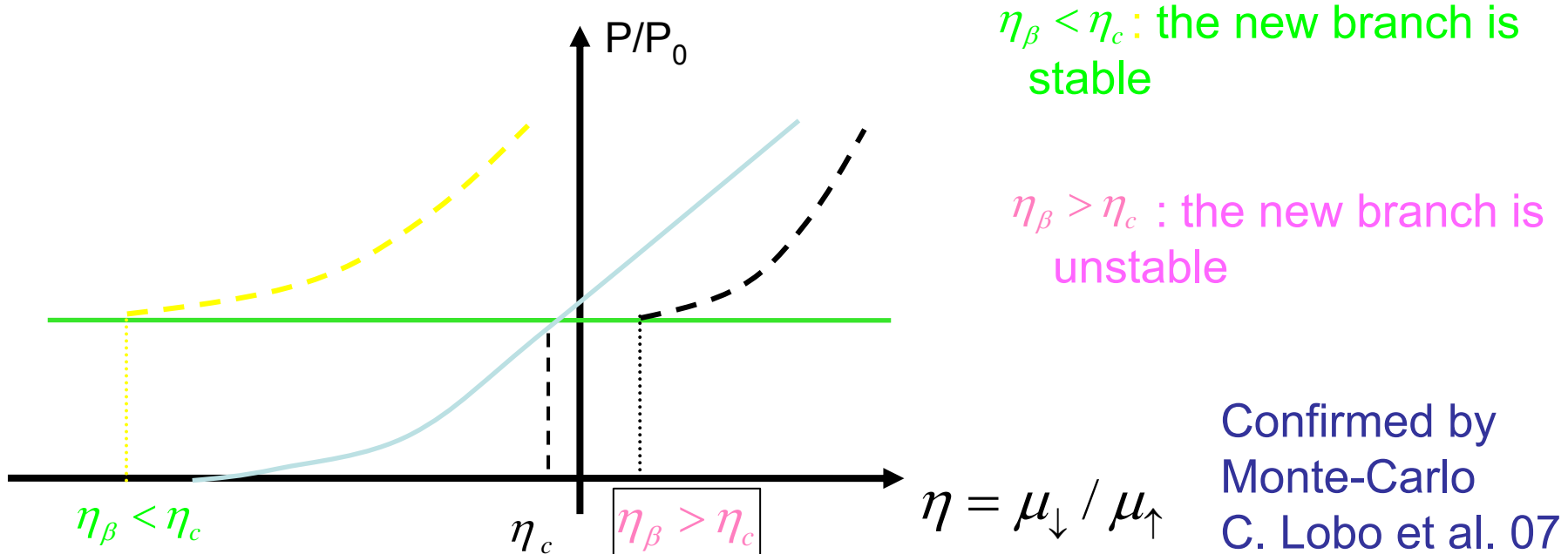
Theoretical evidence for an intermediate phase

General properties of a mixed branch?

Step 1: calculate the energy E of a single impurity atom immersed in a Fermi sea ($E = \mu_{\downarrow}$, with $n_{\downarrow} = 0^+$)

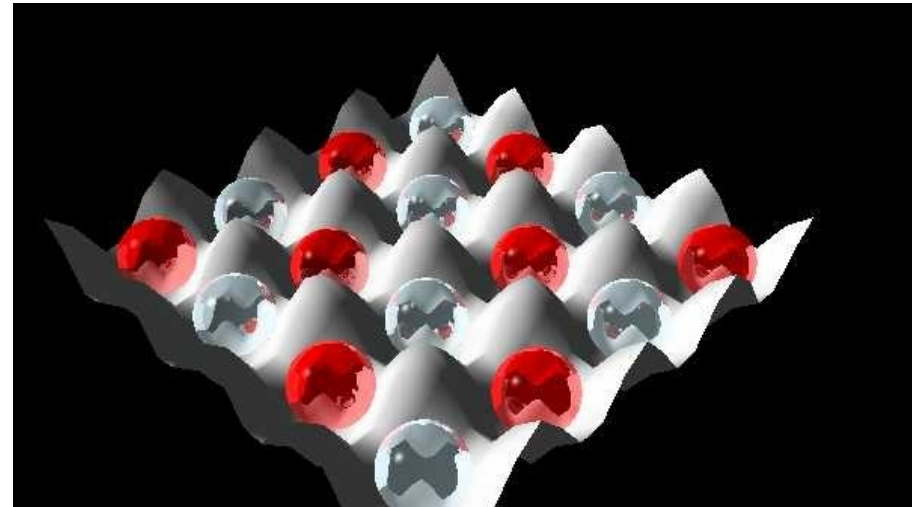
For $a = \infty$, $E = -0.606 E_{F^*}$ $\Downarrow \eta_{\beta} < -0.606 < \eta_c \sim -0.1$

Step 2: $dP/d\mu_{\sigma} = n > 0$



Open questions and perspectives

- Imbalance of spin populations, properties of mixed phase ?, phase diagram phase separation, role of trap anisotropy (M. Randeria)
- Single particle excitations by Raman transitions, T.L. Dao et al., PRL 07, **98**
- p-wave pairing ?
- **Fermions in optical lattices:
simulation of condensed matter
Hamiltonians**
- Fermionic Hubbard model
- ${}^6\text{Li}$: Transition toward antiferromagnetic order: Néel transition
- Lattices with frustration
- Fermi-Fermi mixtures : pairing with different masses
- Bose-Fermi mixtures



$$T_{\text{Néel}} \sim 30 \text{ nK}$$

F. Werner et al., PRL 05

Thank you for your attention!

