Dynamics of Pairing and Molecule Formation in Ultracold Quantum Gases

Thorsten Köhler

Department of Physics and Astronomy, University College London,

Gower Street, London, WC1E 6BT, United Kingdom

Outline

- Ultracold Feshbach molecules

Outline

- Ultracold Feshbach molecules
- Diatomic dynamics

Outline

- Ultracold Feshbach molecules
- Diatomic dynamics
- Pairing in Bose-Einstein condensates

Outline

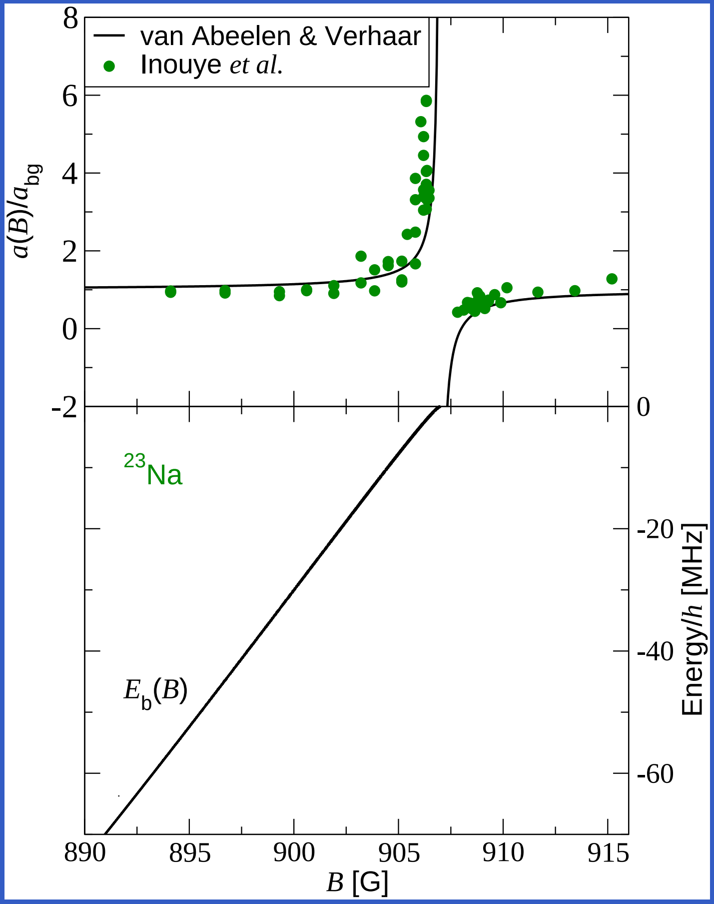
- Ultracold Feshbach molecules
- Diatomic dynamics
- Pairing in Bose-Einstein condensates
- Atom-molecule coherence

Outline

- Ultracold Feshbach molecules
- Diatomic dynamics
- Pairing in Bose-Einstein condensates
- Atom-molecule coherence
- Outlook: Pairing in correlated gases

Ultracold Feshbach molecules

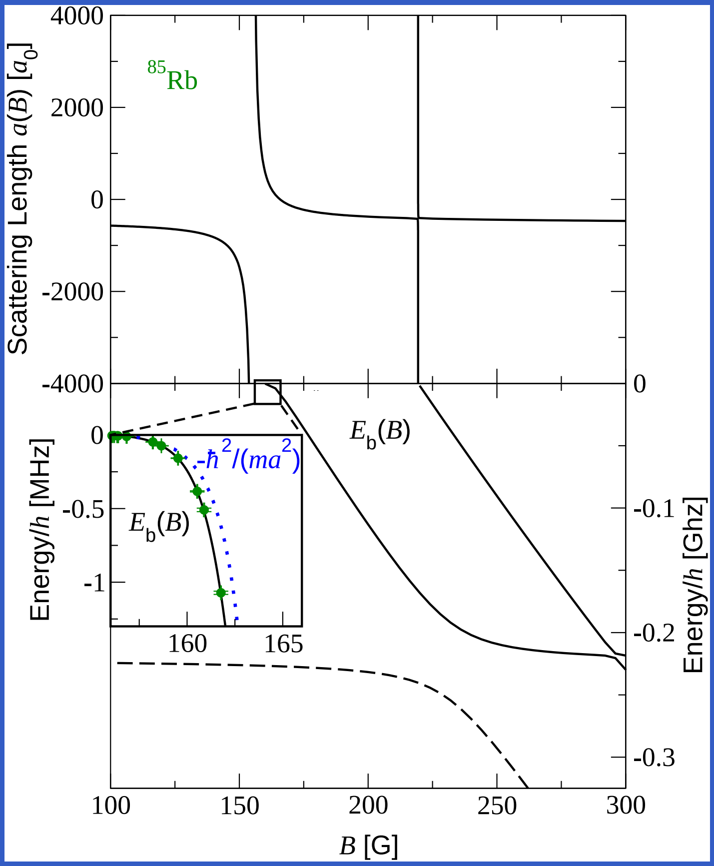
- A singularity of the scattering length indicates emergence of a new vibrational bound state at the collision threshold.



Theory: E. Tiesinga *et al.*, PRA **47**, 4114 (1993)
Experiment: S. Inouyé *et al.*, Nature **392**, 151 (1998)

Ultracold Feshbach molecules

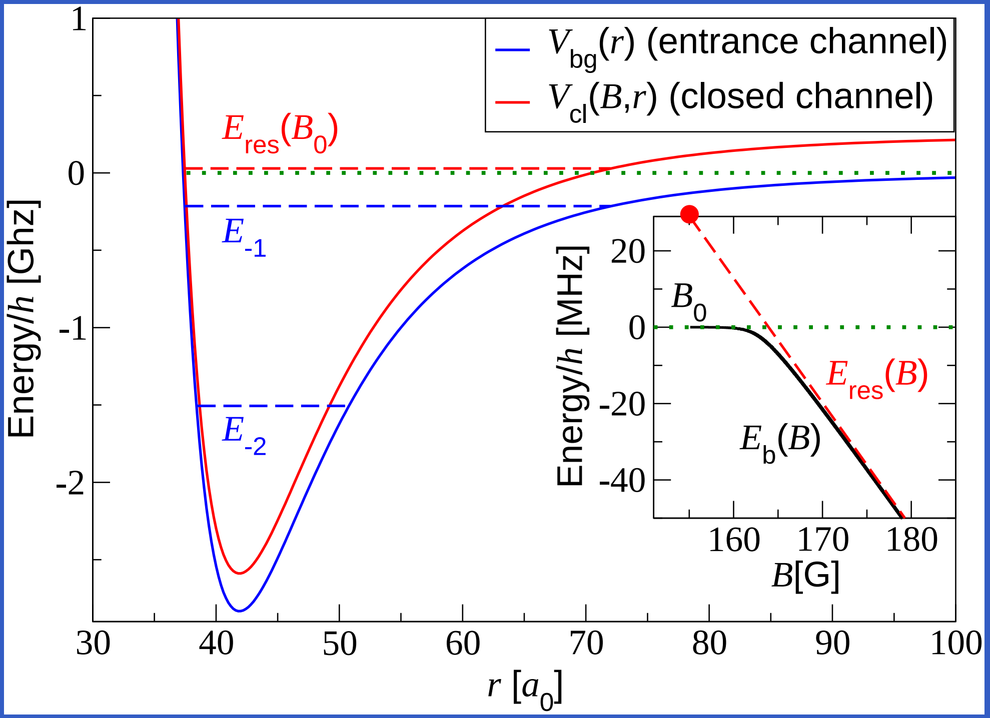
- A singularity of the scattering length indicates emergence of a new vibrational bound state at the collision threshold.
- **Feshbach resonances can be strongly or weakly coupled to the continuum levels.**



N.R. Claussen *et al.*, PRA **67**, 060701(R) (2003)

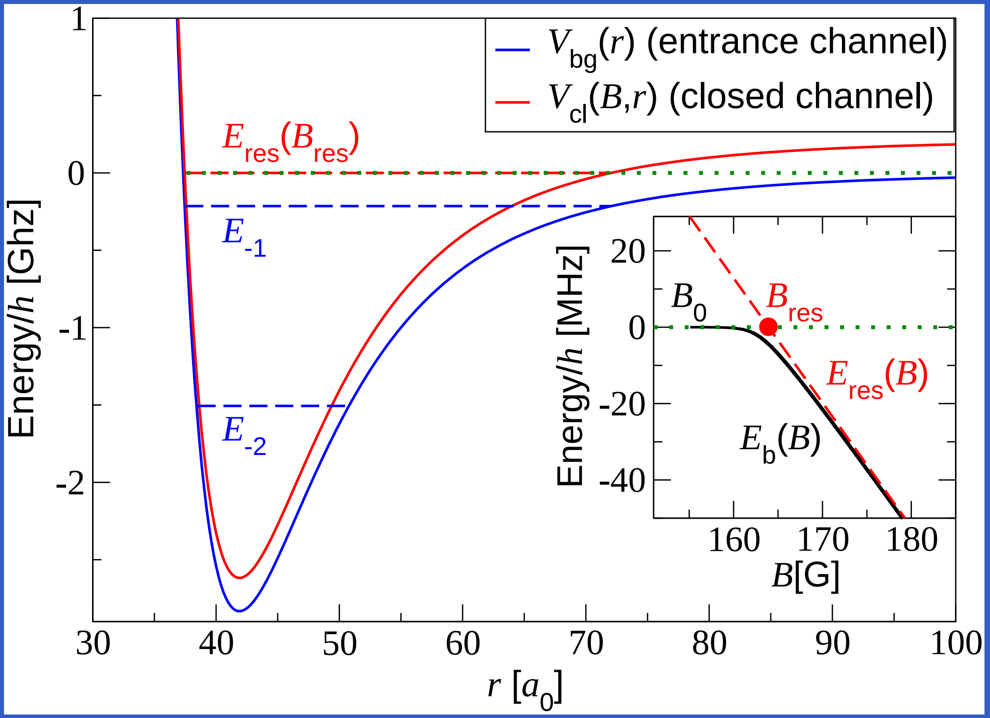
Ultracold Feshbach molecules

- A singularity of the scattering length indicates emergence of a new vibrational bound state at the collision threshold.
- Feshbach resonances can be strongly or weakly coupled to the continuum levels.
- **Feshbach molecules can be associated from pairs of separated atoms using magnetic-field sweeps.**



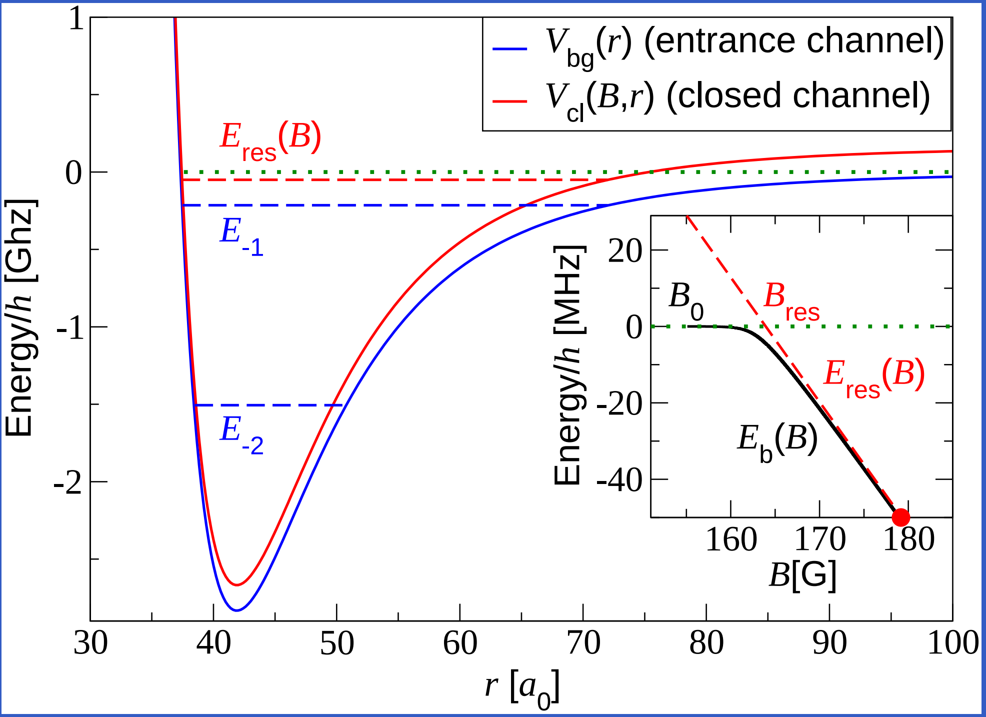
Ultracold Feshbach molecules

- A singularity of the scattering length indicates emergence of a new vibrational bound state at the collision threshold.
- Feshbach resonances can be strongly or weakly coupled to the continuum levels.
- **Feshbach molecules can be associated from pairs of separated atoms using magnetic-field sweeps.**



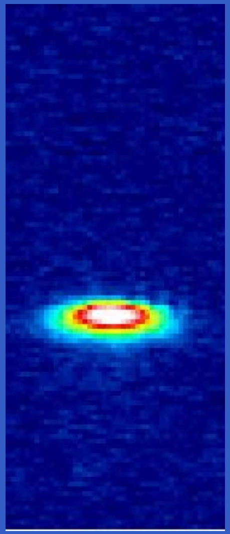
Ultracold Feshbach molecules

- A singularity of the scattering length indicates emergence of a new vibrational bound state at the collision threshold.
- Feshbach resonances can be strongly or weakly coupled to the continuum levels.
- **Feshbach molecules can be associated from pairs of separated atoms using magnetic-field sweeps.**



Ultracold Feshbach molecules

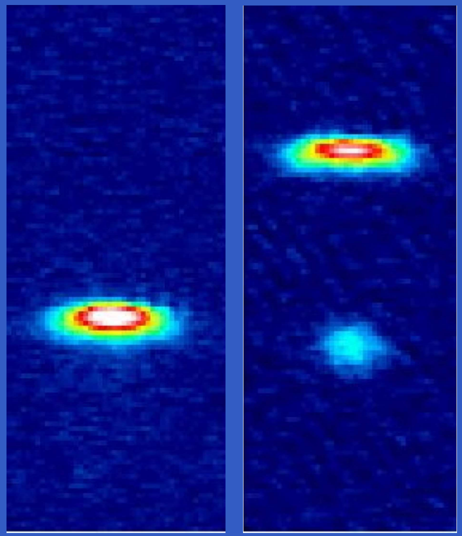
- A singularity of the scattering length indicates emergence of a new vibrational bound state at the collision threshold.
- Feshbach resonances can be strongly or weakly coupled to the continuum levels.
- **Feshbach molecules can be associated from pairs of separated atoms using magnetic-field sweeps.**



Adapted from: J. Herbig *et al.*, Science **301**, 1510 (2003)

Ultracold Feshbach molecules

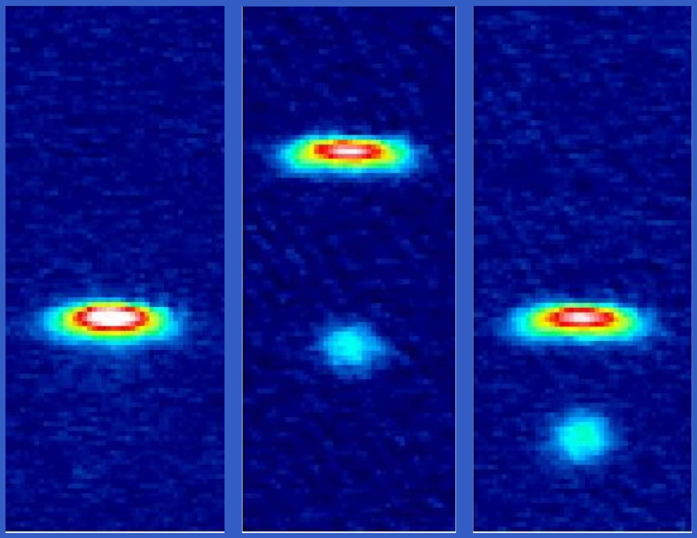
- A singularity of the scattering length indicates emergence of a new vibrational bound state at the collision threshold.
- Feshbach resonances can be strongly or weakly coupled to the continuum levels.
- **Feshbach molecules can be associated from pairs of separated atoms using magnetic-field sweeps.**



Adapted from: J. Herbig *et al.*, Science **301**, 1510 (2003)

Ultracold Feshbach molecules

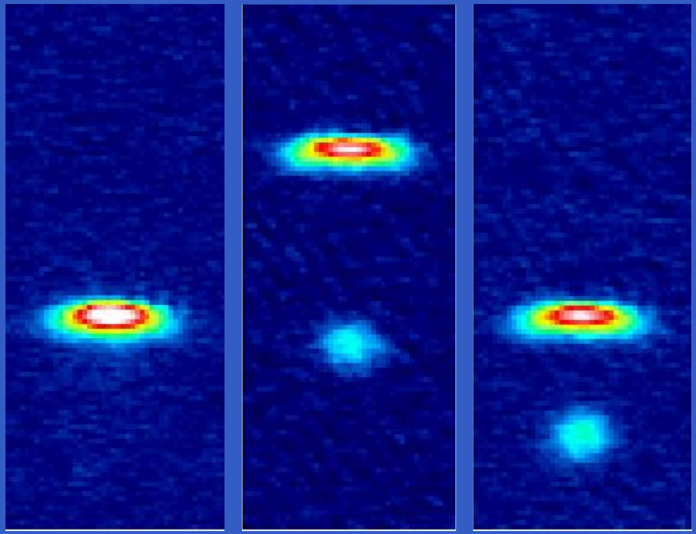
- A singularity of the scattering length indicates emergence of a new vibrational bound state at the collision threshold.
- Feshbach resonances can be strongly or weakly coupled to the continuum levels.
- **Feshbach molecules can be associated from pairs of separated atoms using magnetic-field sweeps.**



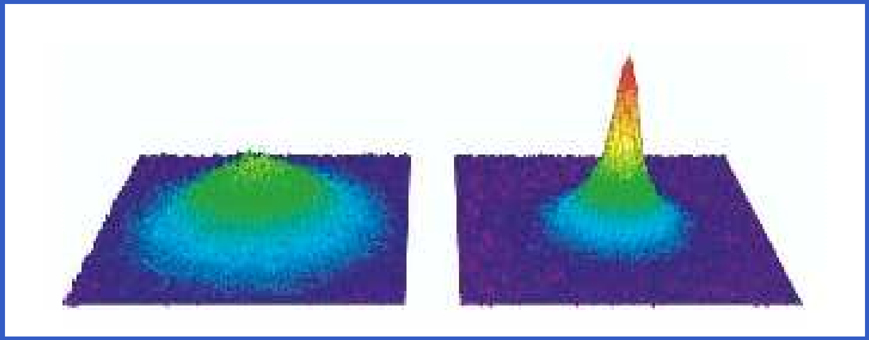
Adapted from: J. Herbig *et al.*, Science **301**, 1510 (2003)

Ultracold Feshbach molecules

- A singularity of the scattering length indicates emergence of a new vibrational bound state at the collision threshold.
- Feshbach resonances can be strongly or weakly coupled to the continuum levels.
- **Feshbach molecules can be associated from pairs of separated atoms using magnetic-field sweeps.**



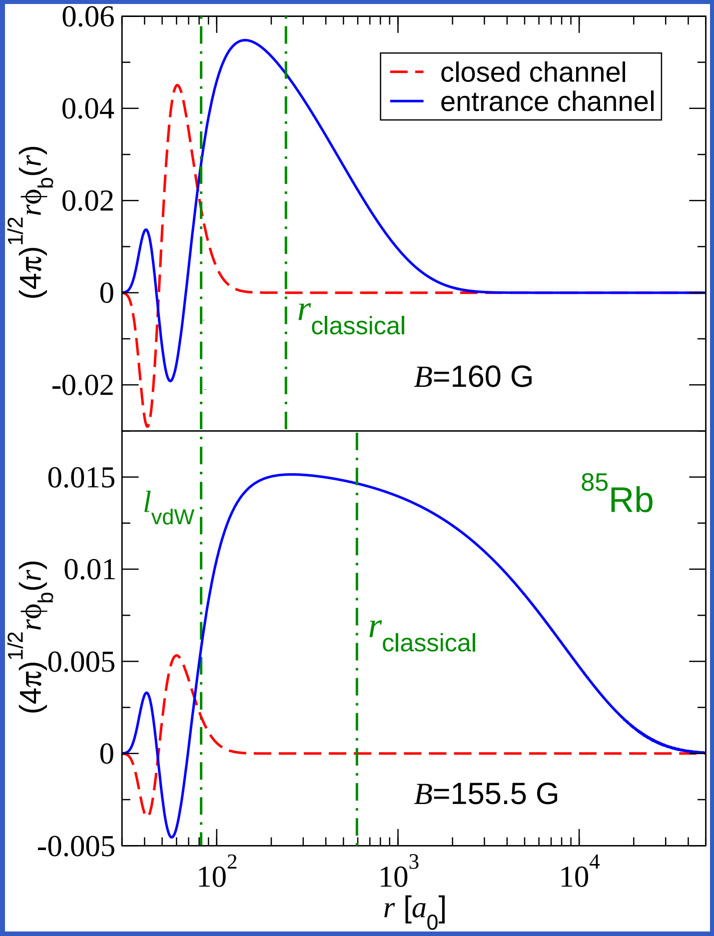
Adapted from: J. Herbig *et al.*, Science **301**, 1510 (2003)



Adapted from: M. Greiner *et al.*, Nature **426**, 537 (2003)

Ultracold Feshbach molecules

- A singularity of the scattering length indicates emergence of a new vibrational bound state at the collision threshold.
- Feshbach resonances can be strongly or weakly coupled to the continuum levels.
- Feshbach molecules can be associated from pairs of separated atoms using magnetic-field sweeps.
- **Near-resonant Feshbach molecules are halo states.**

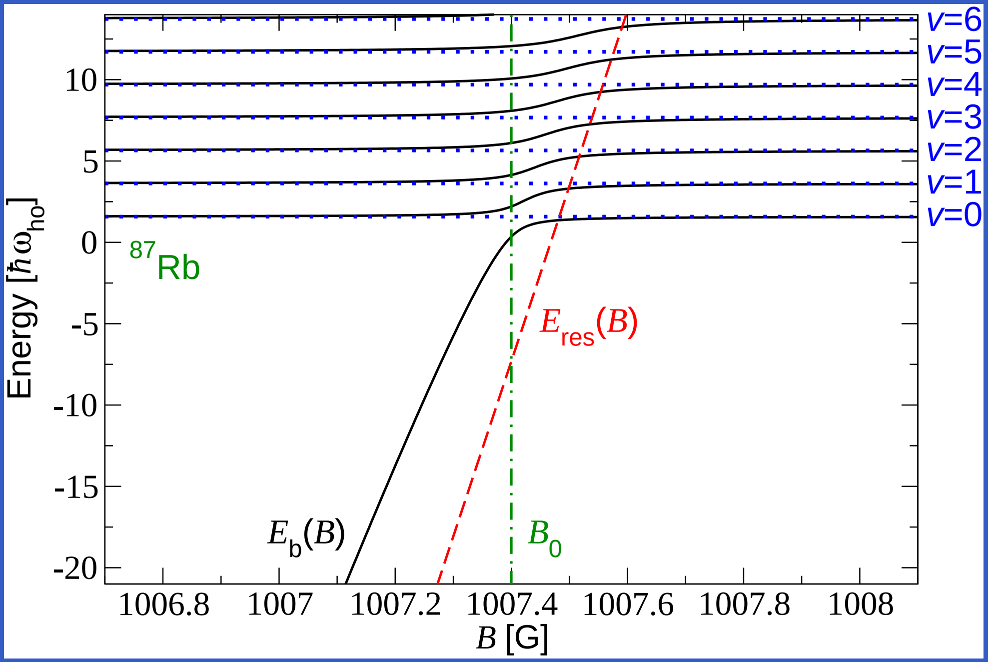


Diatomic dynamics

Association of two atoms in an optical-lattice site

- Magnetic-field variation:

$$B(t) = B_0 + \dot{B}(t - t_0)$$



Experiment: G. Thalhammer, K. Winkler, F. Lang, S. Schmid, R. Grimm, and J. Hecker-Denschlag, PRL **96**, 050402 (2006)

Diatomic dynamics

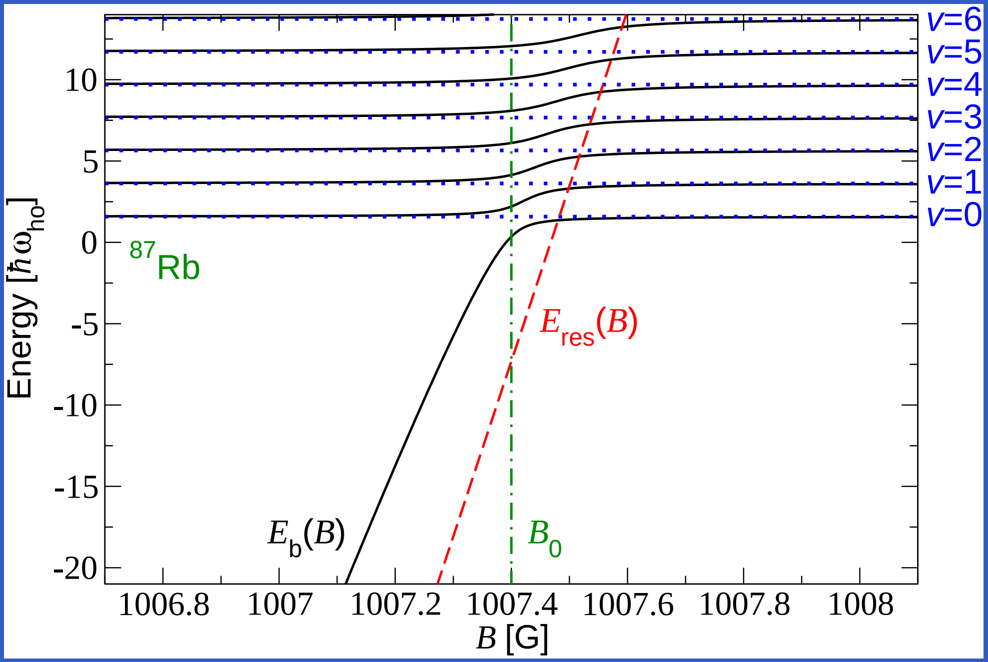
Association of two atoms in an optical-lattice site

- Magnetic-field variation:

$$B(t) = B_0 + \dot{B}(t - t_0)$$

- Time-evolution operator:

$$i\hbar\frac{\partial}{\partial t}U_{\mathrm{2B}}(t, t_{\mathrm{i}}) = H_{\mathrm{2B}}(t)U_{\mathrm{2B}}(t, t_{\mathrm{i}})$$



Experiment: G. Thalhammer, K. Winkler, F. Lang, S. Schmid, R. Grimm, and J. Hecker-Denschlag, PRL **96**, 050402 (2006)

Diatomic dynamics

Association of two atoms in an optical-lattice site

- Magnetic-field variation:

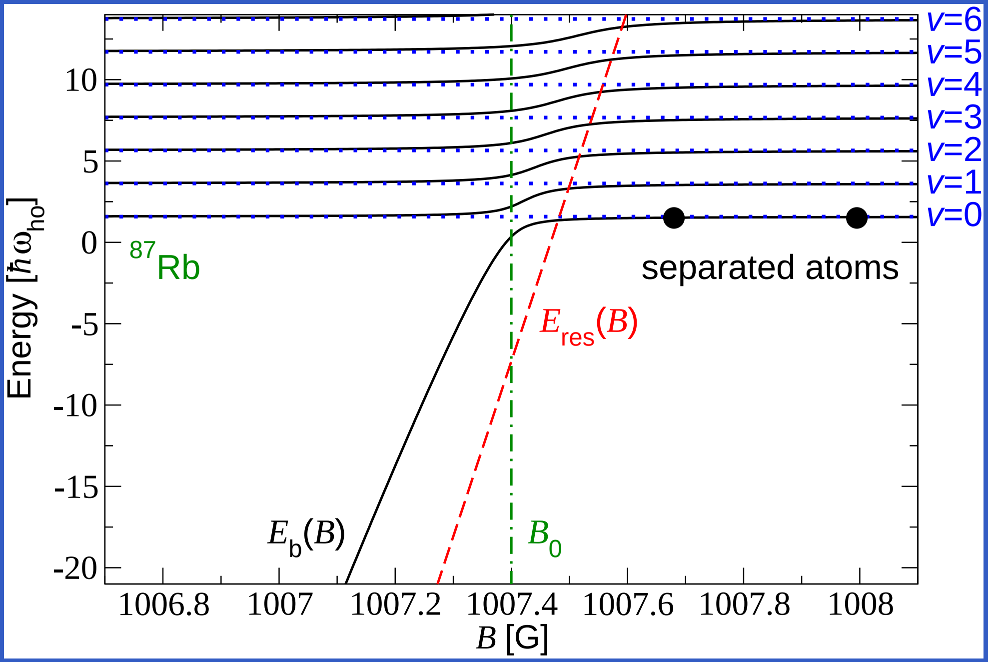
$$B(t) = B_0 + \dot{B}(t - t_0)$$

- Time-evolution operator:

$$i\hbar \frac{\partial}{\partial t} U_{2\mathrm{B}}(t, t_\mathrm{i}) = H_{2\mathrm{B}}(t) U_{2\mathrm{B}}(t, t_\mathrm{i})$$

- Transition probabilities:

$$p_{0,\mathrm{b}} = \left| \langle \phi^{\mathrm{ho}}_{\mathrm{b,f}} | U_{2\mathrm{B}}(t_\mathrm{f}, t_\mathrm{i}) | \phi^{\mathrm{ho}}_{0,\mathrm{i}} \rangle \right|^2$$



Experiment: G. Thalhammer, K. Winkler, F. Lang, S. Schmid, R. Grimm, and J. Hecker-Denschlag, PRL **96**, 050402 (2006)

Diatomic dynamics

Association of two atoms in an optical-lattice site

- Magnetic-field variation:

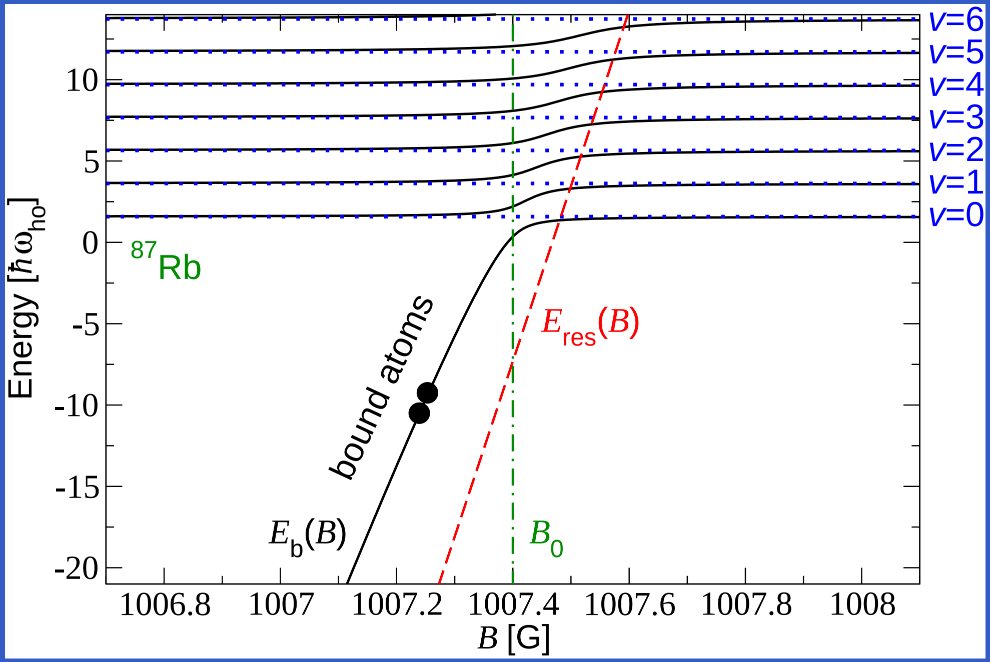
$$B(t) = B_0 + \dot{B}(t - t_0)$$

- Time-evolution operator:

$$i\hbar\frac{\partial}{\partial t}U_{2\mathrm{B}}(t,t_\mathrm{i}) = H_{2\mathrm{B}}(t)U_{2\mathrm{B}}(t,t_\mathrm{i})$$

- Transition probabilities:

$$p_{0,\mathrm{b}} = \left|\langle\phi^{\mathrm{ho}}_{\mathrm{b,f}}|U_{2\mathrm{B}}(t_\mathrm{f},t_\mathrm{i})|\phi^{\mathrm{ho}}_{0,\mathrm{i}}\rangle\right|^2$$



Experiment: G. Thalhammer, K. Winkler, F. Lang, S. Schmid, R. Grimm, and J. Hecker-Denschlag, PRL **96**, 050402 (2006)

Diatomic dynamics

Association of two atoms in an optical-lattice site

- Magnetic-field variation:

$$B(t) = B_0 + \dot{B}(t - t_0)$$

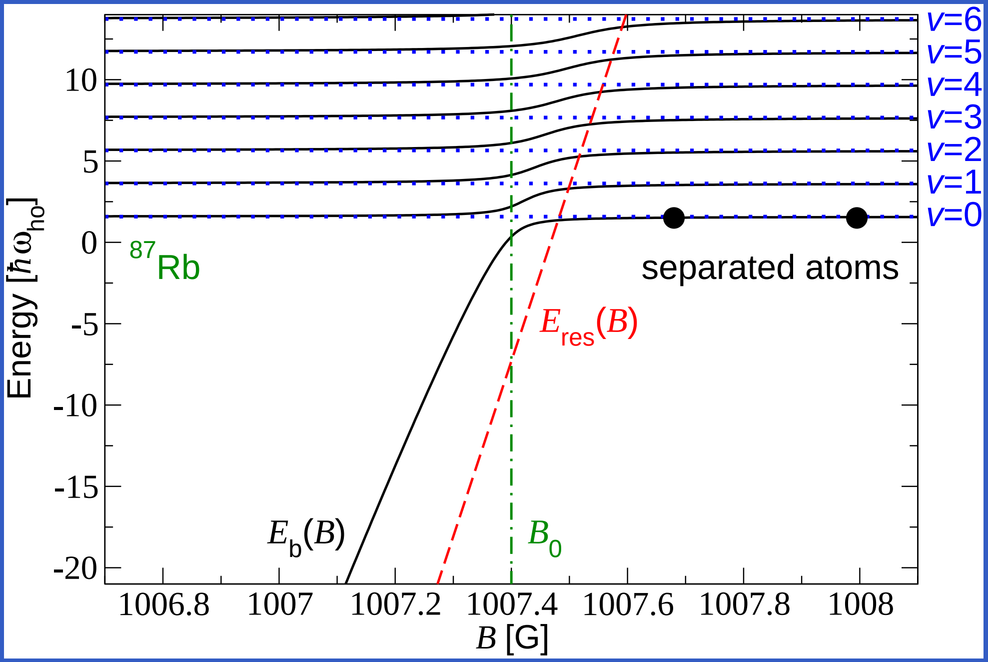
- Time-evolution operator:

$$i\hbar\frac{\partial}{\partial t}U_{2B}(t, t_i) = H_{2B}(t)U_{2B}(t, t_i)$$

- Transition probabilities:

$$p_{0,b} = \left|\langle\phi_{b,f}^{ho}|U_{2B}(t_f, t_i)|\phi_{0,i}^{ho}\rangle\right|^2$$

$$p_{0,0} = \left|\langle\phi_{0,f}^{ho}|U_{2B}(t_f, t_i)|\phi_{0,i}^{ho}\rangle\right|^2$$



Experiment: G. Thalhammer, K. Winkler, F. Lang, S. Schmid, R. Grimm, and J. Hecker-Denschlag, PRL **96**, 050402 (2006)

Diatomic dynamics

Association of two atoms in an optical-lattice site

- Magnetic-field variation:

$$B(t) = B_0 + \dot{B}(t - t_0)$$

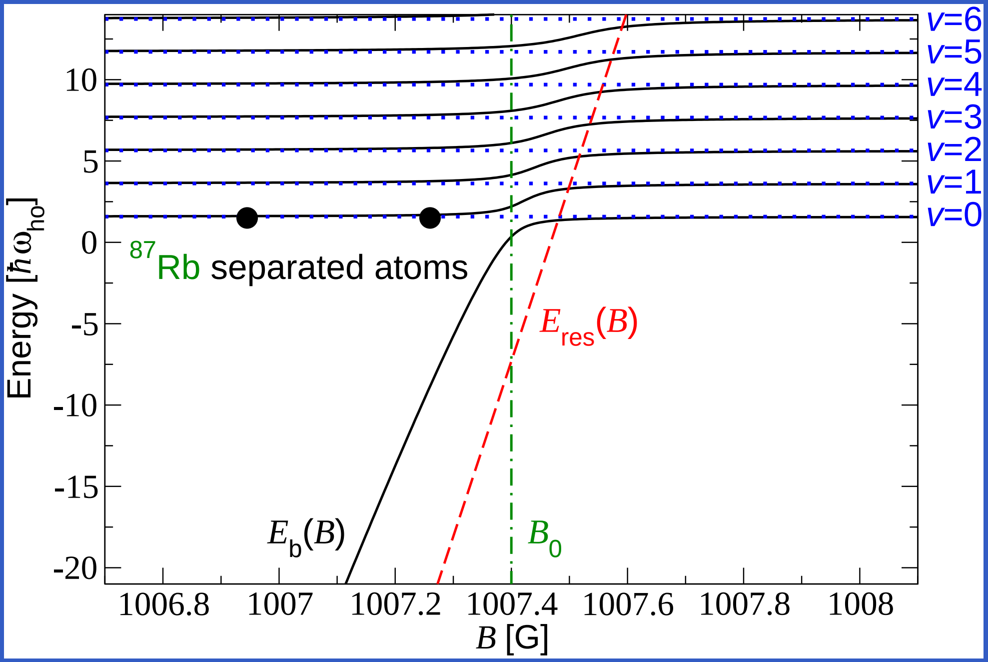
- Time-evolution operator:

$$i\hbar\frac{\partial}{\partial t}U_{2\mathrm{B}}(t,t_{\mathrm{i}}) = H_{2\mathrm{B}}(t)U_{2\mathrm{B}}(t,t_{\mathrm{i}})$$

- Transition probabilities:

$$p_{0,\mathrm{b}} = \left|\langle\phi^{\mathrm{ho}}_{\mathrm{b,f}}|U_{2\mathrm{B}}(t_{\mathrm{f}},t_{\mathrm{i}})|\phi^{\mathrm{ho}}_{0,\mathrm{i}}\rangle\right|^2$$

$$p_{0,0} = \left|\langle\phi^{\mathrm{ho}}_{0,\mathrm{f}}|U_{2\mathrm{B}}(t_{\mathrm{f}},t_{\mathrm{i}})|\phi^{\mathrm{ho}}_{0,\mathrm{i}}\rangle\right|^2$$



Experiment: G. Thalhammer, K. Winkler, F. Lang, S. Schmid, R. Grimm, and J. Hecker-Denschlag, PRL **96**, 050402 (2006)

Diatomic dynamics

Dissociation of two atoms in an optical-lattice site

- Magnetic-field variation:

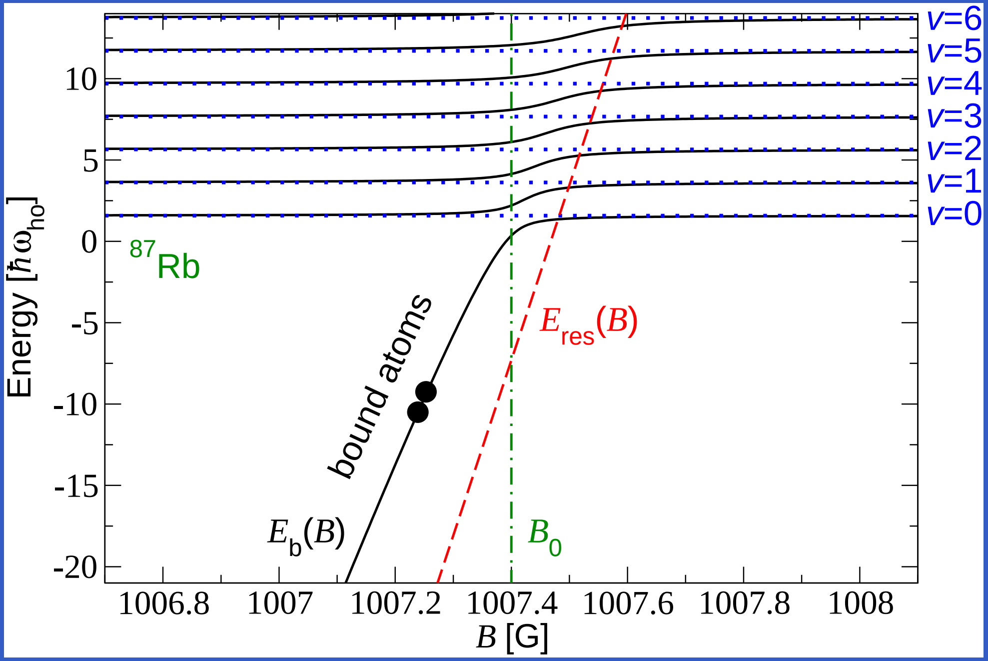
$$B(t) = B_0 + \dot{B}(t - t_0)$$

- Time-evolution operator:

$$i\hbar \frac{\partial}{\partial t} U_{2\mathrm{B}}(t, t_\mathrm{i}) = H_{2\mathrm{B}}(t) U_{2\mathrm{B}}(t, t_\mathrm{i})$$

- Transition probabilities:

$$p_{\mathrm{b},v} = \left| \langle \phi_{v,\mathrm{f}}^{\mathrm{ho}} | U_{2\mathrm{B}}(t_\mathrm{f}, t_\mathrm{i}) | \phi_{\mathrm{b,i}}^{\mathrm{ho}} \rangle \right|^2$$



Experiment: G. Thalhammer, K. Winkler, F. Lang, S. Schmid, R. Grimm, and J. Hecker-Denschlag, PRL **96**, 050402 (2006)

Diatomic dynamics

Dissociation of two atoms in an optical-lattice site

- Magnetic-field variation:

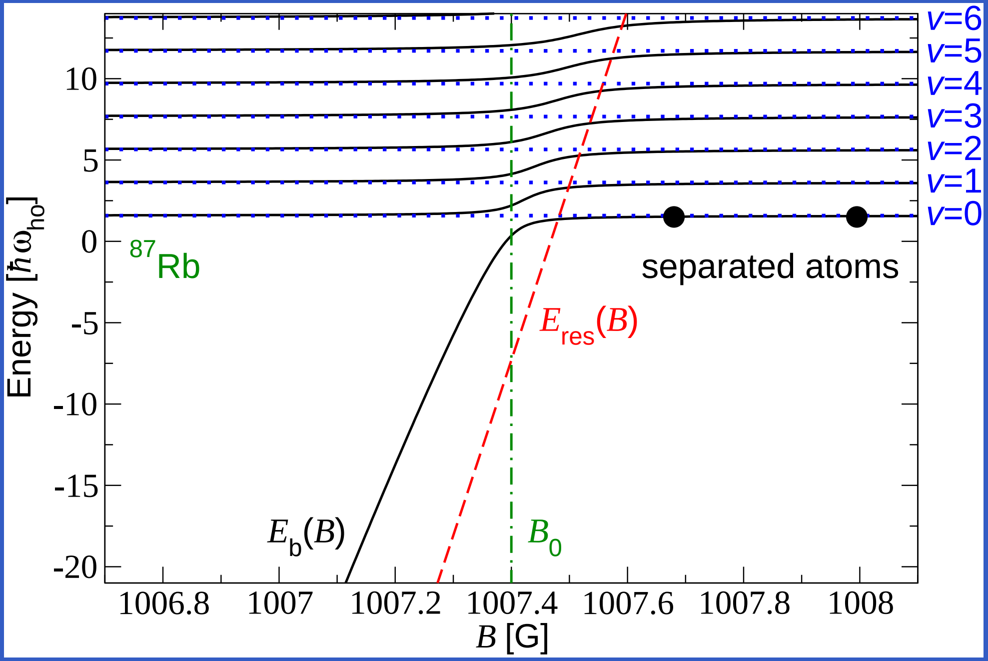
$$B(t) = B_0 + \dot{B}(t - t_0)$$

- Time-evolution operator:

$$i\hbar\frac{\partial}{\partial t}U_{2\mathrm{B}}(t,t_\mathrm{i}) = H_{2\mathrm{B}}(t)U_{2\mathrm{B}}(t,t_\mathrm{i})$$

- Transition probabilities:

$$p_{\mathrm{b},v} = \left|\langle\phi^{\mathrm{ho}}_{v,\mathrm{f}}|U_{2\mathrm{B}}(t_\mathrm{f},t_\mathrm{i})|\phi^{\mathrm{ho}}_{\mathrm{b,i}}\rangle\right|^2$$



Experiment: G. Thalhammer, K. Winkler, F. Lang, S. Schmid, R. Grimm, and J. Hecker-Denschlag, PRL **96**, 050402 (2006)

Diatomic dynamics

Dissociation of two atoms in an optical-lattice site

- Magnetic-field variation:

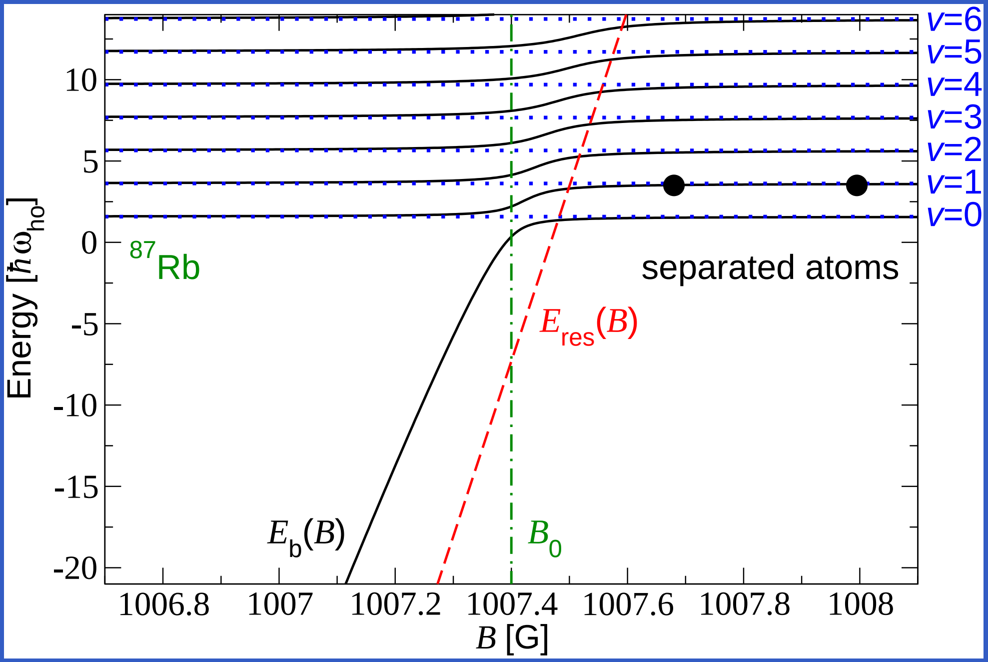
$$B(t) = B_0 + \dot{B}(t - t_0)$$

- Time-evolution operator:

$$i\hbar\frac{\partial}{\partial t}U_{2\mathrm{B}}(t,t_{\mathrm{i}}) = H_{2\mathrm{B}}(t)U_{2\mathrm{B}}(t,t_{\mathrm{i}})$$

- Transition probabilities:

$$p_{\mathrm{b},v} = \left|\langle\phi_{v,\mathrm{f}}^{\mathrm{ho}}|U_{2\mathrm{B}}(t_{\mathrm{f}},t_{\mathrm{i}})|\phi_{\mathrm{b,i}}^{\mathrm{ho}}\rangle\right|^2$$



Experiment: G. Thalhammer, K. Winkler, F. Lang, S. Schmid, R. Grimm, and J. Hecker-Denschlag, PRL **96**, 050402 (2006)

Diatomic dynamics

Dissociation of two atoms in an optical-lattice site

- Magnetic-field variation:

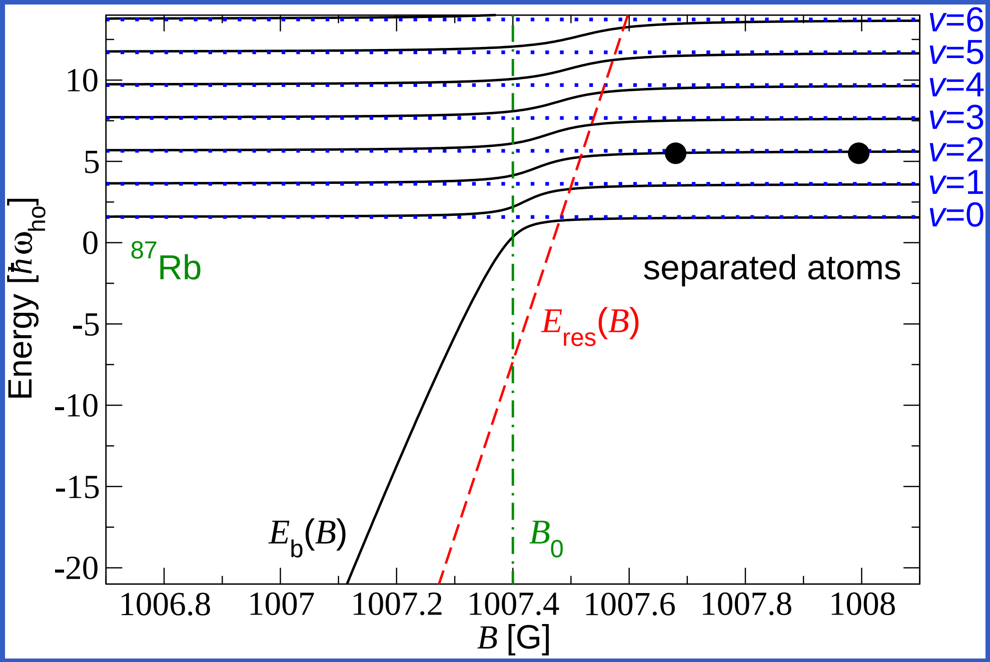
$$B(t) = B_0 + \dot{B}(t - t_0)$$

- Time-evolution operator:

$$i\hbar \frac{\partial}{\partial t} U_{2\mathrm{B}}(t, t_{\mathrm{i}}) = H_{2\mathrm{B}}(t) U_{2\mathrm{B}}(t, t_{\mathrm{i}})$$

- Transition probabilities:

$$p_{\mathrm{b},v} = \left| \langle \phi_{v,\mathrm{f}}^{\mathrm{ho}} | U_{2\mathrm{B}}(t_{\mathrm{f}}, t_{\mathrm{i}}) | \phi_{\mathrm{b,i}}^{\mathrm{ho}} \rangle \right|^2$$



Experiment: G. Thalhammer, K. Winkler, F. Lang, S. Schmid, R. Grimm, and J. Hecker-Denschlag, PRL **96**, 050402 (2006)

Diatomic dynamics

Dissociation of two atoms in an optical-lattice site

- Magnetic-field variation:

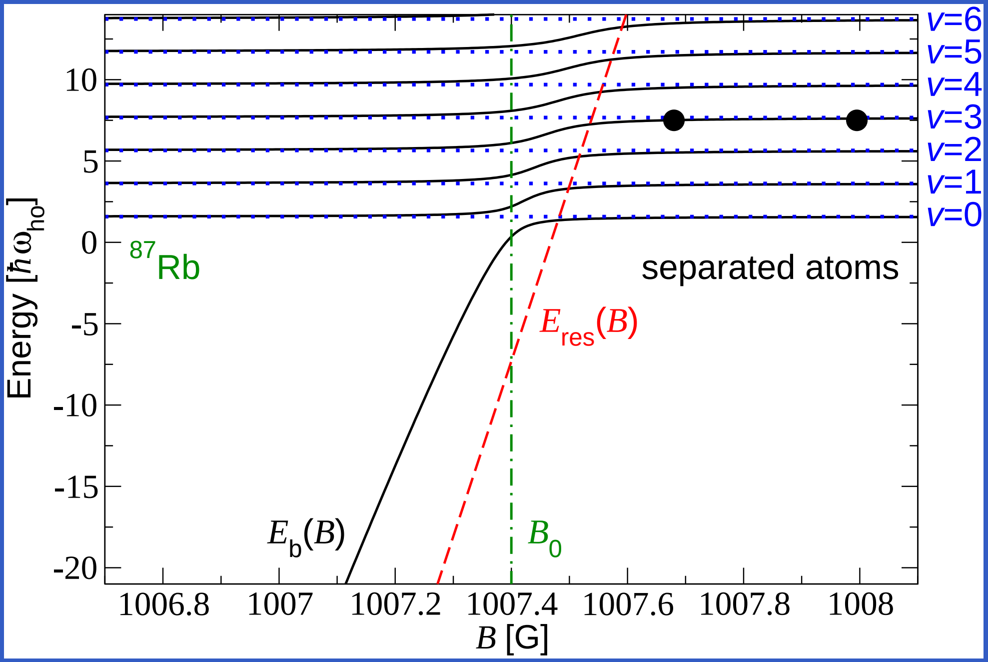
$$B(t) = B_0 + \dot{B}(t - t_0)$$

- Time-evolution operator:

$$i\hbar \frac{\partial}{\partial t} U_{2\mathrm{B}}(t, t_\mathrm{i}) = H_{2\mathrm{B}}(t) U_{2\mathrm{B}}(t, t_\mathrm{i})$$

- Transition probabilities:

$$p_{\mathrm{b},v} = \left| \langle \phi_{v,\mathrm{f}}^{\mathrm{ho}} | U_{2\mathrm{B}}(t_\mathrm{f}, t_\mathrm{i}) | \phi_{\mathrm{b,i}}^{\mathrm{ho}} \rangle \right|^2$$



Experiment: G. Thalhammer, K. Winkler, F. Lang, S. Schmid, R. Grimm, and J. Hecker-Denschlag, PRL **96**, 050402 (2006)

Diatomic dynamics

Dissociation of two atoms in an optical-lattice site

- Magnetic-field variation:

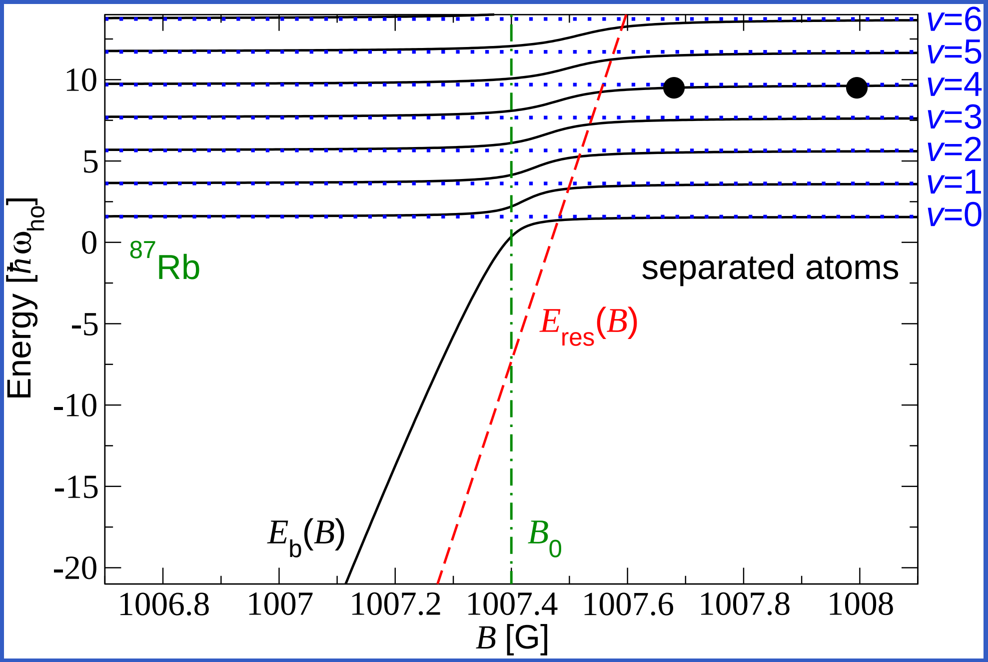
$$B(t) = B_0 + \dot{B}(t - t_0)$$

- Time-evolution operator:

$$i\hbar\frac{\partial}{\partial t}U_{2\mathrm{B}}(t, t_{\mathrm{i}}) = H_{2\mathrm{B}}(t)U_{2\mathrm{B}}(t, t_{\mathrm{i}})$$

- Transition probabilities:

$$p_{\mathrm{b},v} = \left|\langle\phi^{\mathrm{ho}}_{v,\mathrm{f}}|U_{2\mathrm{B}}(t_{\mathrm{f}}, t_{\mathrm{i}})|\phi^{\mathrm{ho}}_{\mathrm{b,i}}\rangle\right|^2$$



Experiment: G. Thalhammer, K. Winkler, F. Lang, S. Schmid, R. Grimm, and J. Hecker-Denschlag, PRL **96**, 050402 (2006)

Diatomic dynamics

Dissociation of two atoms in an optical-lattice site

- Magnetic-field variation:

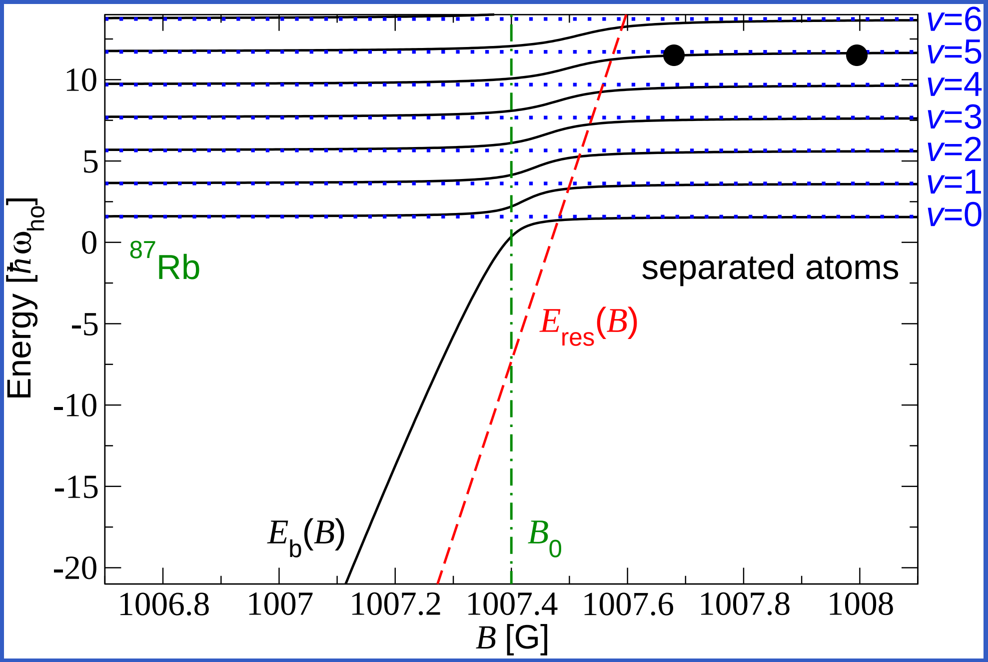
$$B(t) = B_0 + \dot{B}(t - t_0)$$

- Time-evolution operator:

$$i\hbar \frac{\partial}{\partial t} U_{2\mathrm{B}}(t, t_{\mathrm{i}}) = H_{2\mathrm{B}}(t) U_{2\mathrm{B}}(t, t_{\mathrm{i}})$$

- Transition probabilities:

$$p_{\mathrm{b},v} = \left| \langle \phi^{\mathrm{ho}}_{v,\mathrm{f}} | U_{2\mathrm{B}}(t_{\mathrm{f}}, t_{\mathrm{i}}) | \phi^{\mathrm{ho}}_{\mathrm{b,i}} \rangle \right|^2$$



Experiment: G. Thalhammer, K. Winkler, F. Lang, S. Schmid, R. Grimm, and J. Hecker-Denschlag, PRL **96**, 050402 (2006)

Diatomic dynamics

Dissociation of two atoms in an optical-lattice site

- Magnetic-field variation:

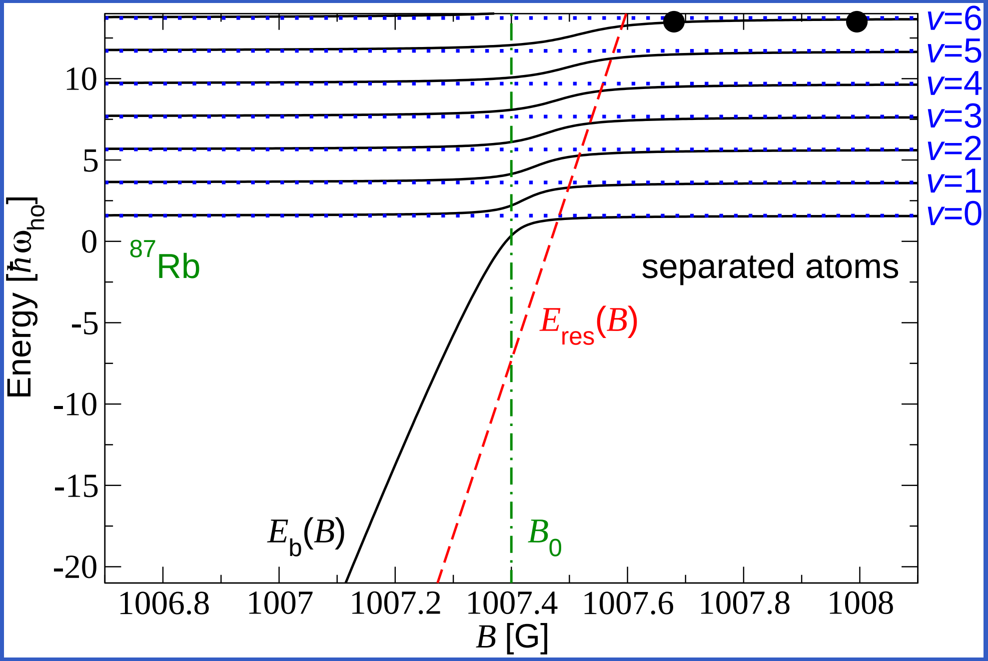
$$B(t) = B_0 + \dot{B}(t - t_0)$$

- Time-evolution operator:

$$i\hbar\frac{\partial}{\partial t}U_{2\mathrm{B}}(t, t_{\mathrm{i}}) = H_{2\mathrm{B}}(t)U_{2\mathrm{B}}(t, t_{\mathrm{i}})$$

- Transition probabilities:

$$p_{\mathrm{b},v} = \left|\langle\phi^{\mathrm{ho}}_{v,\mathrm{f}}|U_{2\mathrm{B}}(t_{\mathrm{f}}, t_{\mathrm{i}})|\phi^{\mathrm{ho}}_{\mathrm{b,i}}\rangle\right|^2$$



Experiment: G. Thalhammer, K. Winkler, F. Lang, S. Schmid, R. Grimm, and J. Hecker-Denschlag, PRL **96**, 050402 (2006)

Diatomic dynamics

Transition probabilities for harmonic confinement

- Landau-Zener formulae:

$$\lim_{\substack{t_{\mathrm{i}}\to-\infty\\ t_{\mathrm{f}}\to\infty}} p_{0,0} = e^{-2\pi\delta_{\mathrm{LZ}}}$$

L.D. Landau, Phys. Z. Sowjetunion **2**, 46 (1932)
C. Zener, Proc. R. Soc. London, Ser. A **137**, 696 (1932)

Diatomic dynamics

Transition probabilities for harmonic confinement

- Landau-Zener formulae:

$$\lim_{\substack{t_\mathrm{i}\to-\infty\\ t_\mathrm{f}\to\infty}} p_{0,0} = e^{-2\pi\delta_\mathrm{LZ}}$$

$$\lim_{\substack{t_\mathrm{i}\to-\infty\\ t_\mathrm{f}\to\infty}} p_{0,\mathrm{b}} = 1 - e^{-2\pi\delta_\mathrm{LZ}}$$

L.D. Landau, Phys. Z. Sowjetunion **2**, 46 (1932)
C. Zener, Proc. R. Soc. London, Ser. A **137**, 696 (1932)

Diatomic dynamics

Transition probabilities for harmonic confinement

- Landau-Zener formulae:

$$\lim_{\substack{t_{\mathrm{i}} \to -\infty \\ t_{\mathrm{f}} \to \infty}} p_{0,0} = e^{-2\pi\delta_{\mathrm{LZ}}}$$

$$\lim_{\substack{t_{\mathrm{i}} \to -\infty \\ t_{\mathrm{f}} \to \infty}} p_{0,\mathrm{b}} = 1 - e^{-2\pi\delta_{\mathrm{LZ}}}$$

$$\lim_{\substack{t_{\mathrm{i}} \to -\infty \\ t_{\mathrm{f}} \to \infty}} p_{\mathrm{b},0} = \lim_{\substack{t_{\mathrm{i}} \to -\infty \\ t_{\mathrm{f}} \to \infty}} p_{0,\mathrm{b}}$$

Y.N. Demkov and V.I. Osherov, Sov. Phys. JETP **26**, 916 (1968)
F.H. Mies, E. Tiesinga, and P.S. Julienne, PRA **61**, 022721 (2000)

Diatomic dynamics

Transition probabilities for harmonic confinement

- Landau-Zener formulae:

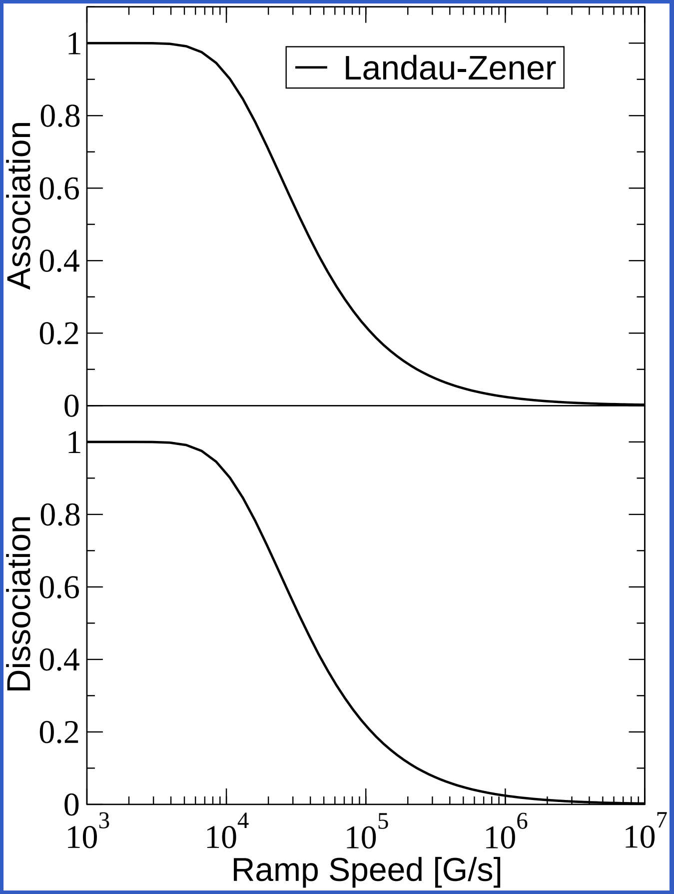
$$\lim_{\substack{t_\mathrm{i}\to-\infty\\ t_\mathrm{f}\to\infty}} p_{0,0} = e^{-2\pi\delta_\mathrm{LZ}}$$

$$\lim_{\substack{t_\mathrm{i}\to-\infty\\ t_\mathrm{f}\to\infty}} p_{0,\mathrm{b}} = 1 - e^{-2\pi\delta_\mathrm{LZ}}$$

$$\lim_{\substack{t_\mathrm{i}\to-\infty\\ t_\mathrm{f}\to\infty}} p_{\mathrm{b},0} = \lim_{\substack{t_\mathrm{i}\to-\infty\\ t_\mathrm{f}\to\infty}} p_{0,\mathrm{b}}$$

- Landau-Zener coefficient ($a_\mathrm{ho} = [\hbar/(m\omega_\mathrm{ho})]^{1/2}$):

$$\delta_\mathrm{LZ}^\mathrm{ho} = \frac{\sqrt{6}\hbar}{\pi m a_\mathrm{ho}^3}\left|\frac{a_\mathrm{bg}\Delta B}{\dot{B}}\right|$$



This Landau-Zener method: P.S. Julienne, E. Tiesinga, and TK, J. Mod. Opt. **51**, 1787 (2004)

Diatomic dynamics

Transition probabilities for harmonic confinement

- Landau-Zener formulae:

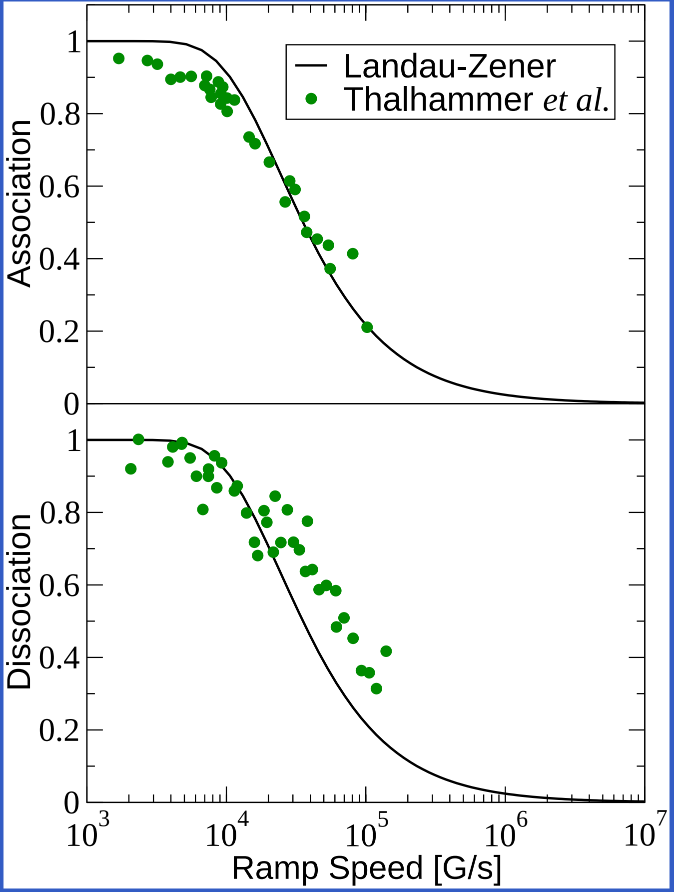
$$\lim_{\substack{t_{\rm i}\to-\infty \\ t_{\rm f}\to\infty}} p_{0,0} = e^{-2\pi\delta_{\rm LZ}}$$

$$\lim_{\substack{t_{\rm i}\to-\infty \\ t_{\rm f}\to\infty}} p_{0,\rm b} = 1 - e^{-2\pi\delta_{\rm LZ}}$$

$$\lim_{\substack{t_{\rm i}\to-\infty \\ t_{\rm f}\to\infty}} p_{\rm b,0} = \lim_{\substack{t_{\rm i}\to-\infty \\ t_{\rm f}\to\infty}} p_{0,\rm b}$$

- Landau-Zener coefficient ($a_{\rm ho} = [\hbar/(m\omega_{\rm ho})]^{1/2}$):

$$\delta^{\rm ho}_{\rm LZ} = \frac{\sqrt{6}\hbar}{\pi m a^3_{\rm ho}} \left| \frac{a_{\rm bg}\Delta B}{\dot{B}} \right|$$



This Landau-Zener method: P.S. Julienne, E. Tiesinga, and TK, J. Mod. Opt. **51**, 1787 (2004)
Experiment: G. Thalhammer, K. Winkler, F. Lang, S. Schmid, R. Grimm, and J. Hecker-Denschlag, PRL **96**, 050402 (2006)

Diatomic dynamics

Dissociation-energy spectrum

- Closed-channel dominance (^{87}Rb at $B_0 = 685\,$G):

$$\eta = \frac{E_{\mathrm{vdW}}}{\Gamma_{\mathrm{res}}(E_{\mathrm{vdW}})} = 463$$

This experiment: S. Dürr, T. Volz, and G. Rempe, PRA **70**, 031601 (2004)

Diatomic dynamics

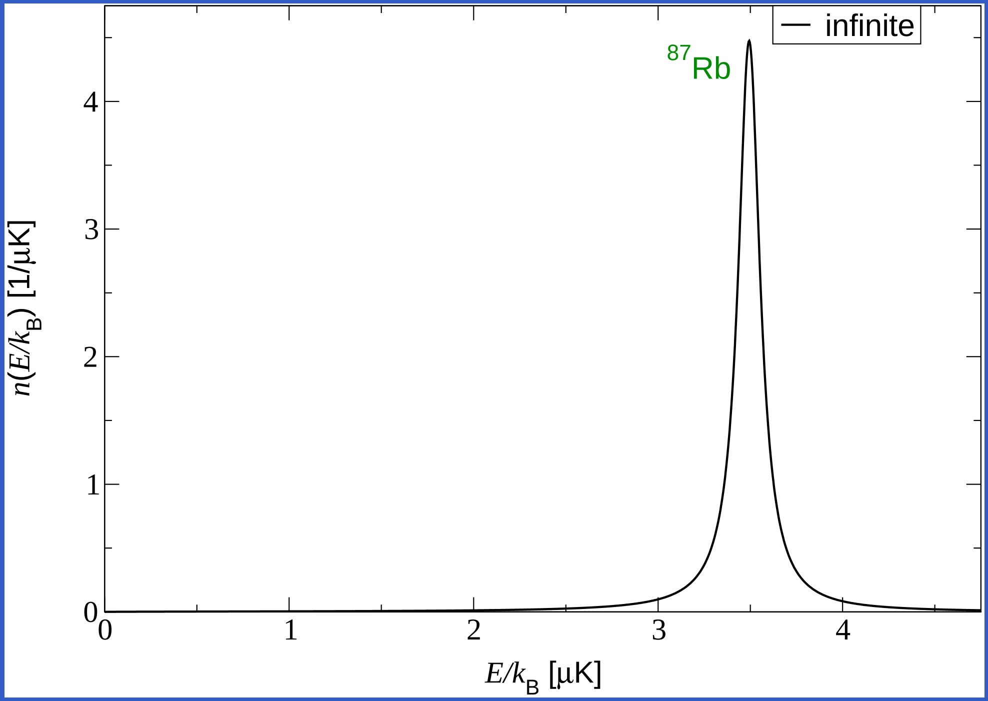
Dissociation-energy spectrum

- Closed-channel dominance (^{87}Rb at $B_0 = 685\,$G):

$$\eta = \frac{E_{\mathrm{vdW}}}{\Gamma_{\mathrm{res}}(E_{\mathrm{vdW}})} = 463$$

- Energy spectrum for an instantaneous switch from B_{i} to B_{f} across B_0:

$$n(E)dE = \int d\Omega \, |\langle\phi_{\mathbf{p},\mathrm{f}}|\phi_{\mathrm{b,i}}\rangle|^2 \, p^2 dp$$



This experiment: S. Dürr, T. Volz, and G. Rempe, PRA **70**, 031601 (2004)
This calculation: T.M. Hanna, K. Góral, E. Witkowska, and TK, PRA **74**, 023618 (2006)

Diatomic dynamics

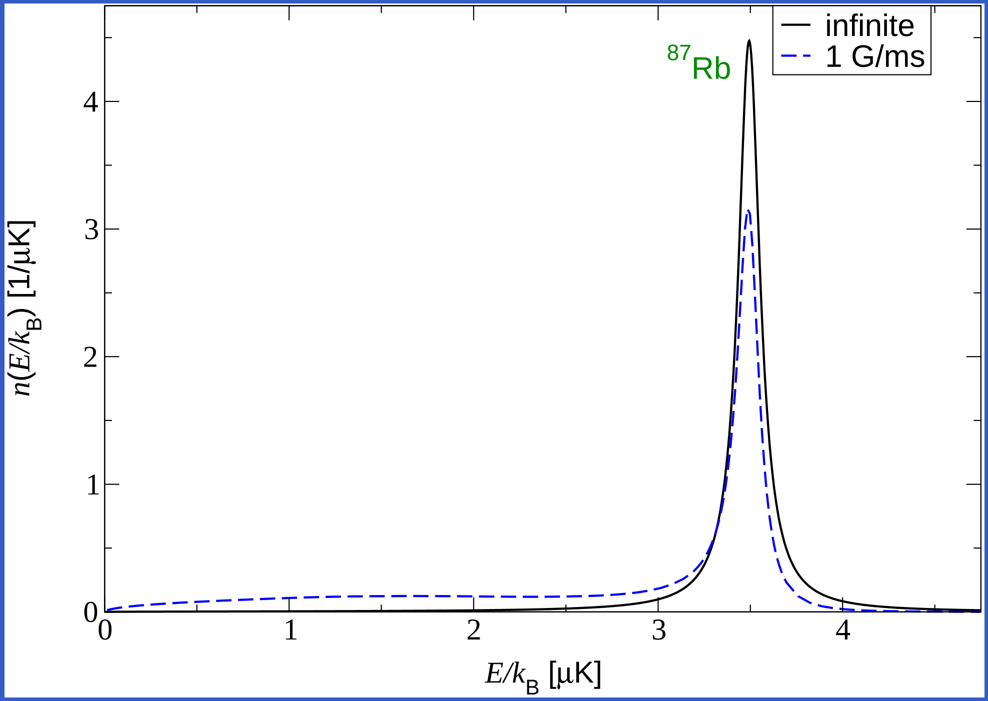
Dissociation-energy spectrum

- Closed-channel dominance (^{87}Rb at $B_0 = 685\,$G):

$$\eta = \frac{E_{\mathrm{vdW}}}{\Gamma_{\mathrm{res}}(E_{\mathrm{vdW}})} = 463$$

- Energy spectrum for an instantaneous switch from B_{i} to B_{f} across B_0:

$$n(E)dE = \int d\Omega \, |\langle \phi_{\mathbf{p},\mathrm{f}} | \phi_{\mathrm{b,i}} \rangle|^2 \, p^2 dp$$



This experiment: S. Dürr, T. Volz, and G. Rempe, PRA **70**, 031601 (2004)
This calculation: T.M. Hanna, K. Góral, E. Witkowska, and TK, PRA **74**, 023618 (2006)

Diatomic dynamics

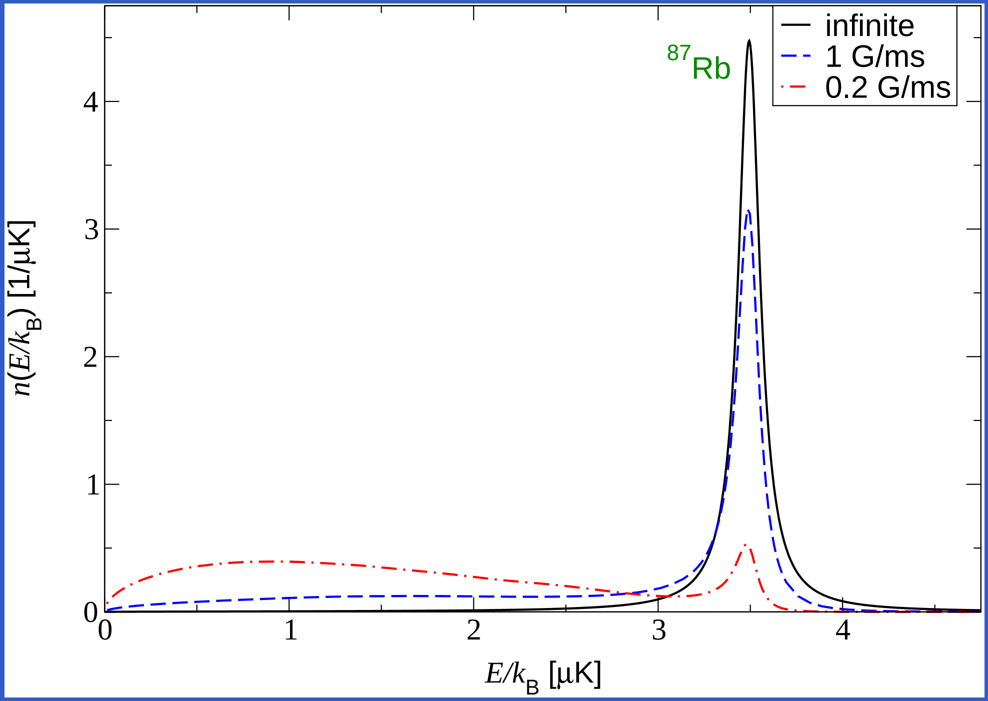
Dissociation-energy spectrum

- Closed-channel dominance (^{87}Rb at $B_0 = 685\,$G):

$$\eta = \frac{E_{\mathrm{vdW}}}{\Gamma_{\mathrm{res}}(E_{\mathrm{vdW}})} = 463$$

- Energy spectrum for an instantaneous switch from B_{i} to B_{f} across B_0:

$$n(E)dE = \int d\Omega \, |\langle \phi_{\mathbf{p},\mathrm{f}} | \phi_{\mathrm{b,i}} \rangle|^2 \, p^2 dp$$



This experiment: S. Dürr, T. Volz, and G. Rempe, PRA **70**, 031601 (2004)
This calculation: T.M. Hanna, K. Góral, E. Witkowska, and TK, PRA **74**, 023618 (2006)

Diatomic dynamics

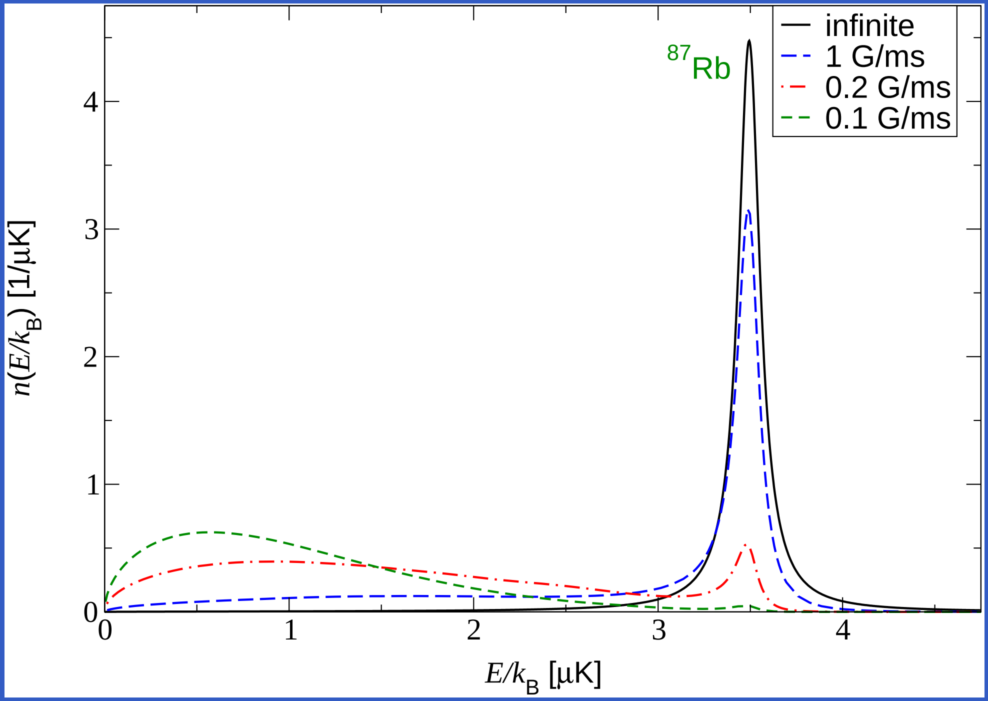
Dissociation-energy spectrum

- Closed-channel dominance (^{87}Rb at $B_0 = 685\,$G):

$$\eta = \frac{E_{\rm vdW}}{\Gamma_{\rm res}(E_{\rm vdW})} = 463$$

- Energy spectrum for an instantaneous switch from $B_{\rm i}$ to $B_{\rm f}$ across B_0:

$$n(E)dE = \int d\Omega \left|\langle\phi_{\mathbf{p},\rm f}|\phi_{\rm b,i}\rangle\right|^2 p^2 dp$$



This experiment: S. Dürr, T. Volz, and G. Rempe, PRA **70**, 031601 (2004)
This calculation: T.M. Hanna, K. Góral, E. Witkowska, and TK, PRA **74**, 023618 (2006)

Diatomic dynamics

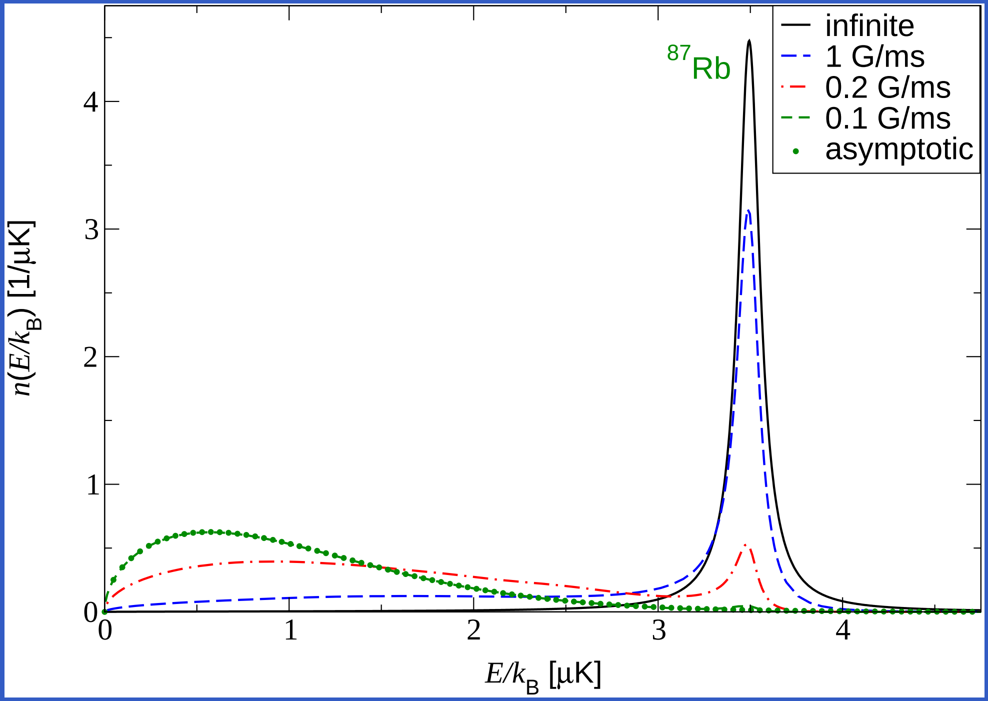
Dissociation-energy spectrum

- Closed-channel dominance (^{87}Rb at $B_0 = 685\,$G):

$$\eta = \frac{E_{\mathrm{vdW}}}{\Gamma_{\mathrm{res}}(E_{\mathrm{vdW}})} = 463$$

- Energy spectrum for an instantaneous switch from B_i to B_f across B_0:

$$n(E)dE = \int d\Omega\, |\langle \phi_{\mathbf{p},\mathrm{f}} | \phi_{\mathrm{b,i}} \rangle|^2\, p^2 dp$$



This experiment: S. Dürr, T. Volz, and G. Rempe, PRA **70**, 031601 (2004)
This calculation: T.M. Hanna, K. Góral, E. Witkowska, and TK, PRA **74**, 023618 (2006)

Diatomic dynamics

Dissociation-energy spectrum

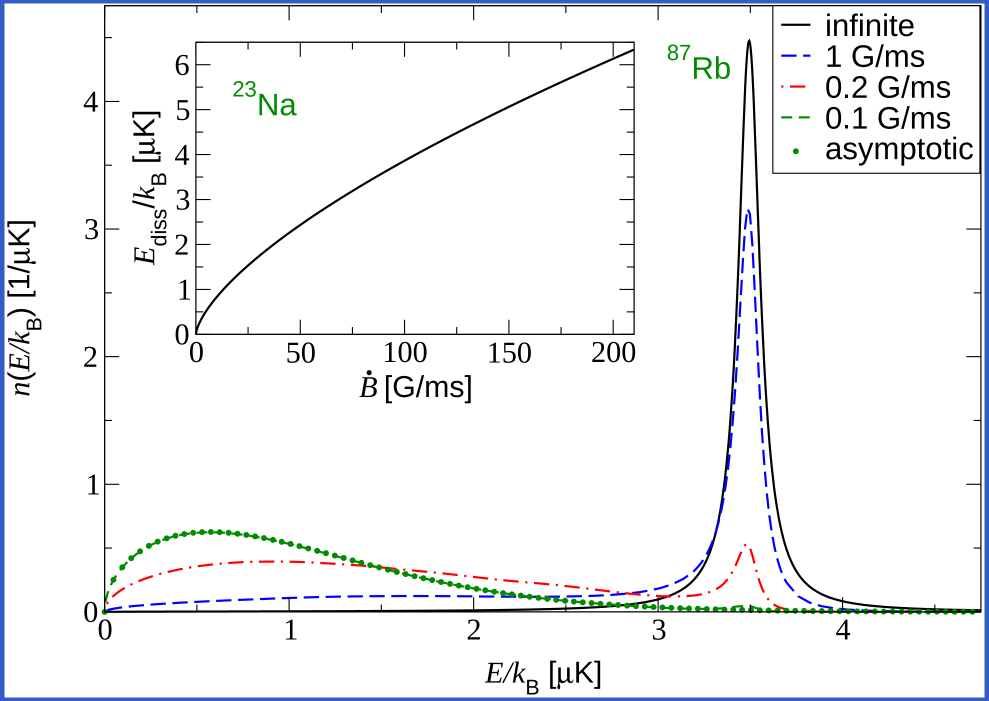
- Closed-channel dominance (^{87}Rb at $B_0 = 685\,$G):

$$\eta = \frac{E_{\rm vdW}}{\Gamma_{\rm res}(E_{\rm vdW})} = 463$$

- Energy spectrum for an instantaneous switch from $B_{\rm i}$ to $B_{\rm f}$ across B_0:

$$n(E)dE = \int d\Omega\,|\langle\phi_{\mathbf{p},\rm f}|\phi_{\rm b,i}\rangle|^2\,p^2dp$$

- Asymptotic energy spectrum:



$$n(E) = -\frac{\partial}{\partial E}\exp\left(-\frac{4}{3}\sqrt{\frac{mE}{\hbar^2}}\frac{|a_{\rm bg}\Delta B|E}{\hbar|\dot{B}|}\right)$$

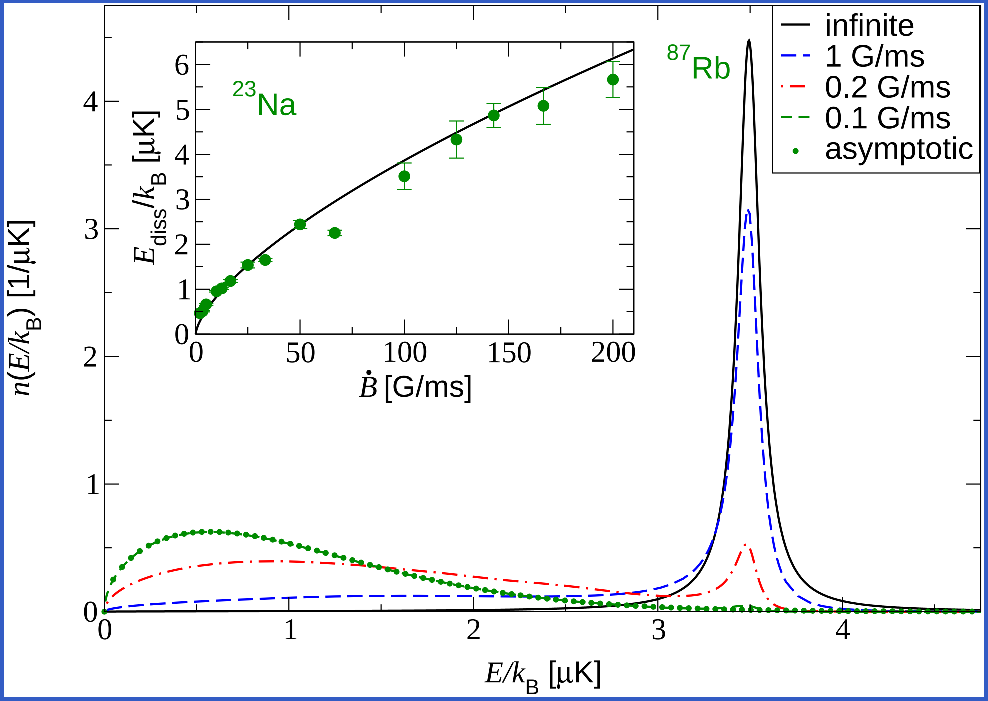
T. Mukaiyama, J.R. Abo-Shaeer, K. Xu, J.K. Chin, and W. Ketterle, PRL **92**, 180402 (2004)
P.S. Julienne (unpublished)

Diatomic dynamics

Dissociation-energy spectrum

- Closed-channel dominance (^{87}Rb at $B_0 = 685\,$G):

$$\eta = \frac{E_{\rm vdW}}{\Gamma_{\rm res}(E_{\rm vdW})} = 463$$



- Energy spectrum for an instantaneous switch from $B_{\rm i}$ to $B_{\rm f}$ across B_0:

$$n(E)dE = \int d\Omega\, |\langle \phi_{\mathbf{p},\rm f}|\phi_{\rm b,i}\rangle|^2\, p^2 dp$$

- Asymptotic energy spectrum:

$$n(E) = -\frac{\partial}{\partial E} \exp\left(-\frac{4}{3}\sqrt{\frac{mE}{\hbar^2}}\,\frac{|a_{\rm bg}\Delta B|E}{\hbar|\dot{B}|}\right)$$

T. Mukaiyama, J.R. Abo-Shaeer, K. Xu, J.K. Chin, and W. Ketterle, PRL **92**, 180402 (2004)

Diatomic dynamics

Association of atom pairs in a periodic box

- Transition probability:

$$p_{0,\mathrm{b}} = \frac{(2\pi\hbar)^3}{\mathcal{V}} \left| \langle \phi_{\mathrm{b,f}} | U_{\mathrm{2B}}(t_{\mathrm{f}}, t_{\mathrm{i}}) | 0 \rangle \right|^2$$

Diatomic dynamics

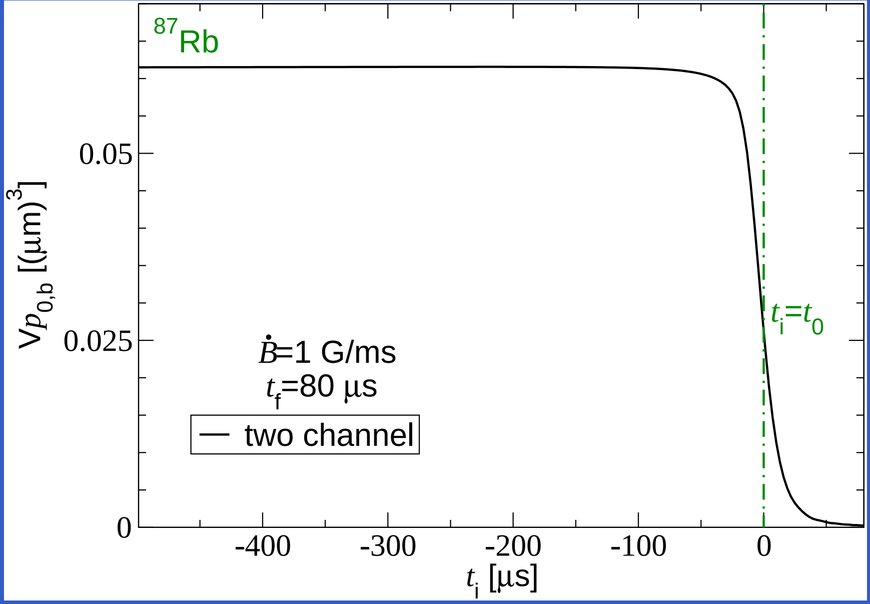
Association of atom pairs in a periodic box

- Transition probability:

$$p_{0,\mathrm{b}} = \frac{(2\pi\hbar)^3}{\mathcal{V}} \left|\langle\phi_{\mathrm{b,f}}|U_{2\mathrm{B}}(t_\mathrm{f}, t_\mathrm{i})|0\rangle\right|^2$$

- Two-channel approach:

$$E_\mathrm{res}(t) = \mu_\mathrm{res}[B(t) - B_\mathrm{res}]$$



This calculation: K. Góral, TK, S.A. Gardiner, E. Tiesinga, and P.S. Julienne, J. Phys. B **37**, 3457 (2004)

Diatomic dynamics

Association of atom pairs in a periodic box

- Transition probability:

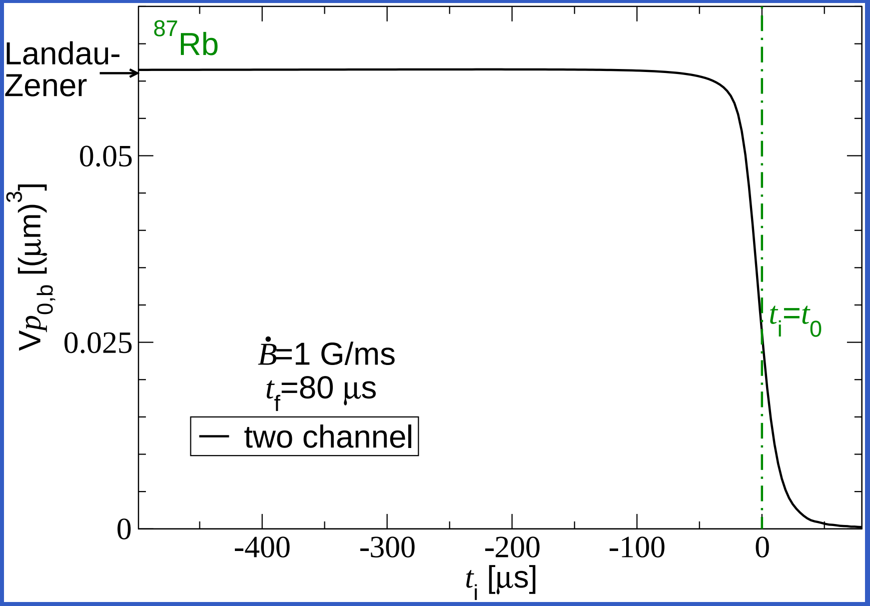
$$p_{0,\mathrm{b}} = \frac{(2\pi\hbar)^3}{\mathcal{V}} \left|\langle\phi_{\mathrm{b,f}}|U_{2\mathrm{B}}(t_\mathrm{f}, t_\mathrm{i})|0\rangle\right|^2$$

- Two-channel approach:

$$E_\mathrm{res}(t) = \mu_\mathrm{res}[B(t) - B_\mathrm{res}]$$

- Landau-Zener parameter:

$$\delta_\mathrm{LZ}^\mathrm{box} = \frac{4\pi\hbar}{m\mathcal{V}} \left|\frac{a_\mathrm{bg}\Delta B}{\dot{B}}\right|$$



This calculation: K. Góral, TK, S.A. Gardiner, E. Tiesinga, and P.S. Julienne, J. Phys. B **37**, 3457 (2004)

Diatomic dynamics

Association of atom pairs in a periodic box

- Transition probability:

$$p_{0,\mathrm{b}} = \frac{(2\pi\hbar)^3}{\mathcal{V}} \left|\langle\phi_{\mathrm{b,f}}|U_{2\mathrm{B}}(t_\mathrm{f},t_\mathrm{i})|0\rangle\right|^2$$

- Two-channel approach:

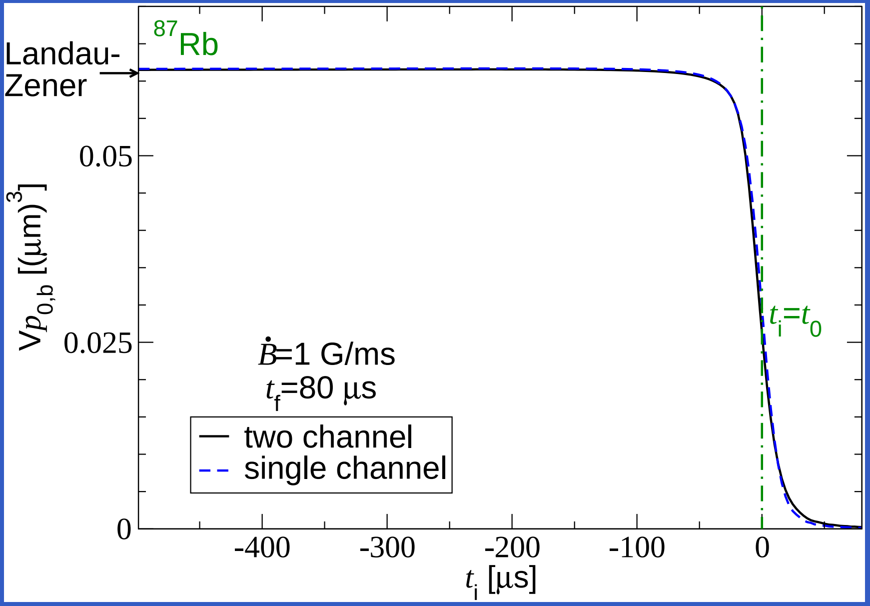
$$E_\mathrm{res}(t) = \mu_\mathrm{res}[B(t) - B_\mathrm{res}]$$

- Landau-Zener parameter:

$$\delta_\mathrm{LZ}^\mathrm{box} = \frac{4\pi\hbar}{m\mathcal{V}}\left|\frac{a_\mathrm{bg}\Delta B}{\dot{B}}\right|$$

- Single-channel approach:

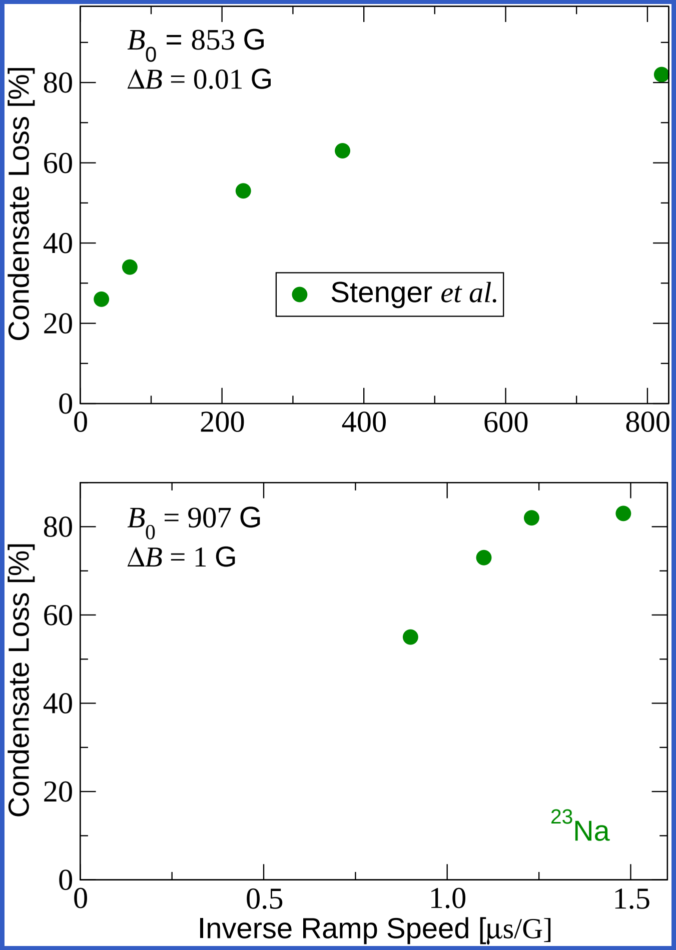
$$a(t) = a_\mathrm{bg}\left[1 - \frac{\Delta B}{B(t)-B_0}\right]$$



See also: T.M. Hanna, K. Góral, E. Witkowska, and TK, PRA **74**, 023618 (2006)

Pairing in Bose-Einstein condensates

Linear magnetic-field sweeps in a Bose-Einstein condensate



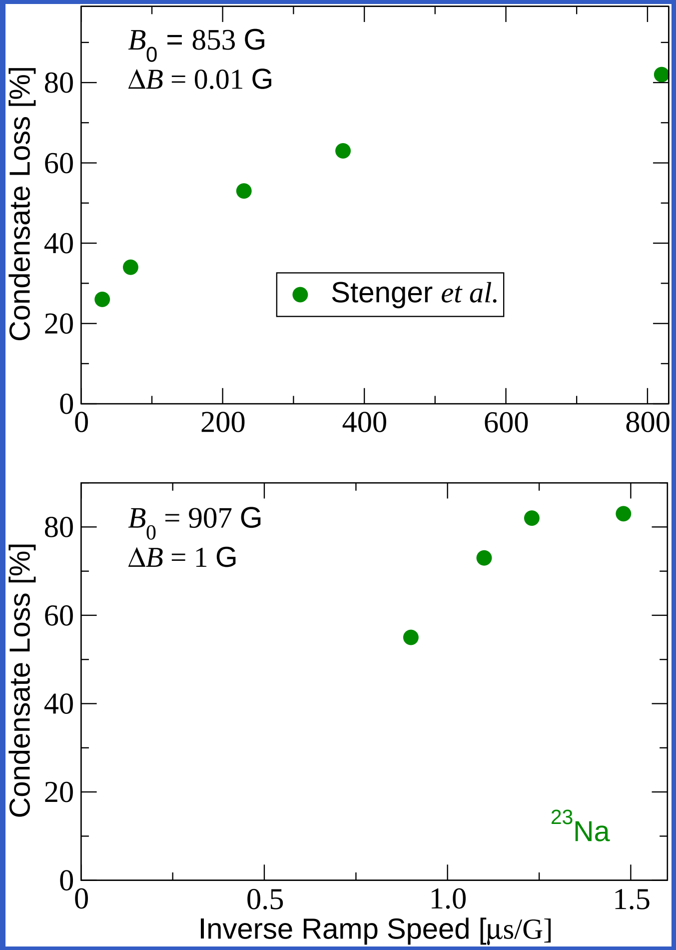
This experiment: J. Stenger *et al.*, PRL **82**, 2422 (1999)

Pairing in Bose-Einstein condensates

Linear magnetic-field sweeps in a Bose-Einstein condensate

- Number of pairs in a gas of N atoms:

$$\binom{N}{2} \approx N^2/2$$



This experiment: J. Stenger *et al.*, PRL **82**, 2422 (1999)

This heuristic approach: K. Góral, TK, S.A. Gardiner, E. Tiesinga, and P.S. Julienne, J. Phys. B **37**, 3457 (2004)

Pairing in Bose-Einstein condensates

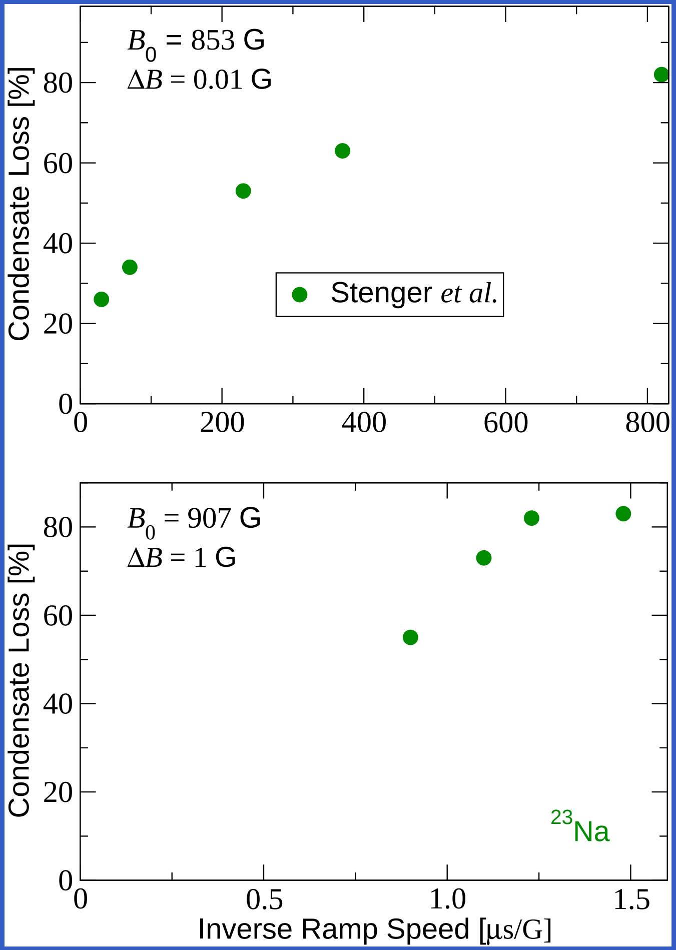
Linear magnetic-field sweeps in a Bose-Einstein condensate

- Number of pairs in a gas of N atoms:

$$\binom{N}{2} \approx N^2/2$$

- Onset of condensate loss in downward and upward sweeps of $E_{\mathrm{res}}(t)$:

$$N_{\mathrm{loss}} \approx N^2 p_{0,\mathrm{b}}$$



This experiment: J. Stenger *et al.*, PRL **82**, 2422 (1999)
This heuristic approach: K. Góral, TK, S.A. Gardiner, E. Tiesinga, and P.S. Julienne, J. Phys. B **37**, 3457 (2004)

Pairing in Bose-Einstein condensates

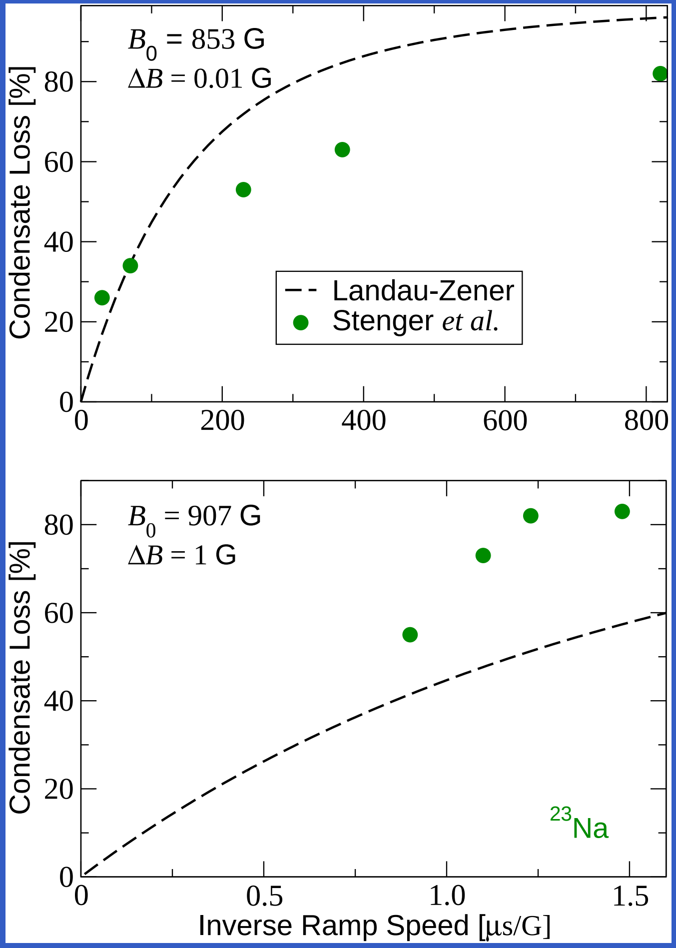
Linear magnetic-field sweeps in a Bose-Einstein condensate

- Number of pairs in a gas of N atoms:

$$\binom{N}{2} \approx N^2/2$$

- Onset of condensate loss in downward and upward sweeps of $E_{\text{res}}(t)$:

$$N_{\text{loss}} \approx N^2 p_{0,\text{b}} \approx N e^{-2\pi\delta_{\text{LZ}}^{\text{eff}}}$$



This experiment: J. Stenger *et al.*, PRL **82**, 2422 (1999)
General idea: F.H. Mies, E. Tiesinga, and P.S. Julienne, PRA **61**, 022721 (2000)

Pairing in Bose-Einstein condensates

Linear magnetic-field sweeps in a Bose-Einstein condensate

- Number of pairs in a gas of N atoms:

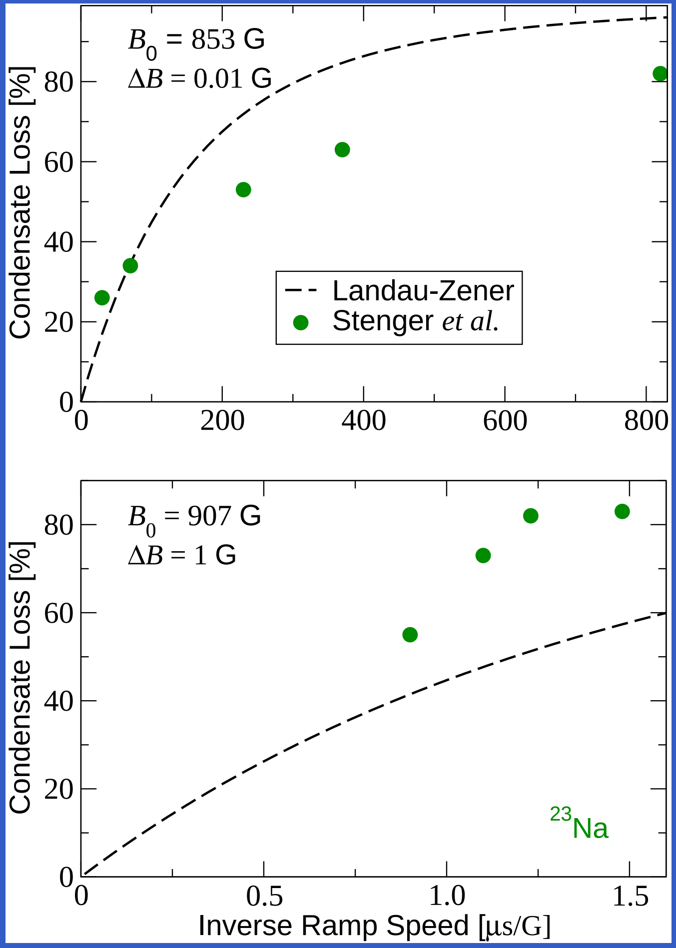
$$\begin{pmatrix} N \\ 2 \end{pmatrix} \approx N^2/2$$

- Onset of condensate loss in downward and upward sweeps of $E_{\mathrm{res}}(t)$:

$$N_{\mathrm{loss}} \approx N^2 p_{0,\mathrm{b}} \approx N e^{-2\pi\delta_{\mathrm{LZ}}^{\mathrm{eff}}}$$

- Effective Landau-Zener coefficient ($n = N/\mathcal{V}$):

$$\delta_{\mathrm{LZ}}^{\mathrm{eff}} = n \frac{4\pi\hbar}{m} \left| \frac{a_{\mathrm{bg}} \Delta B}{\dot{B}} \right| = N \delta_{\mathrm{LZ}}^{\mathrm{box}}$$



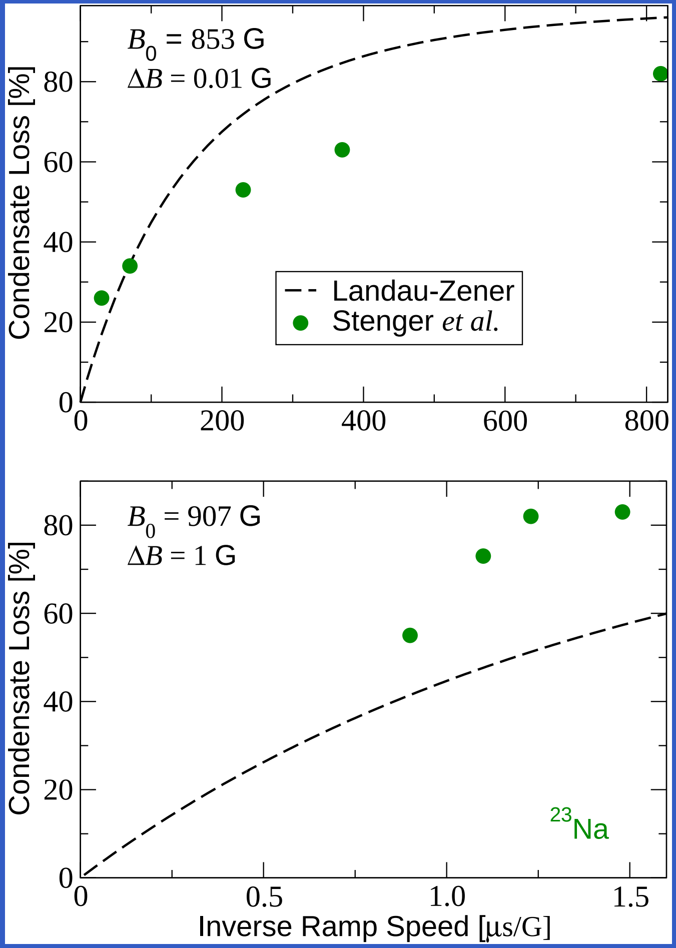
This experiment: J. Stenger *et al.*, PRL **82**, 2422 (1999)
General idea: F.H. Mies, E. Tiesinga, and P.S. Julienne, PRA **61**, 022721 (2000)

Pairing in Bose-Einstein condensates

Many-particle formulation

- Many-particle Hamiltonian:

$$H = H_0 + H_{\text{int}}$$



A.L. Fetter and J.D. Walecka, *Quantum Theory of Many-Particle Systems* (MacGraw-Hill, New York, 1971)

Pairing in Bose-Einstein condensates

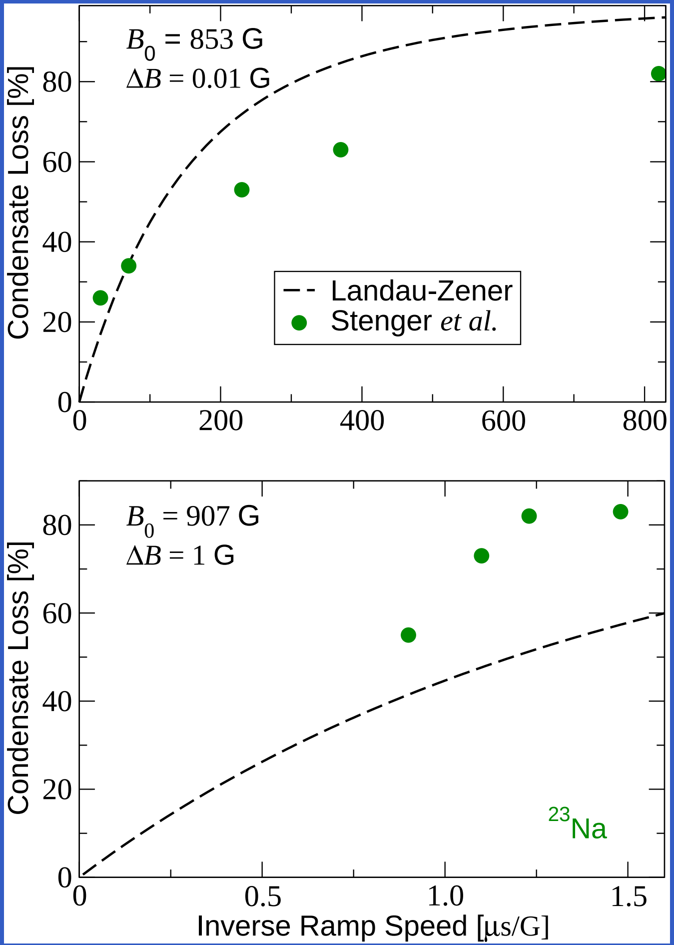
Many-particle formulation

- Many-particle Hamiltonian:

$$H = H_0 + H_{\mathrm{int}}$$

- Free Hamiltonian:

$$H_0 = \sum_\alpha \int d\mathbf{x}\, \psi_\alpha^\dagger(\mathbf{x}) \left[-\frac{\hbar^2}{2m}\boldsymbol{\nabla}^2 + V_\alpha^{\mathrm{ho}}(\mathbf{x}) \right] \psi_\alpha(\mathbf{x})$$



A.L. Fetter and J.D. Walecka, *Quantum Theory of Many-Particle Systems* (MacGraw-Hill, New York, 1971)

Pairing in Bose-Einstein condensates

Many-particle formulation

- Many-particle Hamiltonian:

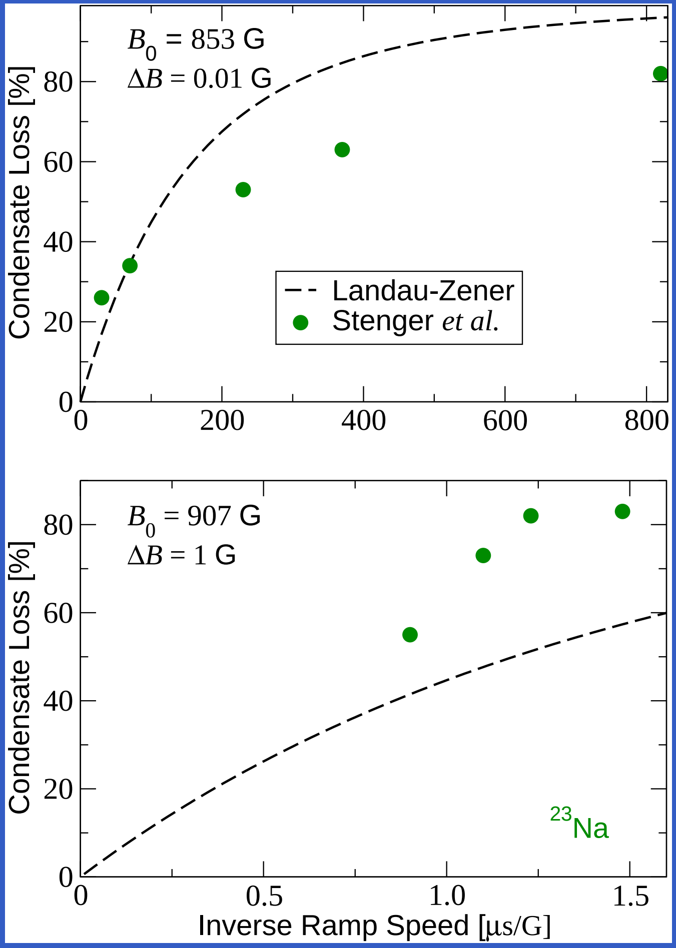
$$H = H_0 + H_{\rm int}$$

- Free Hamiltonian:

$$H_0 = \sum_\alpha \int d\mathbf{x}\, \psi_\alpha^\dagger(\mathbf{x}) \left[-\frac{\hbar^2}{2m}\boldsymbol{\nabla}^2 + V_\alpha^{\rm ho}(\mathbf{x}) \right] \psi_\alpha(\mathbf{x})$$

- Interaction Hamiltonian:

$$H_{\rm int} = \frac{1}{2} \sum_{\alpha\beta,\alpha'\beta'} \int d\mathbf{x} \int d\mathbf{y}\, V_{\alpha\beta,\alpha'\beta'}(\mathbf{x}-\mathbf{y}) \times \psi_\alpha^\dagger(\mathbf{x})\psi_\beta^\dagger(\mathbf{y})\psi_{\beta'}(\mathbf{y})\psi_{\alpha'}(\mathbf{x})$$



A.L. Fetter and J.D. Walecka, *Quantum Theory of Many-Particle Systems* (MacGraw-Hill, New York, 1971)

Pairing in Bose-Einstein condensates

Many-particle formulation

- Many-particle Hamiltonian:

$$H = H_0 + H_{\text{int}}$$

- Free Hamiltonian:

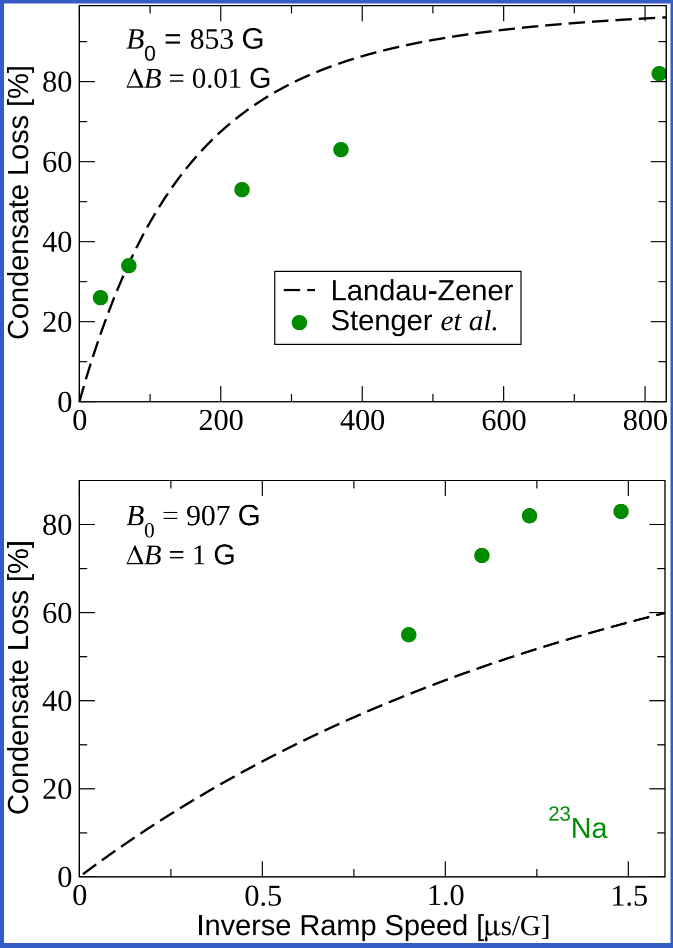
$$H_0 = \sum_\alpha \int d\mathbf{x}\, \psi_\alpha^\dagger(\mathbf{x}) \left[-\frac{\hbar^2}{2m} \nabla^2 + V_\alpha^{\text{ho}}(\mathbf{x}) \right] \psi_\alpha(\mathbf{x})$$

- Interaction Hamiltonian:

$$H_{\text{int}} = \frac{1}{2} \sum_{\alpha\beta,\alpha'\beta'} \int d\mathbf{x} \int d\mathbf{y}\, V_{\alpha\beta,\alpha'\beta'}(\mathbf{x}-\mathbf{y}) \times \psi_\alpha^\dagger(\mathbf{x}) \psi_\beta^\dagger(\mathbf{y}) \psi_{\beta'}(\mathbf{y}) \psi_{\alpha'}(\mathbf{x})$$

- Bose commutation relations:

$$\psi_\alpha(\mathbf{x})\psi_\beta^\dagger(\mathbf{y}) - \psi_\beta^\dagger(\mathbf{y})\psi_\alpha(\mathbf{x}) = \delta_{\alpha\beta}\delta(\mathbf{x}-\mathbf{y})$$

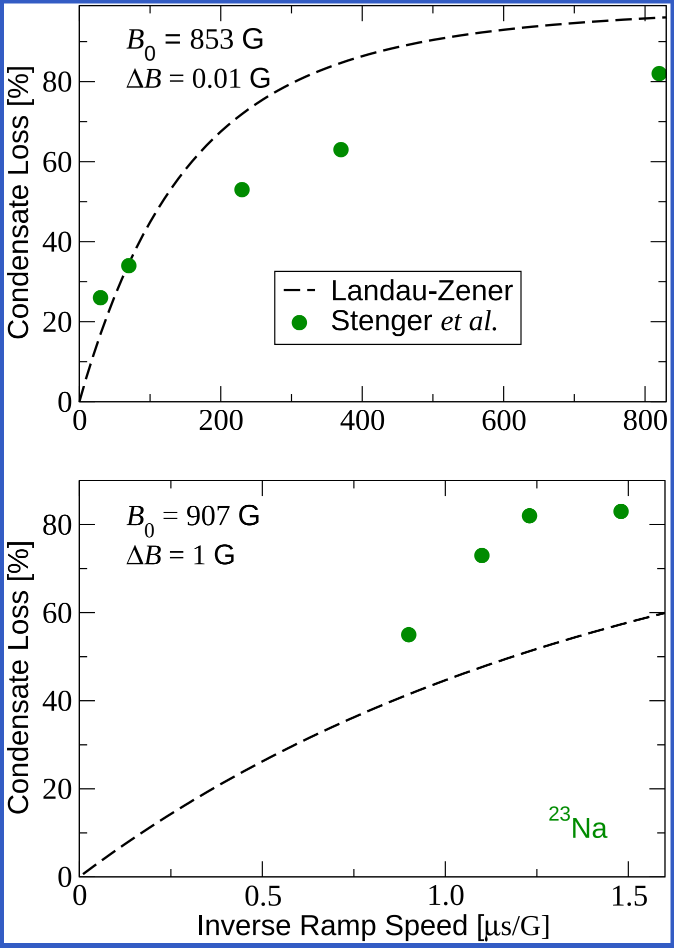


Pairing in Bose-Einstein condensates

Dynamical theory

- Schödinger equation:

$$i\hbar\frac{\partial}{\partial t}\langle\psi^\dagger_\beta(\mathbf{y})\cdots\psi_\alpha(\mathbf{x})\rangle_t = \langle[\psi^\dagger_\beta(\mathbf{y})\cdots\psi_\alpha(\mathbf{x}), H]\rangle_t$$



See, e.g.: J. Fricke, Ann. Phys. (N.Y.) **252**, 479 (1996)
N.P. Proukakis, K. Burnett, and H.T.C. Stoof, PRA **57**, 1230 (1998)

Pairing in Bose-Einstein condensates

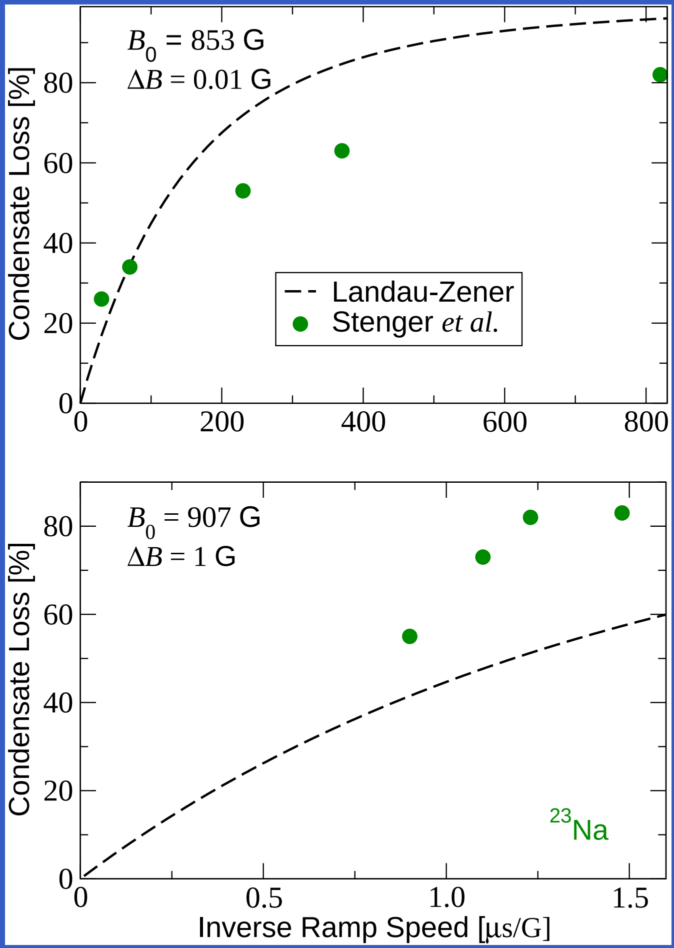
Dynamical theory

- Schödinger equation:

$$i\hbar\frac{\partial}{\partial t}\langle\psi^\dagger_\beta(\mathbf{y})\cdots\psi_\alpha(\mathbf{x})\rangle_t = \langle[\psi^\dagger_\beta(\mathbf{y})\cdots\psi_\alpha(\mathbf{x}), H]\rangle_t$$

- Condensate mean field:

$$\Psi_\alpha(\mathbf{x},t) = \langle\psi_\alpha(\mathbf{x})\rangle_t$$



See, e.g.: J. Fricke, Ann. Phys. (N.Y.) **252**, 479 (1996)
N.P. Proukakis, K. Burnett, and H.T.C. Stoof, PRA **57**, 1230 (1998)

Pairing in Bose-Einstein condensates

Dynamical theory

- Schödinger equation:

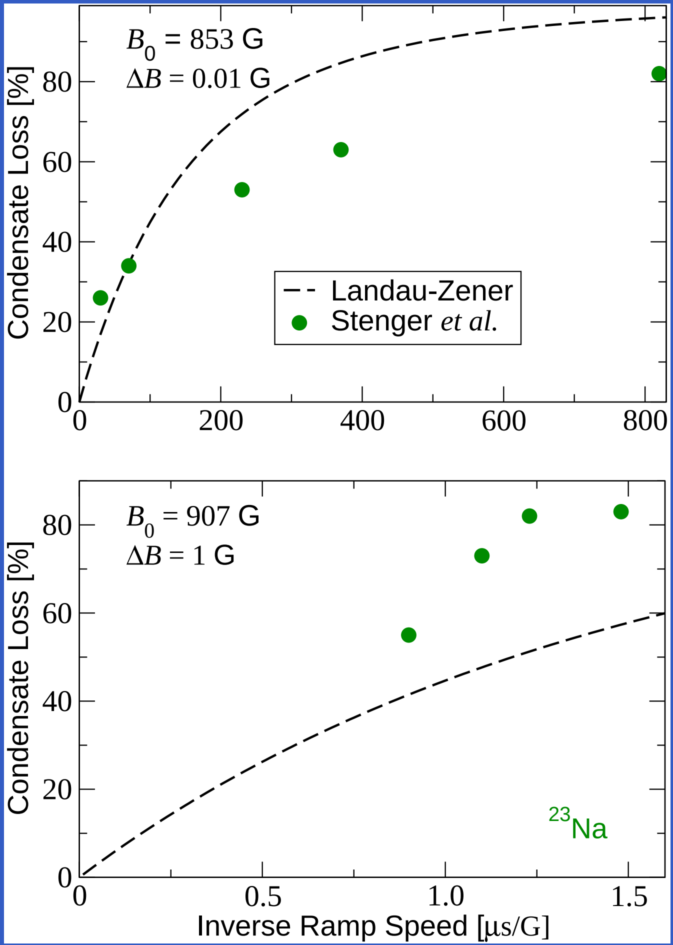
$$i\hbar\frac{\partial}{\partial t}\langle\psi^\dagger_\beta(\mathbf{y})\cdots\psi_\alpha(\mathbf{x})\rangle_t = \langle[\psi^\dagger_\beta(\mathbf{y})\cdots\psi_\alpha(\mathbf{x}), H]\rangle_t$$

- Condensate mean field:

$$\Psi_\alpha(\mathbf{x},t) = \langle\psi_\alpha(\mathbf{x})\rangle_t$$

- Pair function:

$$\Phi_{\alpha\beta}(\mathbf{x},\mathbf{y},t) = \langle\psi_\beta(\mathbf{y})\psi_\alpha(\mathbf{x})\rangle_t - \Psi_\alpha(\mathbf{x},t)\Psi_\beta(\mathbf{y},t)$$



See, e.g.: J. Fricke, Ann. Phys. (N.Y.) **252**, 479 (1996)
N.P. Proukakis, K. Burnett, and H.T.C. Stoof, PRA **57**, 1230 (1998)

Pairing in Bose-Einstein condensates

Dynamical theory

- Schödinger equation:

$$i\hbar\frac{\partial}{\partial t}\langle\psi_\beta^\dagger(\mathbf{y})\cdots\psi_\alpha(\mathbf{x})\rangle_t = \langle[\psi_\beta^\dagger(\mathbf{y})\cdots\psi_\alpha(\mathbf{x}), H]\rangle_t$$

- Condensate mean field:

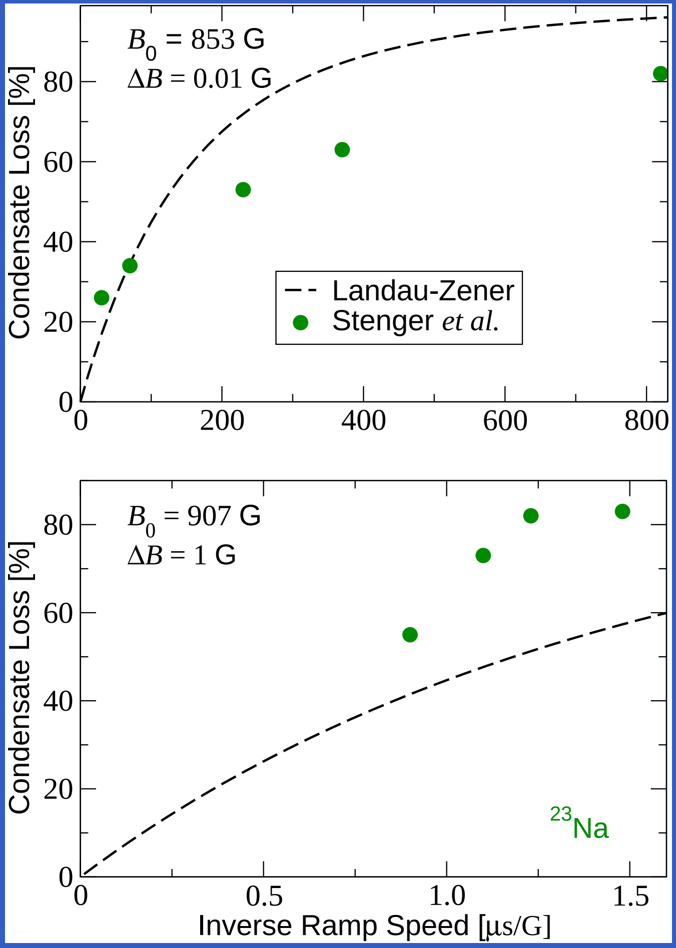
$$\Psi_\alpha(\mathbf{x},t) = \langle\psi_\alpha(\mathbf{x})\rangle_t$$

- Pair function:

$$\Phi_{\alpha\beta}(\mathbf{x},\mathbf{y},t) = \langle\psi_\beta(\mathbf{y})\psi_\alpha(\mathbf{x})\rangle_t - \Psi_\alpha(\mathbf{x},t)\Psi_\beta(\mathbf{y},t)$$

- Density matrix of non-condensed atoms:

$$\Gamma_{\alpha\beta}(\mathbf{x},\mathbf{y},t) = \langle\psi_\beta^\dagger(\mathbf{y})\psi_\alpha(\mathbf{x})\rangle_t - \Psi_\alpha(\mathbf{x},t)\Psi_\beta^*(\mathbf{y},t)$$

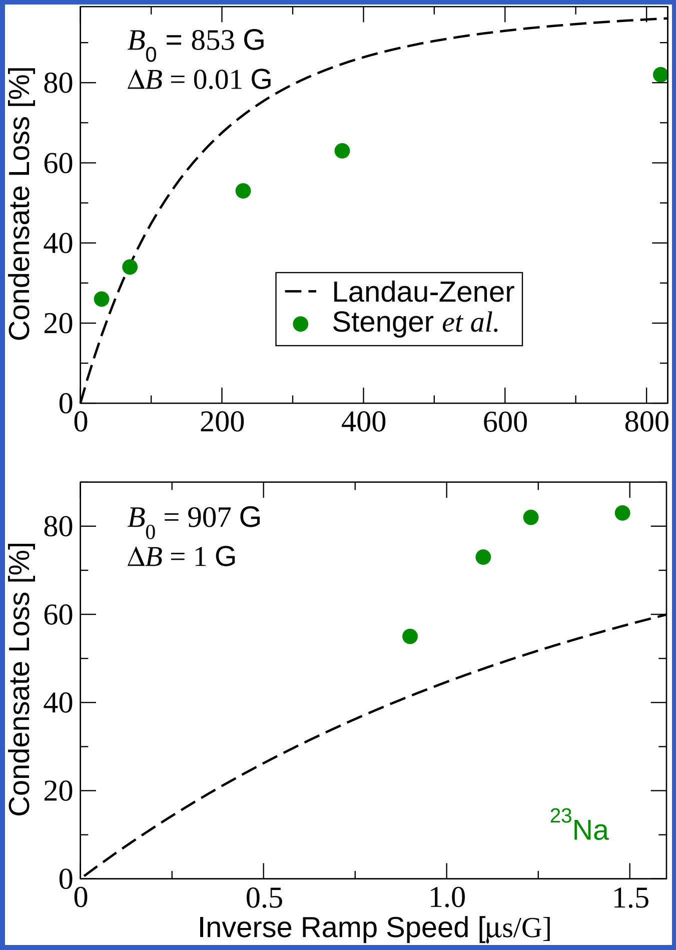


Pairing in Bose-Einstein condensates

Cumulant method for Bose-Einstein condensates

- Dynamical equation for the condensate mean field:

$$i\hbar\dot{\Psi}_\alpha(\mathbf{x},t) = H_\alpha^{1\mathrm{B}}\Psi_\alpha(\mathbf{x},t) + \sum_{\alpha',\beta\beta'} \int d\mathbf{y}\, V_{\alpha\beta,\alpha'\beta'}(\mathbf{r}) \times \Psi_\beta^*(\mathbf{y},t)\langle\psi_{\beta'}(\mathbf{y})\psi_{\alpha'}(\mathbf{x})\rangle_t$$



Pairing in Bose-Einstein condensates

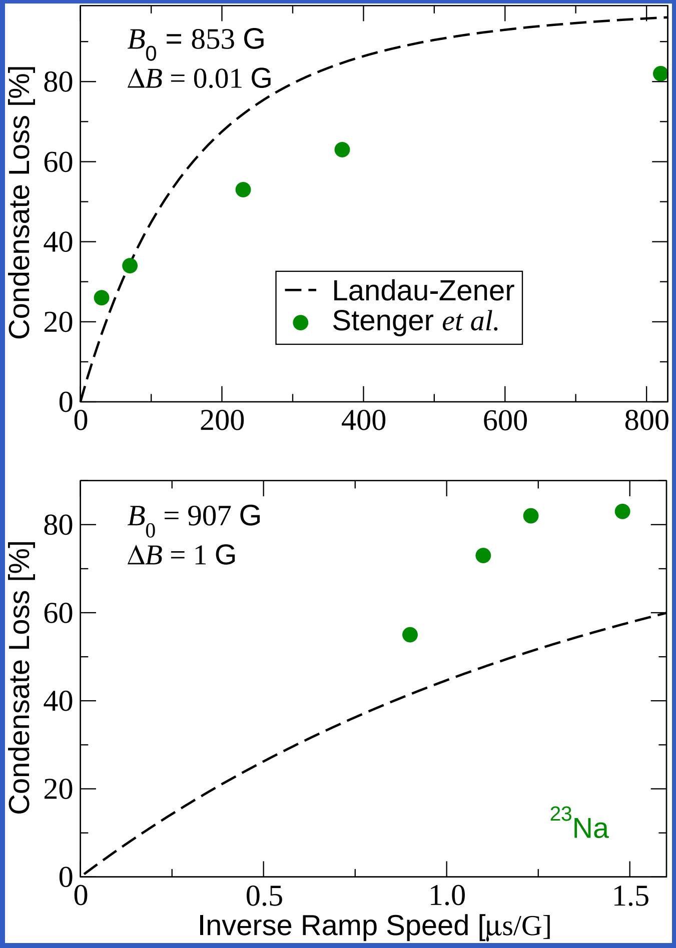
Cumulant method for Bose-Einstein condensates

- Dynamical equation for the condensate mean field:

$$i\hbar\dot{\Psi}_\alpha(\mathbf{x},t) = H_\alpha^{1\mathrm{B}}\Psi_\alpha(\mathbf{x},t) + \sum_{\alpha',\beta\beta'} \int d\mathbf{y}\, V_{\alpha\beta,\alpha'\beta'}(\mathbf{r}) \times \Psi_\beta^*(\mathbf{y},t)\langle\psi_{\beta'}(\mathbf{y})\psi_{\alpha'}(\mathbf{x})\rangle_t$$

- Dynamical equation for the pair function:

$$i\hbar\dot{\Phi}_{\alpha\beta}(\mathbf{x},\mathbf{y},t) = \sum_{\alpha'\beta'} [H_{\alpha\beta,\alpha'\beta'}^{2\mathrm{B}}\Phi_{\alpha'\beta'}(\mathbf{x},\mathbf{y},t) + V_{\alpha\beta,\alpha'\beta'}(\mathbf{r})\Psi_{\alpha'}(\mathbf{x},t)\Psi_{\beta'}(\mathbf{y},t)]$$



Pairing in Bose-Einstein condensates

Cumulant method for Bose-Einstein condensates

- Dynamical equation for the condensate mean field:

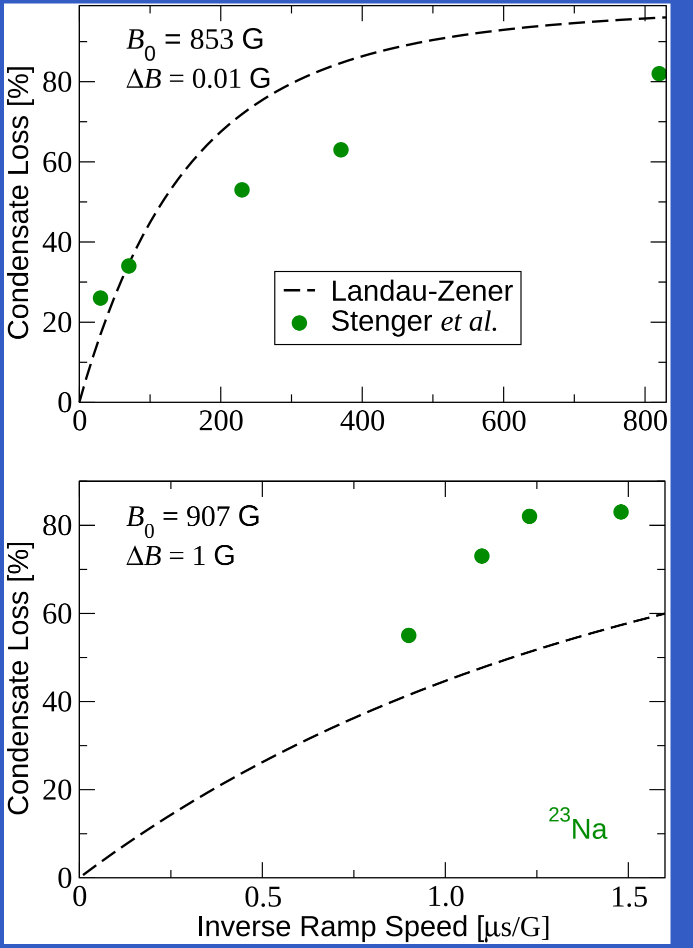
$$i\hbar\dot{\Psi}_\alpha(\mathbf{x},t) = H^{1\mathrm{B}}_\alpha \Psi_\alpha(\mathbf{x},t) + \sum_{\alpha',\beta\beta'} \int d\mathbf{y}\, V_{\alpha\beta,\alpha'\beta'}(\mathbf{r}) \times \Psi^*_\beta(\mathbf{y},t)\langle\psi_{\beta'}(\mathbf{y})\psi_{\alpha'}(\mathbf{x})\rangle_t$$

- Dynamical equation for the pair function:

$$i\hbar\dot{\Phi}_{\alpha\beta}(\mathbf{x},\mathbf{y},t) = \sum_{\alpha'\beta'} [H^{2\mathrm{B}}_{\alpha\beta,\alpha'\beta'} \Phi_{\alpha'\beta'}(\mathbf{x},\mathbf{y},t) + V_{\alpha\beta,\alpha'\beta'}(\mathbf{r})\Psi_{\alpha'}(\mathbf{x},t)\Psi_{\beta'}(\mathbf{y},t)]$$

- Positivity of the one-body density matrix:

$$\Gamma_{\alpha\beta}(\mathbf{x},\mathbf{y},t) = \sum_\gamma \int d\mathbf{z}\, \Phi_{\alpha\gamma}(\mathbf{x},\mathbf{z},t)[\Phi_{\beta\gamma}(\mathbf{y},\mathbf{z},t)]^*$$



This method: TK and K. Burnett, PRA **65**, 033601 (2002)
TK, T. Gasenzer, and K. Burnett, PRA **67**, 013601 (2003)

Pairing in Bose-Einstein condensates

Cumulant method for Bose-Einstein condensates

- Dynamical equation for the condensate mean field:

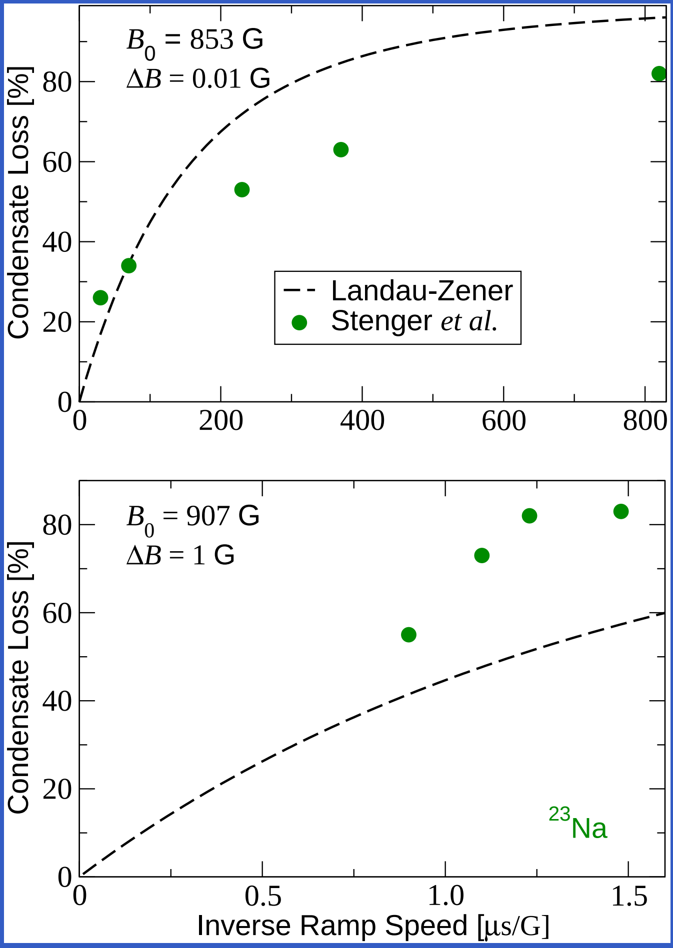
$$i\hbar\dot{\Psi}_\alpha(\mathbf{x},t) = H^{1B}_\alpha \Psi_\alpha(\mathbf{x},t) + \sum_{\alpha',\beta\beta'} \int d\mathbf{y}\, V_{\alpha\beta,\alpha'\beta'}(\mathbf{r}) \times \Psi^*_\beta(\mathbf{y},t)\langle\psi_{\beta'}(\mathbf{y})\psi_{\alpha'}(\mathbf{x})\rangle_t$$

- Dynamical equation for the pair function:

$$i\hbar\dot{\Phi}_{\alpha\beta}(\mathbf{x},\mathbf{y},t) = \sum_{\alpha'\beta'} [H^{2B}_{\alpha\beta,\alpha'\beta'}\Phi_{\alpha'\beta'}(\mathbf{x},\mathbf{y},t) + V_{\alpha\beta,\alpha'\beta'}(\mathbf{r})\Psi_{\alpha'}(\mathbf{x},t)\Psi_{\beta'}(\mathbf{y},t)]$$

- Number conservation:

$$\sum_\alpha \int d\mathbf{x}\, \left[|\Psi_\alpha(\mathbf{x},t)|^2 + \Gamma_{\alpha\alpha}(\mathbf{x},\mathbf{x},t)\right] = N$$



This method: TK and K. Burnett, PRA **65**, 033601 (2002)
TK, T. Gasenzer, and K. Burnett, PRA **67**, 013601 (2003)

Pairing in Bose-Einstein condensates

Gross-Pitaevskii limit

- Dynamical equation for the condensate mean field:

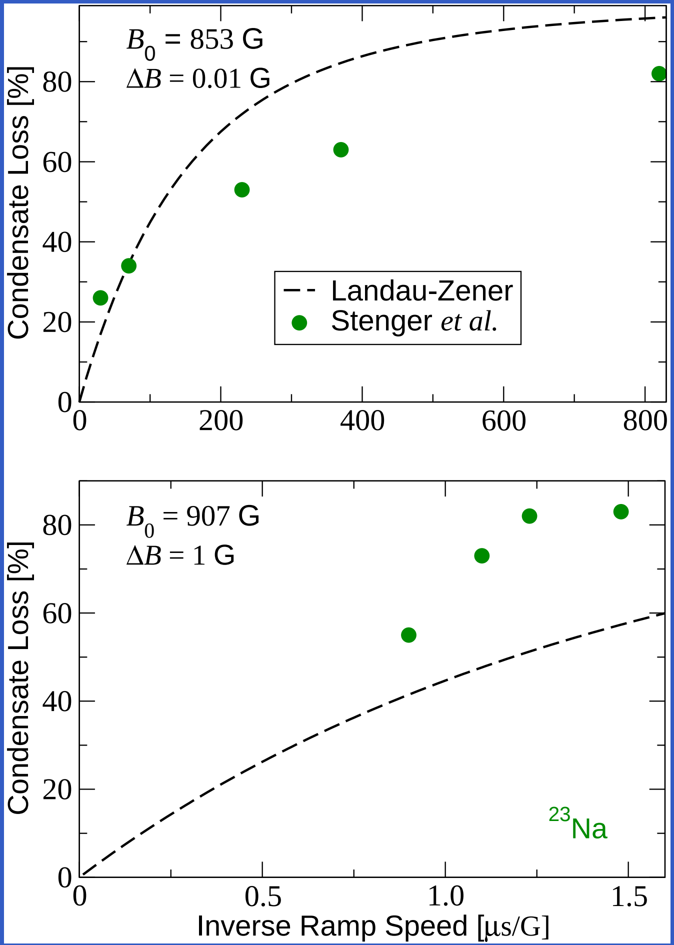
$$i\hbar\dot{\Psi}_\alpha(\mathbf{x},t) = H^{1B}_\alpha \Psi_\alpha(\mathbf{x},t) + \sum_{\alpha',\beta\beta'} \int d\mathbf{y}\, V_{\alpha\beta,\alpha'\beta'}(\mathbf{r}) \times \Psi^*_\beta(\mathbf{y},t)\langle\psi_{\beta'}(\mathbf{y})\psi_{\alpha'}(\mathbf{x})\rangle_t$$

- Dynamical equation for the pair function:

$$i\hbar\dot{\Phi}_{\alpha\beta}(\mathbf{x},\mathbf{y},t) = \sum_{\alpha'\beta'} [H^{2B}_{\alpha\beta,\alpha'\beta'}\Phi_{\alpha'\beta'}(\mathbf{x},\mathbf{y},t) + V_{\alpha\beta,\alpha'\beta'}(\mathbf{r})\Psi_{\alpha'}(\mathbf{x},t)\Psi_{\beta'}(\mathbf{y},t)]$$

- One-component Bose-Einstein condensate:

$$\Psi_\alpha(\mathbf{x},t) = \Psi(\mathbf{x},t)$$



E.P. Gross, Nuovo Cimento **20**, 454 (1961)
L.P. Pitaevskii, Sov. Phys. JETP **13**, 451 (1961)

Pairing in Bose-Einstein condensates

Molecular mean-field approach

- Dynamical equation for the condensate mean field:

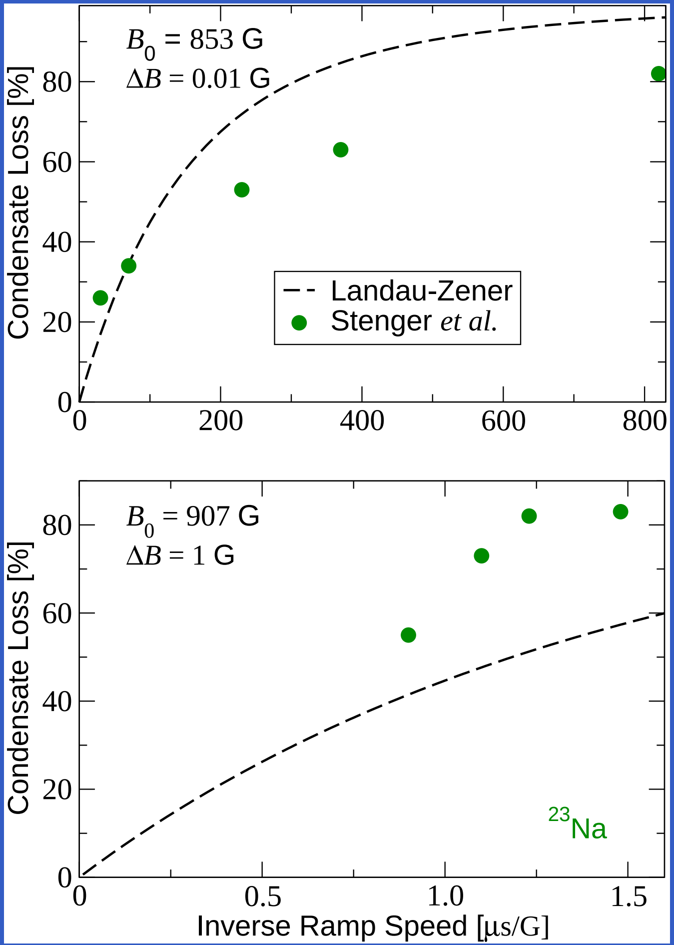
$$i\hbar\dot{\Psi}_\alpha(\mathbf{x},t) = H_\alpha^{1\mathrm{B}}\Psi_\alpha(\mathbf{x},t) + \sum_{\alpha',\beta\beta'}\int d\mathbf{y}\, V_{\alpha\beta,\alpha'\beta'}(\mathbf{r}) \times\Psi_\beta^*(\mathbf{y},t)\langle\psi_{\beta'}(\mathbf{y})\psi_{\alpha'}(\mathbf{x})\rangle_t$$

- Dynamical equation for the pair function:

$$i\hbar\dot{\Phi}_{\alpha\beta}(\mathbf{x},\mathbf{y},t) = \sum_{\alpha'\beta'}[H^{2\mathrm{B}}_{\alpha\beta,\alpha'\beta'}\Phi_{\alpha'\beta'}(\mathbf{x},\mathbf{y},t) + V_{\alpha\beta,\alpha'\beta'}(\mathbf{r})\Psi_{\alpha'}(\mathbf{x},t)\Psi_{\beta'}(\mathbf{y},t)]$$

- Closed-channel molecules:

$$\Phi_{\alpha\beta}(\mathbf{x},\mathbf{y},t) = \Phi_{\mathrm{cl}}(\mathbf{x},\mathbf{y},t) = \Psi_{\mathrm{res}}(\mathbf{R},t)\phi_{\mathrm{res}}(\mathbf{r})$$



E. Timmermans, P. Tommasini, M. Hussein, and A. Kerman, Phys. Rep. **315**, 199 (1999)
P.D. Drummond, K.V. Kheruntsyan, and H. He, PRL **81**, 3055 (1998)

Pairing in Bose-Einstein condensates

Molecular mean-field approach

- Dynamical equation for the condensate mean field:

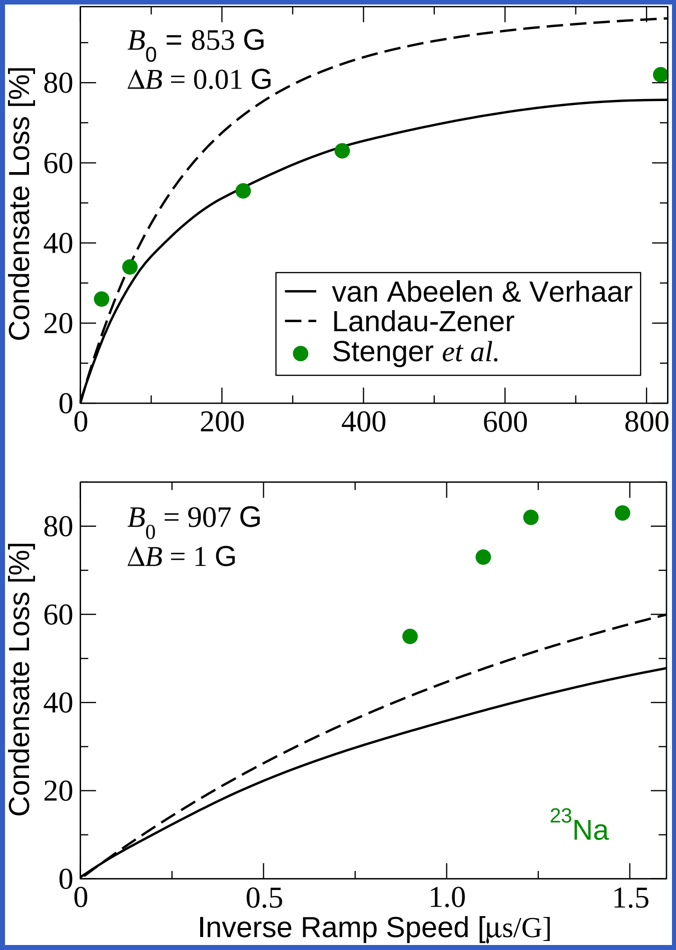
$$i\hbar\dot{\Psi}_\alpha(\mathbf{x},t) = H_\alpha^{1\mathrm{B}}\Psi_\alpha(\mathbf{x},t) + \sum_{\alpha',\beta\beta'} \int d\mathbf{y}\, V_{\alpha\beta,\alpha'\beta'}(\mathbf{r}) \times \Psi_\beta^*(\mathbf{y},t)\langle\psi_{\beta'}(\mathbf{y})\psi_{\alpha'}(\mathbf{x})\rangle_t$$

- Dynamical equation for the pair function:

$$i\hbar\dot{\Phi}_{\alpha\beta}(\mathbf{x},\mathbf{y},t) = \sum_{\alpha'\beta'}[H^{2\mathrm{B}}_{\alpha\beta,\alpha'\beta'}\Phi_{\alpha'\beta'}(\mathbf{x},\mathbf{y},t) + V_{\alpha\beta,\alpha'\beta'}(\mathbf{r})\Psi_{\alpha'}(\mathbf{x},t)\Psi_{\beta'}(\mathbf{y},t)]$$

- Closed-channel molecules:

$$\Phi_{\alpha\beta}(\mathbf{x},\mathbf{y},t) = \Phi_{\mathrm{cl}}(\mathbf{x},\mathbf{y},t) = \Psi_{\mathrm{res}}(\mathbf{R},t)\phi_{\mathrm{res}}(\mathbf{r})$$



F.A. van Abeelen and B.J. Verhaar, PRL **83**, 1550 (1999)

Pairing in Bose-Einstein condensates

Inclusion of the entrance-channel pair function

- Dynamical equation for the condensate mean field:

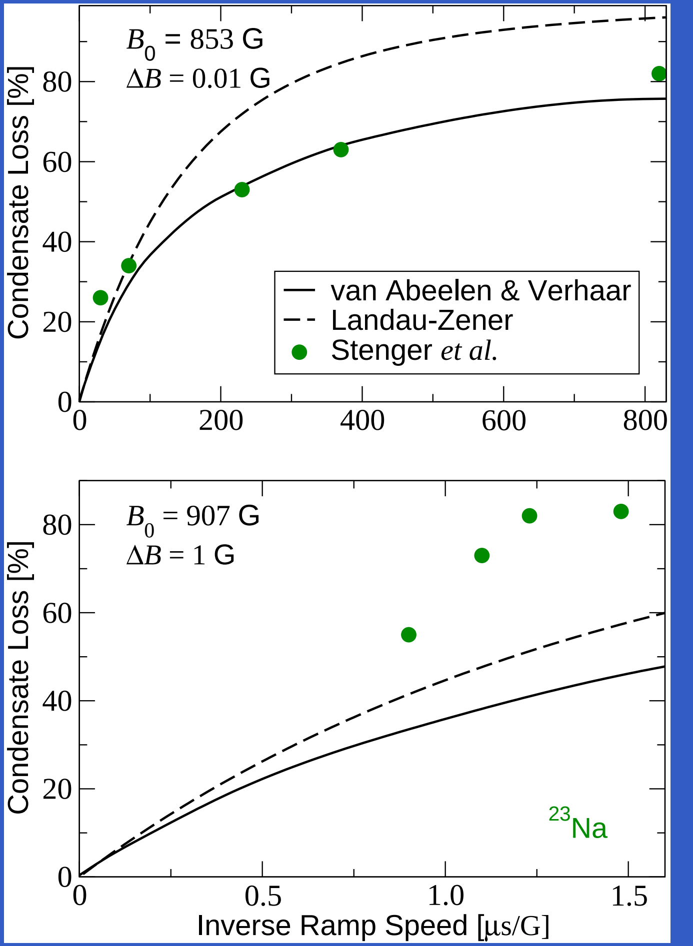
$$i\hbar\dot{\Psi}_\alpha(\mathbf{x},t) = H_\alpha^{1\mathrm{B}}\Psi_\alpha(\mathbf{x},t) + \sum_{\alpha',\beta\beta'} \int d\mathbf{y}\, V_{\alpha\beta,\alpha'\beta'}(\mathbf{r}) \times \Psi_\beta^*(\mathbf{y},t)\langle\psi_{\beta'}(\mathbf{y})\psi_{\alpha'}(\mathbf{x})\rangle_t$$

- Dynamical equation for the pair function:

$$i\hbar\dot{\Phi}_{\alpha\beta}(\mathbf{x},\mathbf{y},t) = \sum_{\alpha'\beta'} [H_{\alpha\beta,\alpha'\beta'}^{2\mathrm{B}}\Phi_{\alpha'\beta'}(\mathbf{x},\mathbf{y},t) + V_{\alpha\beta,\alpha'\beta'}(\mathbf{r})\Psi_{\alpha'}(\mathbf{x},t)\Psi_{\beta'}(\mathbf{y},t)]$$

- Complete pair function:

$$\sum_{\alpha\beta} |\alpha\beta\rangle\Phi_{\alpha\beta} = |\mathrm{bg}\rangle\Phi_{\mathrm{bg}} + |\mathrm{cl}\rangle\Phi_{\mathrm{cl}}$$



HFB approach: M. Holland, J. Park, and R. Walser, PRL **86**, 1915 (2001)

Pairing in Bose-Einstein condensates

Entrance-channel pair correlations interfere away!

- Dynamical equation for the condensate mean field:

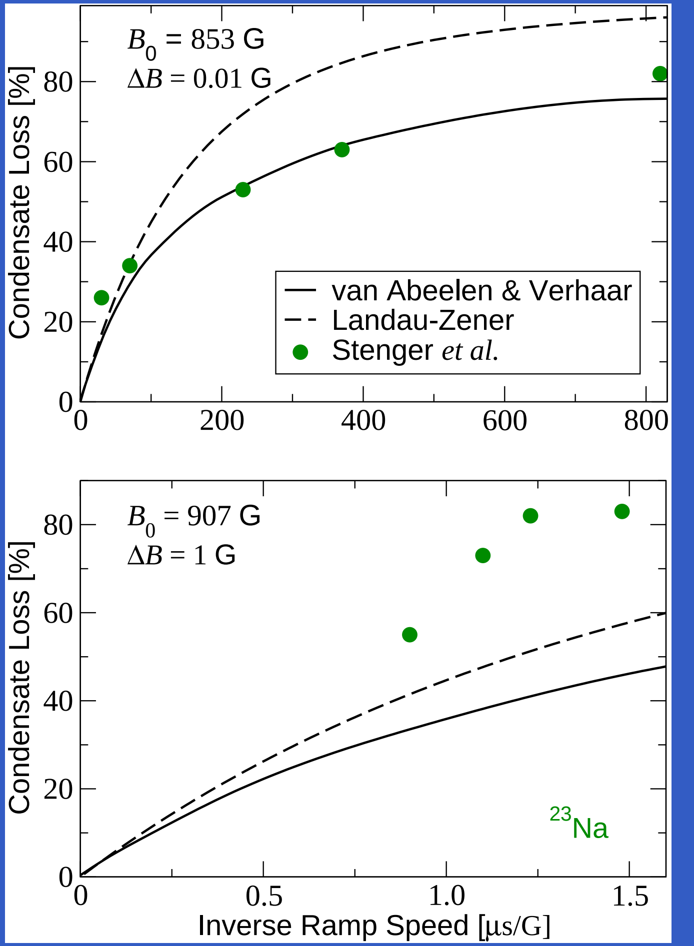
$$i\hbar\dot{\Psi}_\alpha(\mathbf{x},t) = H_\alpha^{1\mathrm{B}}\Psi_\alpha(\mathbf{x},t) + \sum_{\alpha',\beta\beta'}\int d\mathbf{y}\, V_{\alpha\beta,\alpha'\beta'}(\mathbf{r}) \times \Psi_\beta^*(\mathbf{y},t)\langle\psi_{\beta'}(\mathbf{y})\psi_{\alpha'}(\mathbf{x})\rangle_t$$

- Dynamical equation for the pair function:

$$i\hbar\dot{\Phi}_{\alpha\beta}(\mathbf{x},\mathbf{y},t) = \sum_{\alpha'\beta'}\left[H^{2\mathrm{B}}_{\alpha\beta,\alpha'\beta'}\Phi_{\alpha'\beta'}(\mathbf{x},\mathbf{y},t) + V_{\alpha\beta,\alpha'\beta'}(\mathbf{r})\Psi_{\alpha'}(\mathbf{x},t)\Psi_{\beta'}(\mathbf{y},t)\right]$$

- Complete pair function:

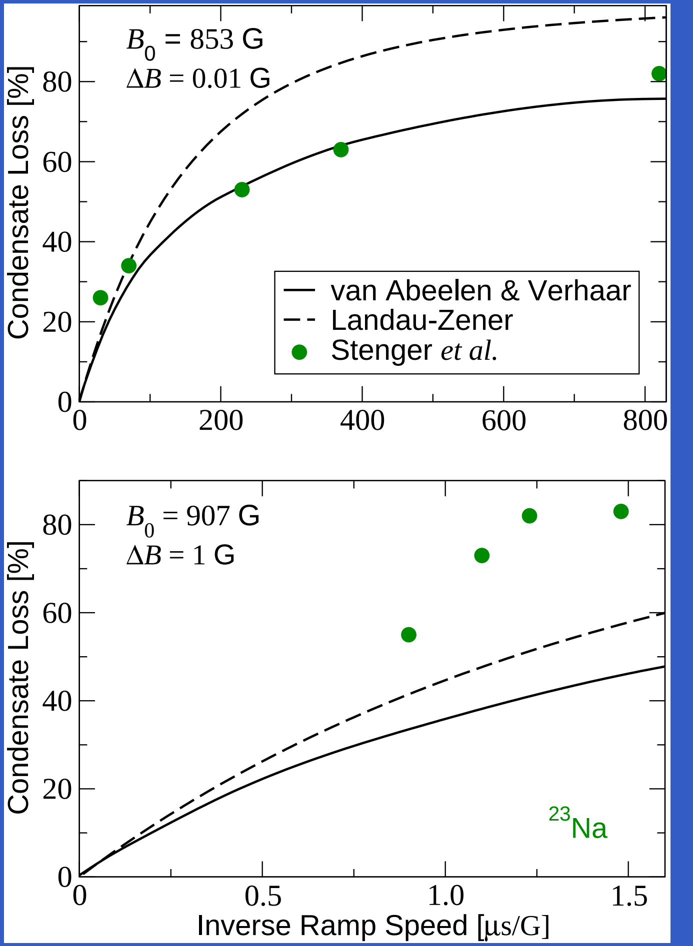
$$\sum_{\alpha\beta}|\alpha\beta\rangle\Phi_{\alpha\beta} = |\mathrm{bg}\rangle\Phi_{\mathrm{bg}} + |\mathrm{cl}\rangle\Phi_{\mathrm{cl}}$$



K. Góral, TK, S.A. Gardiner, E. Tiesinga, and P.S. Julienne, J. Phys. B **37**, 3457 (2004)

Pairing in Bose-Einstein condensates

Feshbach molecules decay due to collisional relaxation!



- Dynamical equation for the condensate mean field:

$$i\hbar\dot{\Psi}_\alpha(\mathbf{x},t) = H_\alpha^{1\mathrm{B}}\Psi_\alpha(\mathbf{x},t) + \sum_{\alpha',\beta\beta'}\int d\mathbf{y}\, V_{\alpha\beta,\alpha'\beta'}(\mathbf{r}) \times \Psi_\beta^*(\mathbf{y},t)\langle\psi_{\beta'}(\mathbf{y})\psi_{\alpha'}(\mathbf{x})\rangle_t$$

- Dynamical equation for the pair function:

$$i\hbar\dot{\Phi}_{\alpha\beta}(\mathbf{x},\mathbf{y},t) = \sum_{\alpha'\beta'}\left[H^{2\mathrm{B}}_{\alpha\beta,\alpha'\beta'}\Phi_{\alpha'\beta'}(\mathbf{x},\mathbf{y},t) + V_{\alpha\beta,\alpha'\beta'}(\mathbf{r})\Psi_{\alpha'}(\mathbf{x},t)\Psi_{\beta'}(\mathbf{y},t)\right]$$

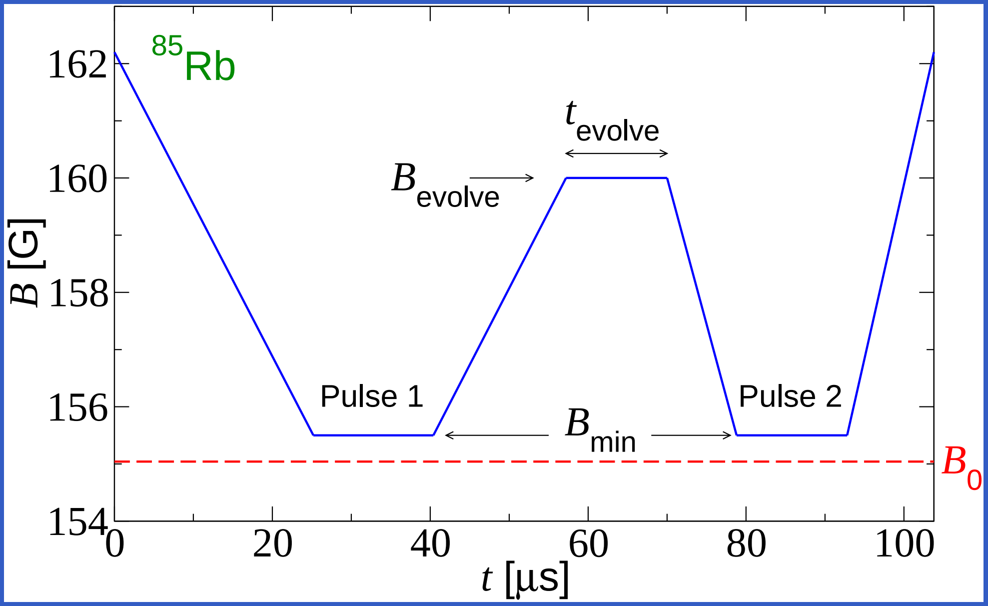
- Complete pair function:

$$\sum_{\alpha\beta}|\alpha\beta\rangle\Phi_{\alpha\beta} = |\mathrm{bg}\rangle\Phi_{\mathrm{bg}} + |\mathrm{cl}\rangle\Phi_{\mathrm{cl}}$$

P. Soldán, M.T. Cvitaš, J.M. Hutson, P. Honvault, and J.-M. Launay, PRL **89**, 153201 (2002)
V.A. Yurovsky, A. Ben-Reuven, P.S. Julienne, and C.J. Williams, PRA **62**, 043605 (2000)

Atom-molecule coherence

Magnetic-field pulse sequence



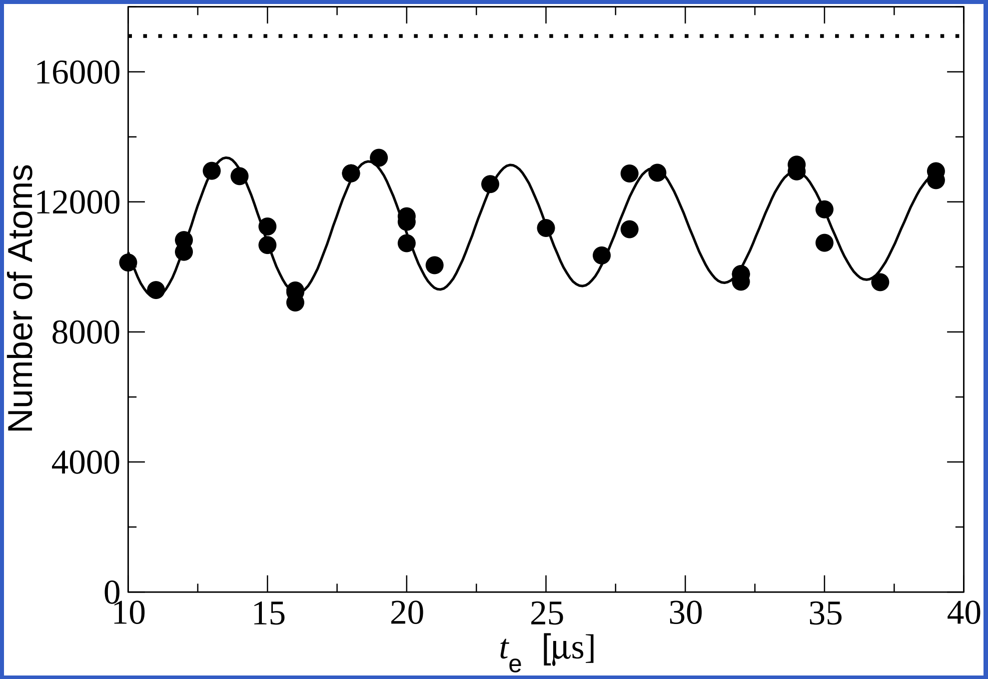
E.A. Donley, N.R. Claussen, S.T. Thompson, and C.E. Wieman, Nature **412**, 295 (2002)

Atom-molecule coherence

Experimental observations

Interference fringes in:

- a remnant Bose-Einstein condensate



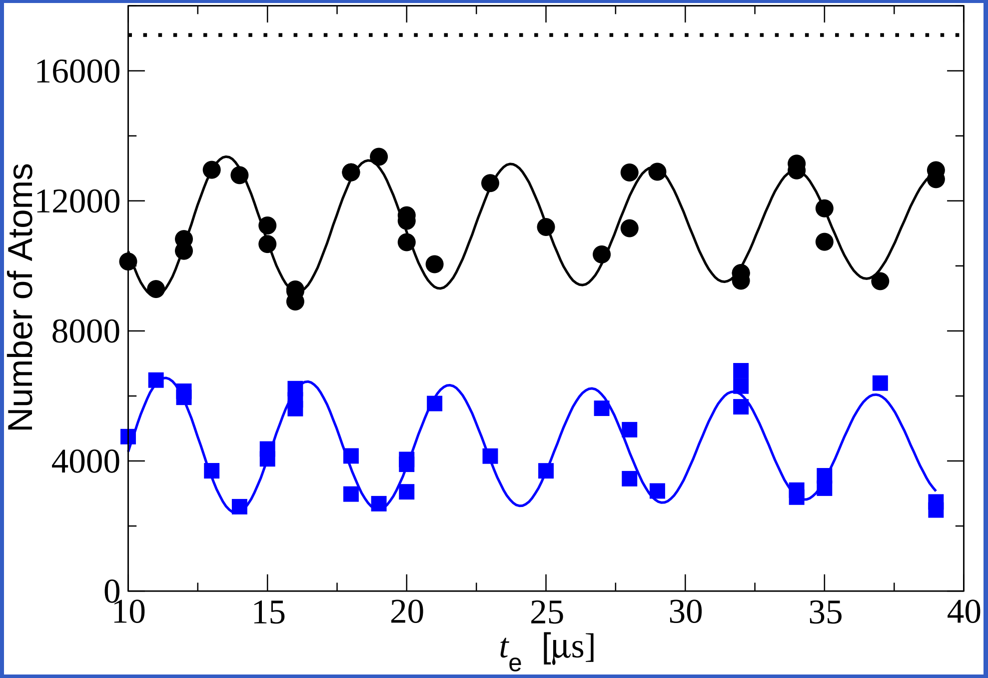
E.A. Donley, N.R. Claussen, S.T. Thompson, and C.E. Wieman, Nature **412**, 295 (2002)

Atom-molecule coherence

Experimental observations

Interference fringes in:

- a remnant Bose-Einstein condensate
- a “burst” of correlated atom pairs



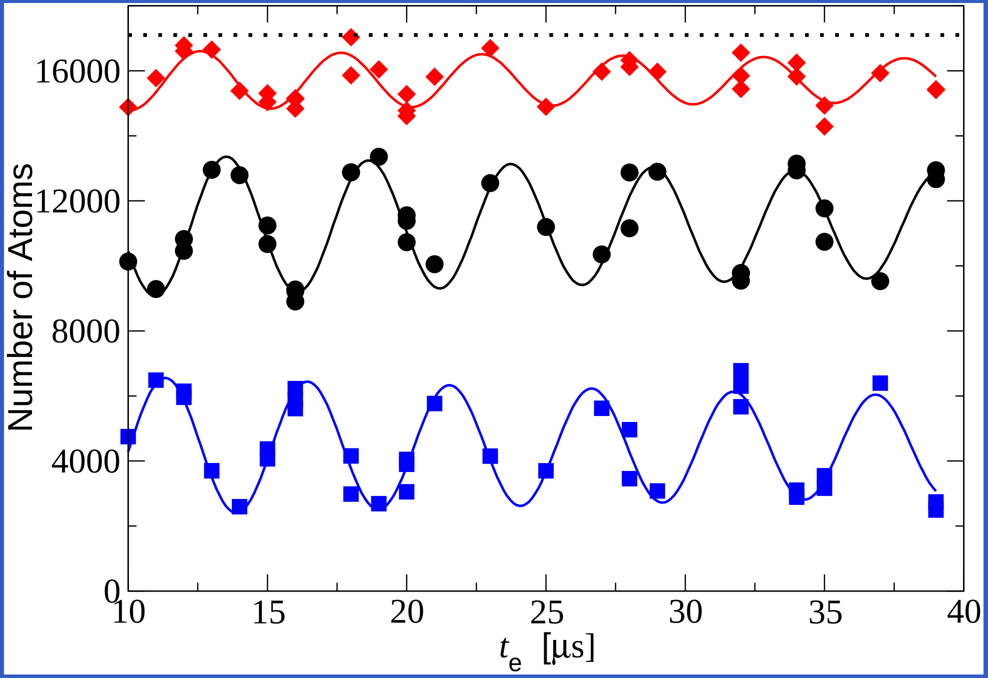
E.A. Donley, N.R. Claussen, S.T. Thompson, and C.E. Wieman, Nature **412**, 295 (2002)

Atom-molecule coherence

Experimental observations

Interference fringes in:

- a remnant Bose-Einstein condensate
- a “burst” of correlated atom pairs
- a component of undetected “missing atoms”



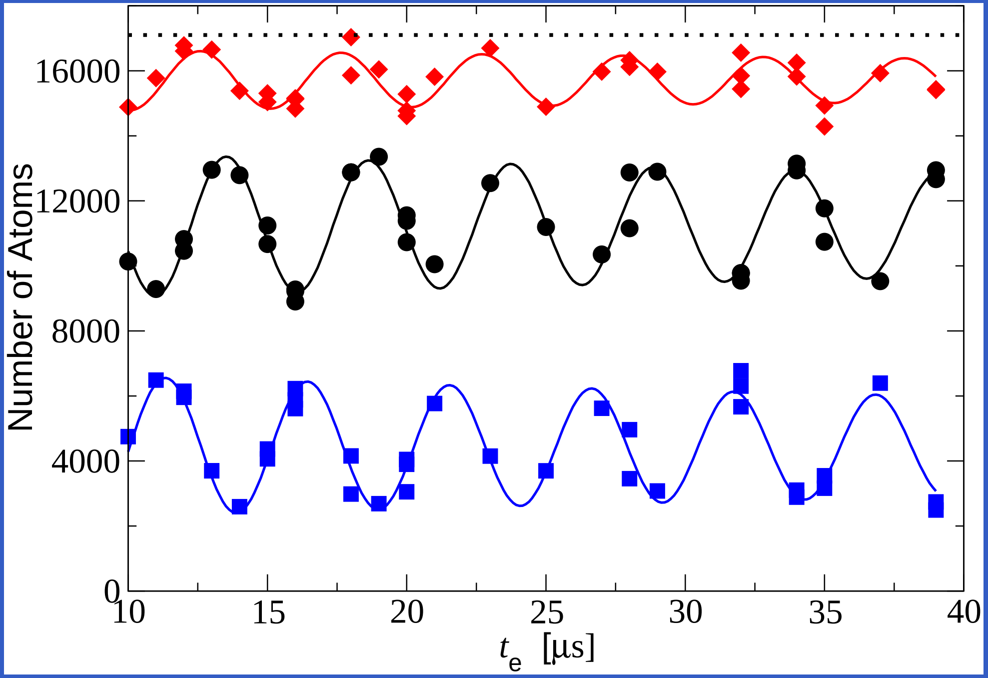
E.A. Donley, N.R. Claussen, S.T. Thompson, and C.E. Wieman, Nature **412**, 295 (2002)

Atom-molecule coherence

Experimental observations

Interference fringes in:

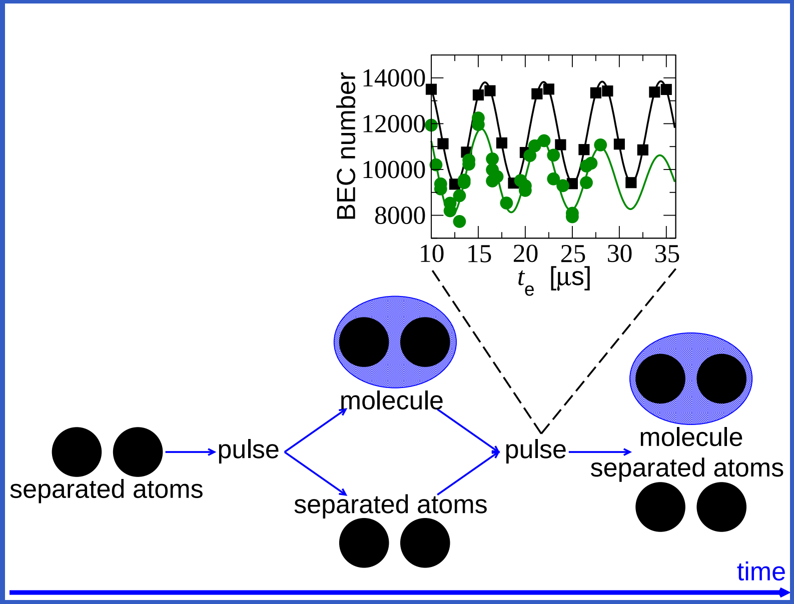
- a remnant Bose-Einstein condensate
- a “burst” of correlated atom pairs
- a component of undetected diatomic Feshbach molecules?



E.A. Donley, N.R. Claussen, S.T. Thompson, and C.E. Wieman, Nature **412**, 295 (2002)

Atom-molecule coherence

Ramsey interferometry with atoms and Feshbach molecules



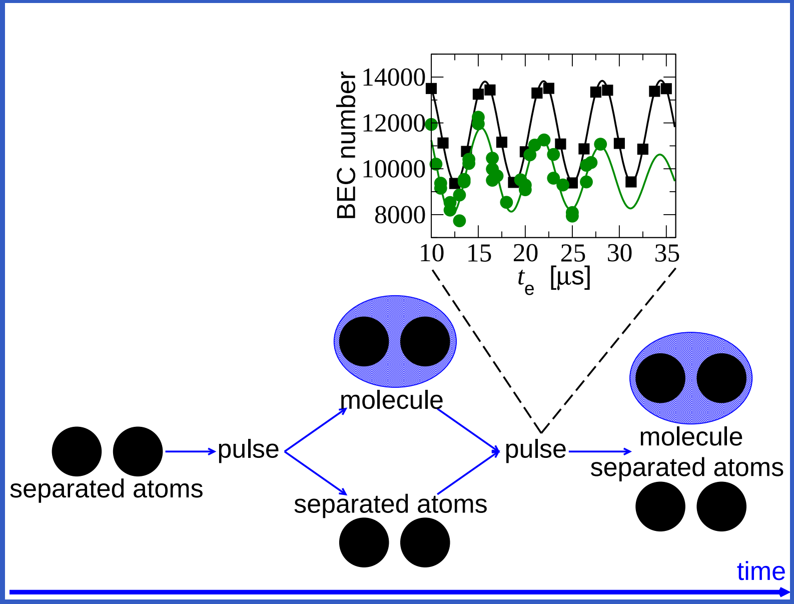
E.A. Donley *et al.*, Nature **412**, 295 (2002)
P. Zoller, Nature **417**, 493 (2002)

Atom-molecule coherence

Explanation in terms of diatomic dynamics

- Time-evolution operator:

$$i\hbar\frac{\partial}{\partial t}U_{2\mathrm{B}}(t,t_{\mathrm{i}}) = H_{2\mathrm{B}}(t)U_{2\mathrm{B}}(t,t_{\mathrm{i}})$$



E.A. Donley *et al.*, Nature **412**, 295 (2002)
P. Zoller, Nature **417**, 493 (2002)

Atom-molecule coherence

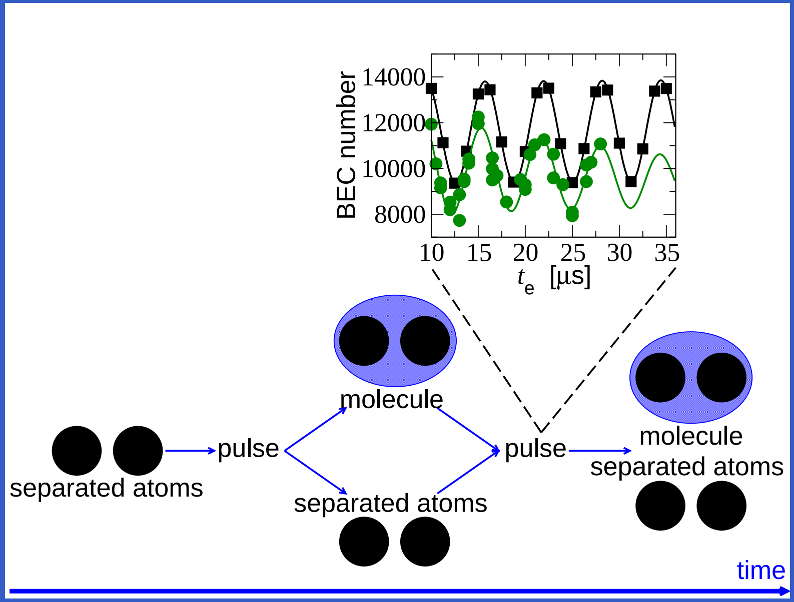
Explanation in terms of diatomic dynamics

- Time-evolution operator:

$$i\hbar\frac{\partial}{\partial t}U_{2\mathrm{B}}(t,t_{\mathrm{i}})=H_{2\mathrm{B}}(t)U_{2\mathrm{B}}(t,t_{\mathrm{i}})$$

- Factorisation:

$$U_{2\mathrm{B}}(t_{\mathrm{f}},t_{\mathrm{i}})=U_2(t_{\mathrm{f}},t_2)U_{\mathrm{e}}(t_{\mathrm{e}})U_1(t_1,t_{\mathrm{i}})$$



E.A. Donley *et al.*, Nature **412**, 295 (2002)
P. Zoller, Nature **417**, 493 (2002)

Atom-molecule coherence

Explanation in terms of diatomic dynamics

- Time-evolution operator:

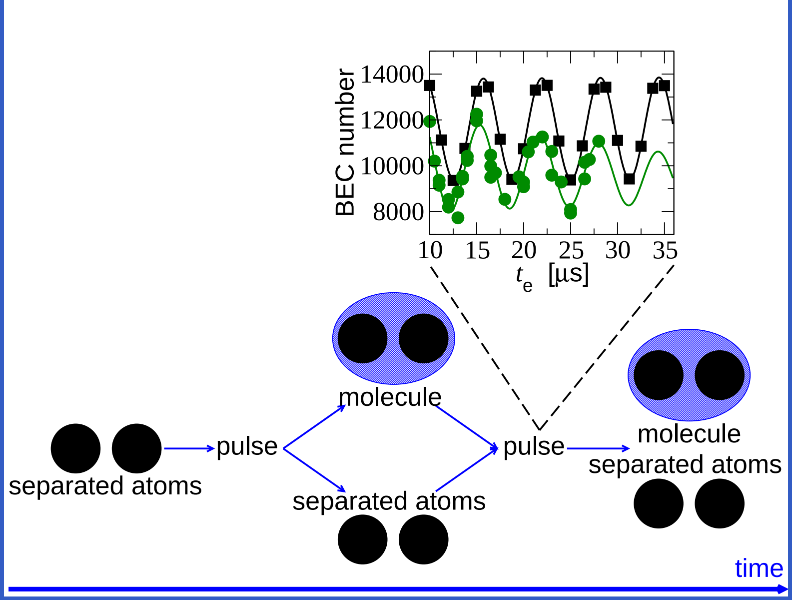
$$i\hbar\frac{\partial}{\partial t}U_{2\mathrm{B}}(t,t_\mathrm{i}) = H_{2\mathrm{B}}(t)U_{2\mathrm{B}}(t,t_\mathrm{i})$$

- Factorisation:

$$U_{2\mathrm{B}}(t_\mathrm{f},t_\mathrm{i}) = U_2(t_\mathrm{f},t_2)U_\mathrm{e}(t_\mathrm{e})U_1(t_1,t_\mathrm{i})$$

- Spectral decomposition ($\omega_{-1} = E_\mathrm{b}/\hbar$):

$$U_\mathrm{e}(t_\mathrm{e}) = \sum_{v=-1}^{\infty} |\phi_v^\mathrm{e}\rangle e^{-i\omega_v^\mathrm{e} t_\mathrm{e}} \langle\phi_v^\mathrm{e}|$$



E.A. Donley *et al.*, Nature **412**, 295 (2002)
P. Zoller, Nature **417**, 493 (2002)

Atom-molecule coherence

Explanation in terms of diatomic dynamics

- Time-evolution operator:

$$i\hbar\frac{\partial}{\partial t}U_{2\mathrm{B}}(t,t_{\mathrm{i}}) = H_{2\mathrm{B}}(t)U_{2\mathrm{B}}(t,t_{\mathrm{i}})$$

- Factorisation:

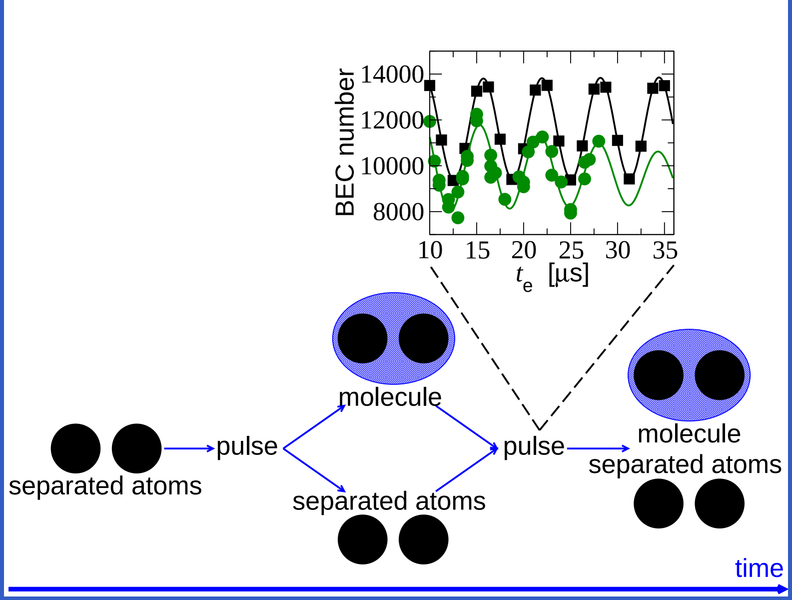
$$U_{2\mathrm{B}}(t_{\mathrm{f}},t_{\mathrm{i}}) = U_2(t_{\mathrm{f}},t_2)U_{\mathrm{e}}(t_{\mathrm{e}})U_1(t_1,t_{\mathrm{i}})$$

- Spectral decomposition ($\omega_{-1} = E_{\mathrm{b}}/\hbar$):

$$U_{\mathrm{e}}(t_{\mathrm{e}}) = \sum_{v=-1}^{\infty} |\phi_v^{\mathrm{e}}\rangle e^{-i\omega_v^{\mathrm{e}} t_{\mathrm{e}}} \langle\phi_v^{\mathrm{e}}|$$

- Transition amplitude:

$$\langle\phi_0^{\mathrm{f}}|U_{2\mathrm{B}}(t_{\mathrm{f}},t_{\mathrm{i}})|\phi_0^{\mathrm{i}}\rangle = D(t_{\mathrm{e}}) + Ae^{-i\omega_{-1}^{\mathrm{e}} t_{\mathrm{e}}}$$



E.A. Donley *et al.*, Nature **412**, 295 (2002)
P. Zoller, Nature **417**, 493 (2002)

Atom-molecule coherence

Explanation in terms of diatomic dynamics

- Time-evolution operator:

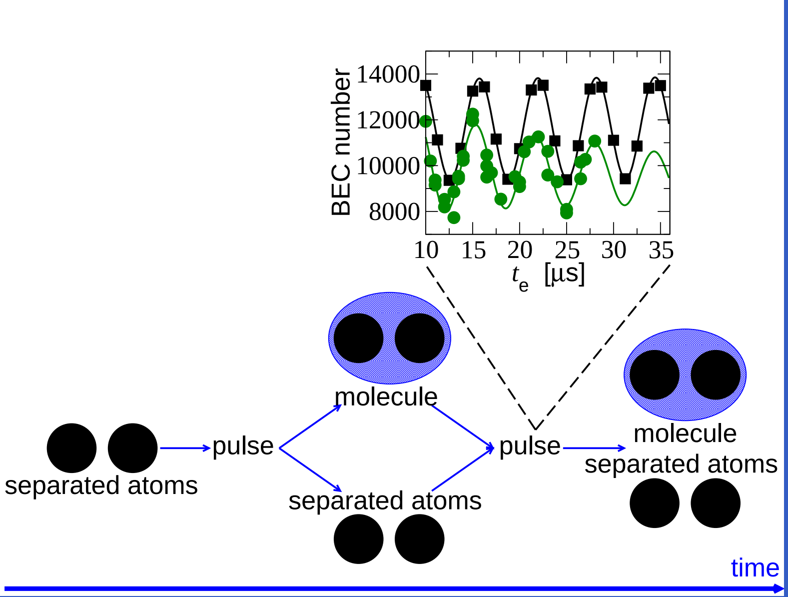
$$i\hbar\frac{\partial}{\partial t}U_{2\mathrm{B}}(t,t_{\mathrm{i}}) = H_{2\mathrm{B}}(t)U_{2\mathrm{B}}(t,t_{\mathrm{i}})$$

- Factorisation:

$$U_{2\mathrm{B}}(t_{\mathrm{f}},t_{\mathrm{i}}) = U_2(t_{\mathrm{f}},t_2)U_{\mathrm{e}}(t_{\mathrm{e}})U_1(t_1,t_{\mathrm{i}})$$

- Spectral decomposition ($\omega_{-1} = E_{\mathrm{b}}/\hbar$):

$$U_{\mathrm{e}}(t_{\mathrm{e}}) = \sum_{v=-1}^{\infty} |\phi_v^{\mathrm{e}}\rangle e^{-i\omega_v^{\mathrm{e}} t_{\mathrm{e}}} \langle\phi_v^{\mathrm{e}}|$$



E.A. Donley *et al.*, Nature **412**, 295 (2002)
P. Zoller, Nature **417**, 493 (2002)

- Transition amplitude:

$$\langle\phi_0^{\mathrm{f}}|U_{2\mathrm{B}}(t_{\mathrm{f}},t_{\mathrm{i}})|\phi_0^{\mathrm{i}}\rangle = D(t_{\mathrm{e}}) + Ae^{-i\omega_{-1}^{\mathrm{e}} t_{\mathrm{e}}}$$

- Transition probability:

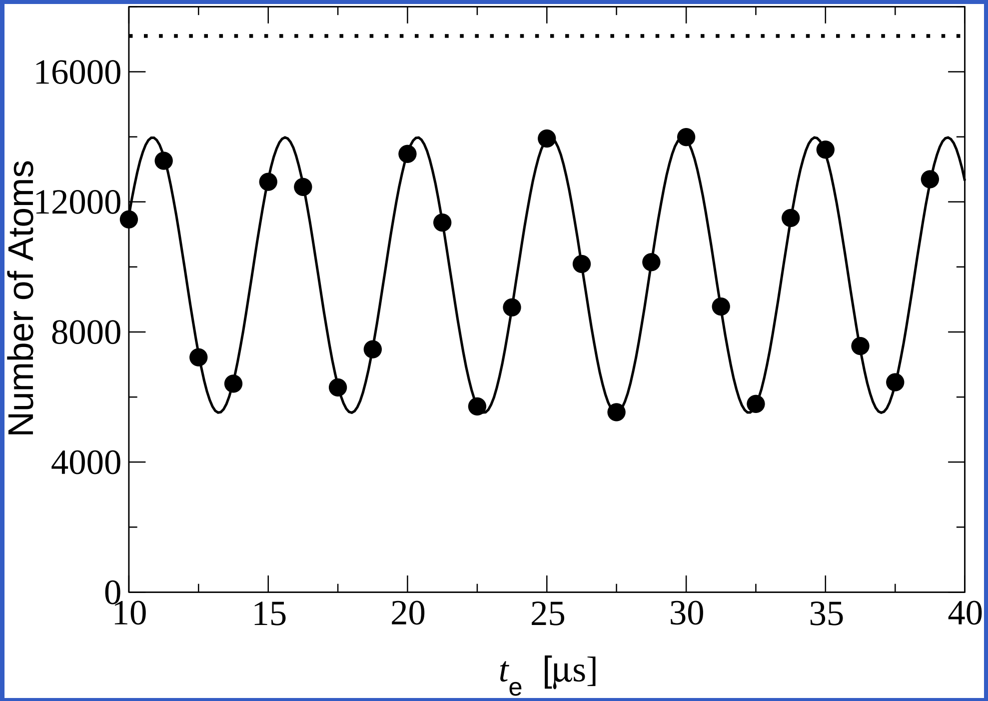
$$p_{0,0} = |D(t_{\mathrm{e}})|^2 + |A|^2 + 2|D(t_{\mathrm{e}})||A|\sin(\omega_{-1}^{\mathrm{e}} t_{\mathrm{e}} + \varphi)$$

Atom-molecule coherence

Three-component Ramsey fringes

- Remnant Bose-Einstein condensate:

$$p_{0,0} = \left| \langle \phi_0^{\mathrm{f}} | U_{2\mathrm{B}}(t_{\mathrm{f}}, t_{\mathrm{i}}) | \phi_0^{\mathrm{i}} \rangle \right|^2$$



This calculation: K. Góral, TK, and K. Burnett, PRA **71**, 023603 (2005)
See also: B. Borca, D. Blume, and C.H. Greene, New J. Phys. **5**, 111 (2003)

Atom-molecule coherence

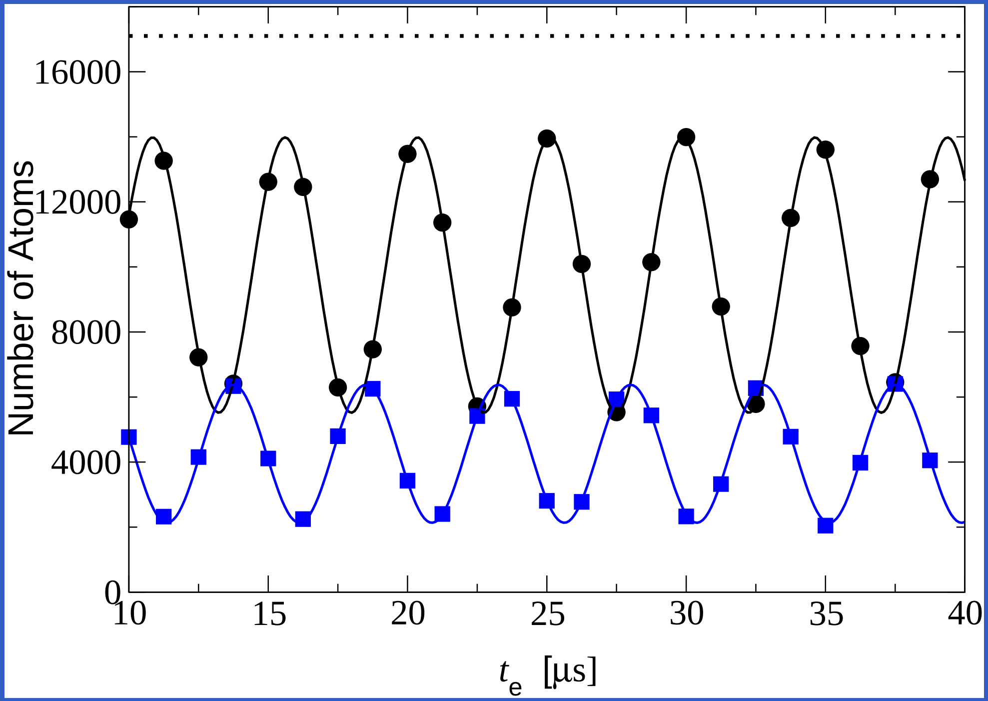
Three-component Ramsey fringes

- Remnant Bose-Einstein condensate:

$$p_{0,0} = \left|\langle\phi_0^{\mathrm{f}}|U_{2\mathrm{B}}(t_{\mathrm{f}}, t_{\mathrm{i}})|\phi_0^{\mathrm{i}}\rangle\right|^2$$

- “Burst” of correlated atom pairs:

$$\sum_{v=0}^{\infty} p_{0,v} = \sum_{v=0}^{\infty} \left|\langle\phi_v^{\mathrm{f}}|U_{2\mathrm{B}}(t_{\mathrm{f}}, t_{\mathrm{i}})|\phi_0^{\mathrm{i}}\rangle\right|^2$$



This calculation: K. Góral, TK, and K. Burnett, PRA **71**, 023603 (2005)
See also: B. Borca, D. Blume, and C.H. Greene, New J. Phys. **5**, 111 (2003)

Atom-molecule coherence

Three-component Ramsey fringes

- Remnant Bose-Einstein condensate:

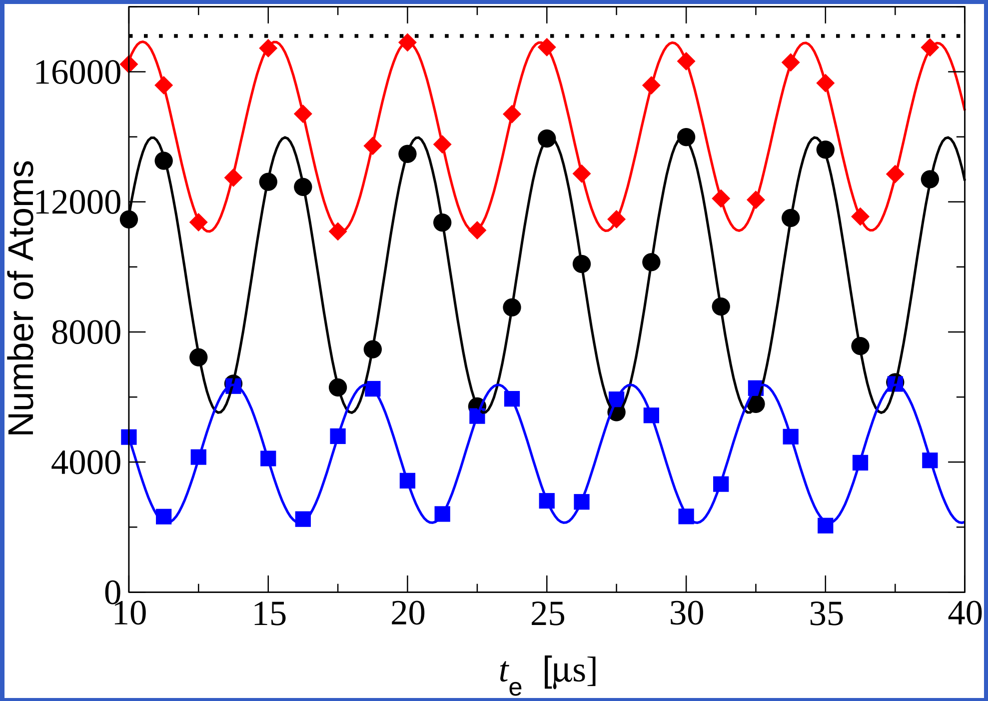
$$p_{0,0} = \left| \langle \phi_0^{\mathrm{f}} | U_{2\mathrm{B}}(t_{\mathrm{f}}, t_{\mathrm{i}}) | \phi_0^{\mathrm{i}} \rangle \right|^2$$

- "Burst" of correlated atom pairs:

$$\sum_{v=0}^{\infty} p_{0,v} = \sum_{v=0}^{\infty} \left| \langle \phi_v^{\mathrm{f}} | U_{2\mathrm{B}}(t_{\mathrm{f}}, t_{\mathrm{i}}) | \phi_0^{\mathrm{i}} \rangle \right|^2$$

- Component of diatomic Feshbach molecules:

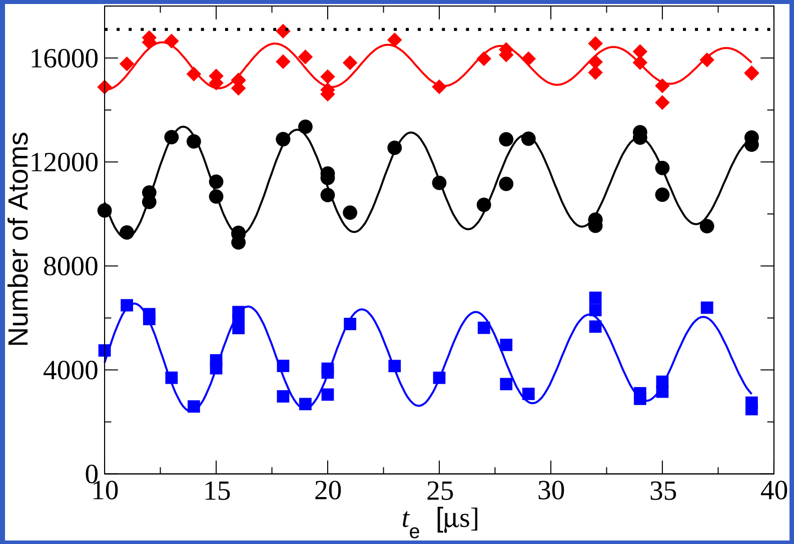
$$p_{0,\mathrm{b}} = \left| \langle \phi_{\mathrm{b}}^{\mathrm{f}} | U_{2\mathrm{B}}(t_{\mathrm{f}}, t_{\mathrm{i}}) | \phi_0^{\mathrm{i}} \rangle \right|^2$$



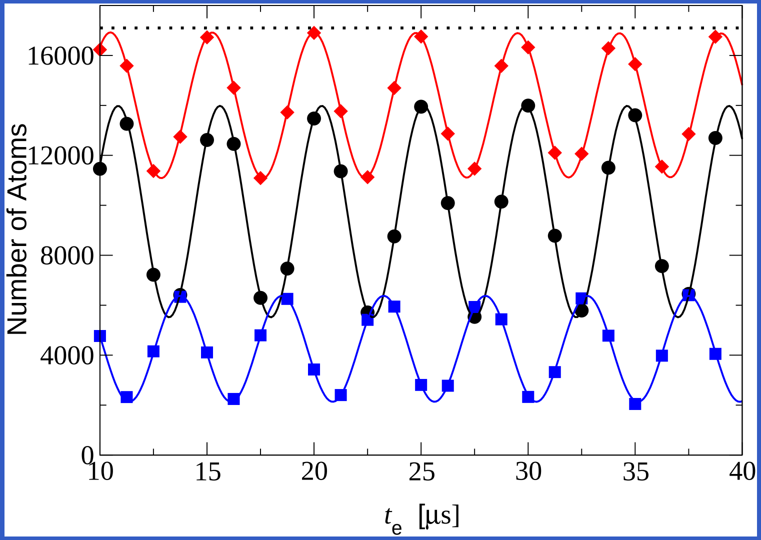
This calculation: K. Góral, TK, and K. Burnett, PRA **71**, 023603 (2005)
See also: B. Borca, D. Blume, and C.H. Greene, New J. Phys. **5**, 111 (2003)

Atom-molecule coherence

Three-component Ramsey fringes



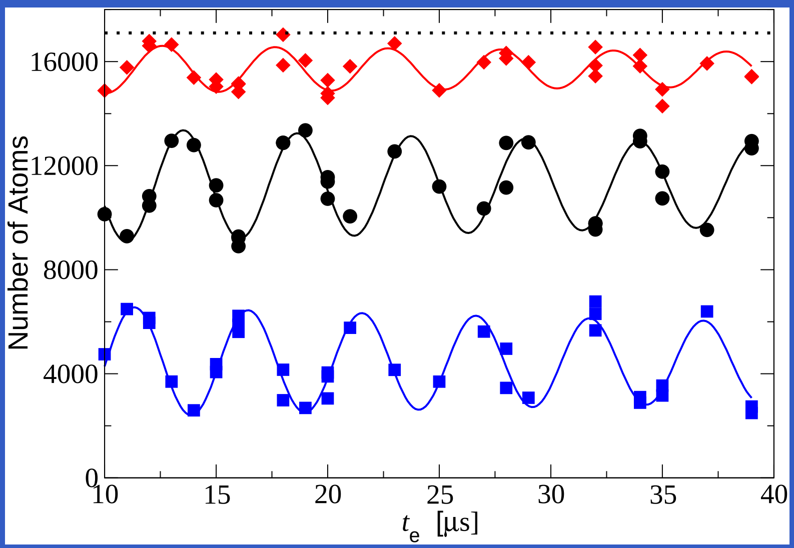
E.A. Donley *et al.*, Nature **412**, 295 (2002)



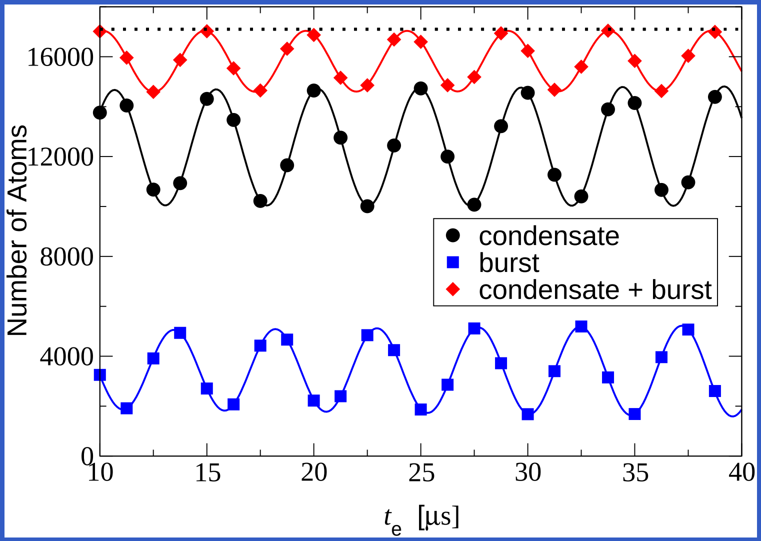
This calculation: K. Góral, TK, and K. Burnett, PRA **71**, 023603 (2005)
See also: B. Borca, D. Blume, and C.H. Greene, New J. Phys. **5**, 111 (2003)

Atom-molecule coherence

Three-component Ramsey fringes



E.A. Donley *et al.*, Nature **412**, 295 (2002)



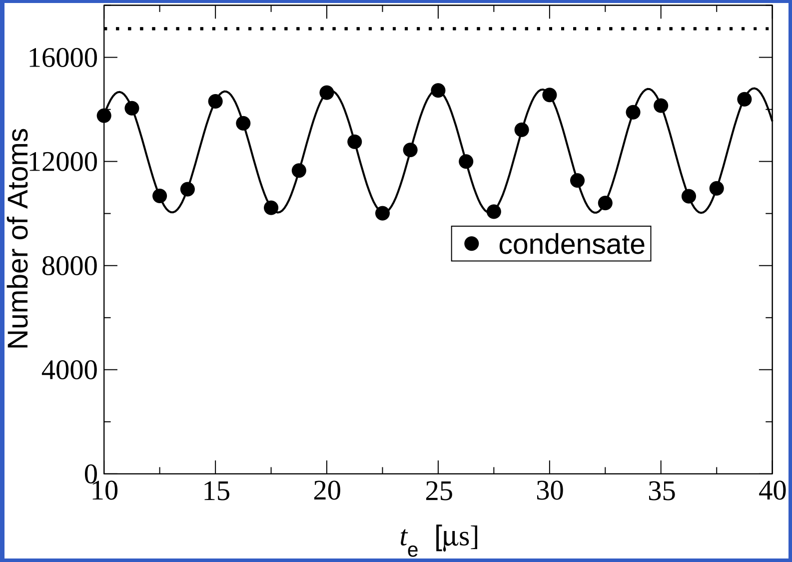
This calculation: K. Góral, TK, and K. Burnett,
PRA **71**, 023603 (2005)
See also: S.J.J.M.F. Kokkelmans and M.J. Holland,
PRL **89**, 180401 (2002)
M. Mackie, K.-A. Suominen, and J. Javanainen,
PRL **89**, 180403 (2002)
TK, T. Gasenzer, and K. Burnett,
PRA **67**, 013601 (2003)

Atom-molecule coherence

Remnant Bose-Einstein condensate

- Non-linear Schrödinger equation:

$$i\hbar\frac{\partial}{\partial t}\Psi(\mathbf{x},t) = \left[-\frac{\hbar^2}{2m}\nabla^2 + V_{\rm trap}(\mathbf{x})\right]\Psi(\mathbf{x},t)$$
$$-\Psi^*(\mathbf{x},t)\int_{t_{\rm i}}^{\infty} d\tau\, \Psi^*(\mathbf{x},\tau)\frac{\partial}{\partial\tau}h(t,\tau)$$



Atom-molecule coherence

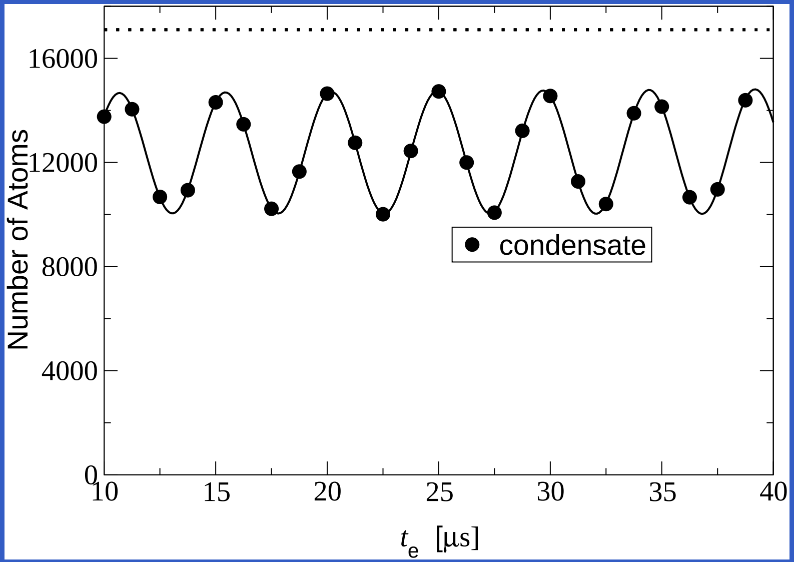
Remnant Bose-Einstein condensate

- Non-linear Schrödinger equation:

$$i\hbar\frac{\partial}{\partial t}\Psi(\mathbf{x},t) = \left[-\frac{\hbar^2}{2m}\nabla^2 + V_{\mathrm{trap}}(\mathbf{x})\right]\Psi(\mathbf{x},t) \\ -\Psi^*(\mathbf{x},t)\int_{t_{\mathrm{i}}}^{\infty} d\tau\,\Psi^*(\mathbf{x},\tau)\frac{\partial}{\partial\tau}h(t,\tau)$$

- Diatomic time evolution:

$$h(t,\tau) = (2\pi\hbar)^3\theta(t-\tau)\langle 0|V(t)U_{2\mathrm{B}}(t,\tau)|0\rangle$$

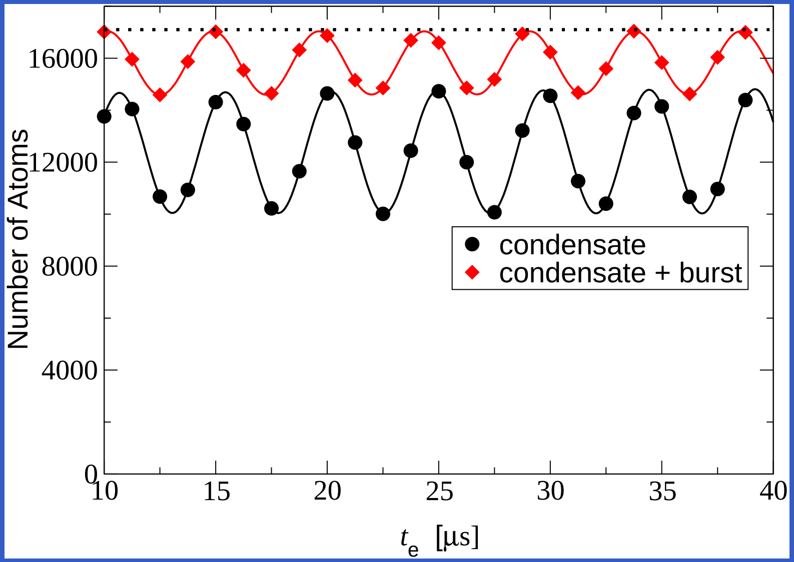


Atom-molecule coherence

Number of diatomic molecules in a gas

- Number operator:

$$\mathrm{N_b} = \frac{1}{2} \sum_{\substack{i,j=1 \\ i \neq j}}^{N} |\phi^{\mathrm{b}}_{ij}\rangle\langle\phi^{\mathrm{b}}_{ij}|$$



Atom-molecule coherence

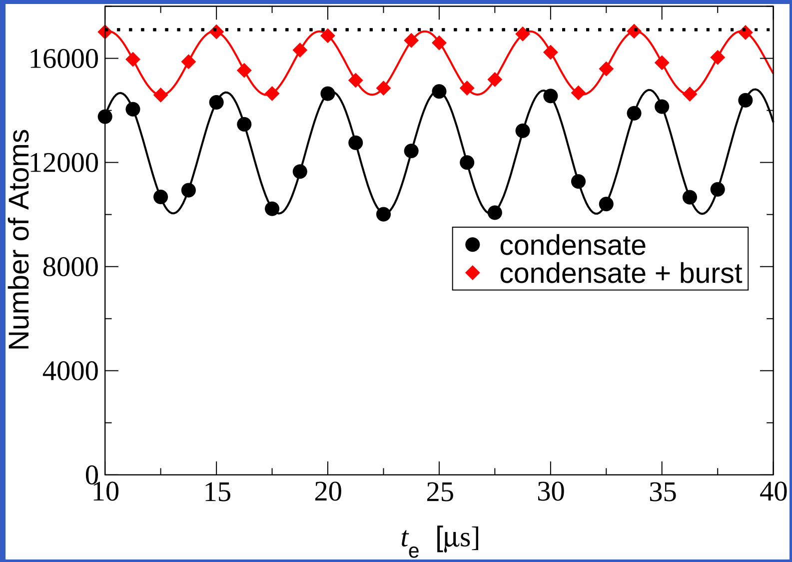
Number of diatomic molecules in a gas

- Number operator:

$$\mathrm{N_b} = \frac{1}{2} \sum_{\substack{i,j=1 \\ i \neq j}}^{N} |\phi_{ij}^{\mathrm{b}}\rangle\langle\phi_{ij}^{\mathrm{b}}|$$

- Two-body correlation function:

$$G^{(2)}(\mathbf{x}', \mathbf{y}'; \mathbf{x}, \mathbf{y}) = \langle \psi^\dagger(\mathbf{x}') \psi^\dagger(\mathbf{y}') \psi(\mathbf{y}) \psi(\mathbf{x}) \rangle$$



Atom-molecule coherence

Number of diatomic molecules in a gas

- Number operator:

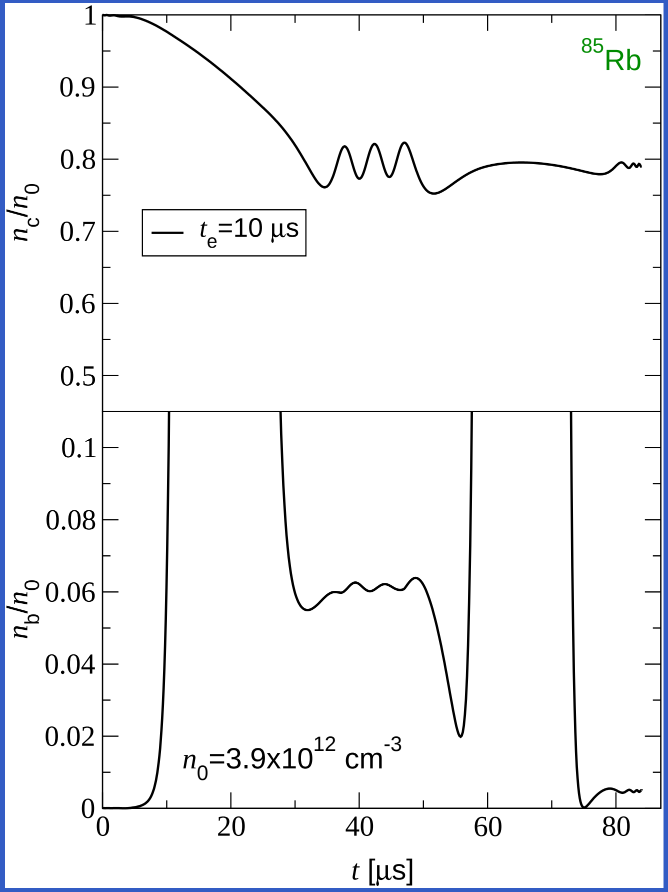
$$\mathsf{N}_\mathrm{b} = \frac{1}{2} \sum_{\substack{i,j=1 \\ i \neq j}}^{N} |\phi_{ij}^\mathrm{b}\rangle\langle\phi_{ij}^\mathrm{b}|$$

- Two-body correlation function:

$$G^{(2)}(\mathbf{x}',\mathbf{y}';\mathbf{x},\mathbf{y}) = \langle\psi^\dagger(\mathbf{x}')\psi^\dagger(\mathbf{y}')\psi(\mathbf{y})\psi(\mathbf{x})\rangle$$

- Number of diatomic molecules:

$$N_\mathrm{b} = \frac{1}{2}\int d\mathbf{r}' \int d\mathbf{r} \int d\mathbf{R}\, \phi_\mathrm{b}(\mathbf{r}')\phi_\mathrm{b}^*(\mathbf{r}) \\ \times G^{(2)}\left(\mathbf{R}+\mathbf{r}'/2, \mathbf{R}-\mathbf{r}'/2; \mathbf{R}+\mathbf{r}/2, \mathbf{R}-\mathbf{r}/2\right)$$



Atom-molecule coherence

Number of diatomic molecules in a gas

- Number operator:

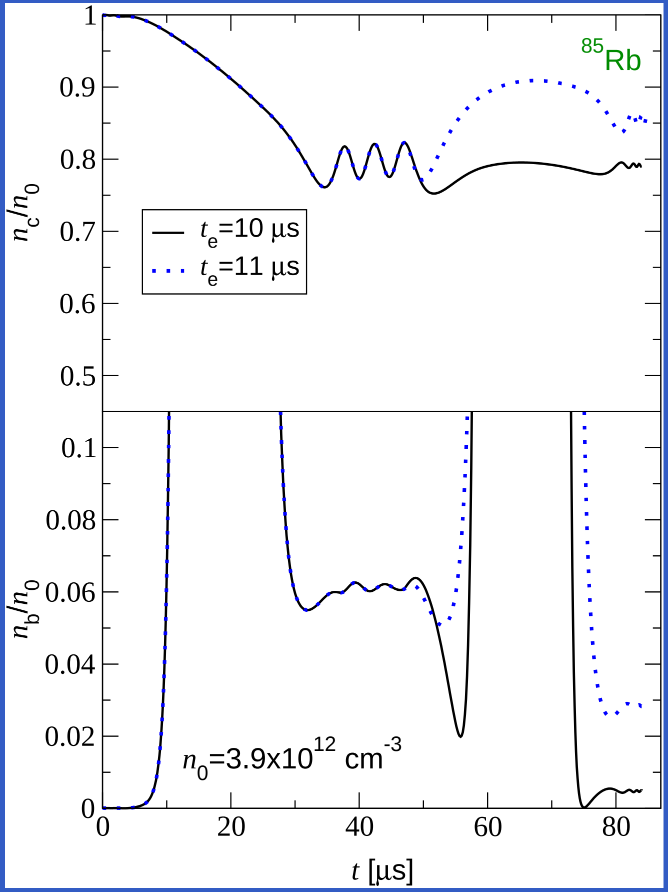
$$\mathrm{N_b} = \frac{1}{2} \sum_{\substack{i,j=1 \\ i \neq j}}^{N} |\phi^{\mathrm{b}}_{ij}\rangle\langle\phi^{\mathrm{b}}_{ij}|$$

- Two-body correlation function:

$$G^{(2)}(\mathbf{x}', \mathbf{y}'; \mathbf{x}, \mathbf{y}) = \langle\psi^\dagger(\mathbf{x}')\psi^\dagger(\mathbf{y}')\psi(\mathbf{y})\psi(\mathbf{x})\rangle$$

- Number of diatomic molecules:

$$N_{\mathrm{b}} = \frac{1}{2} \int d\mathbf{r}' \int d\mathbf{r} \int d\mathbf{R}\, \phi_{\mathrm{b}}(\mathbf{r}')\phi^*_{\mathrm{b}}(\mathbf{r}) \\ \times G^{(2)}\left(\mathbf{R} + \mathbf{r}'/2, \mathbf{R} - \mathbf{r}'/2; \mathbf{R} + \mathbf{r}/2, \mathbf{R} - \mathbf{r}/2\right)$$



Atom-molecule coherence

Number of diatomic molecules in a gas

- Number operator:

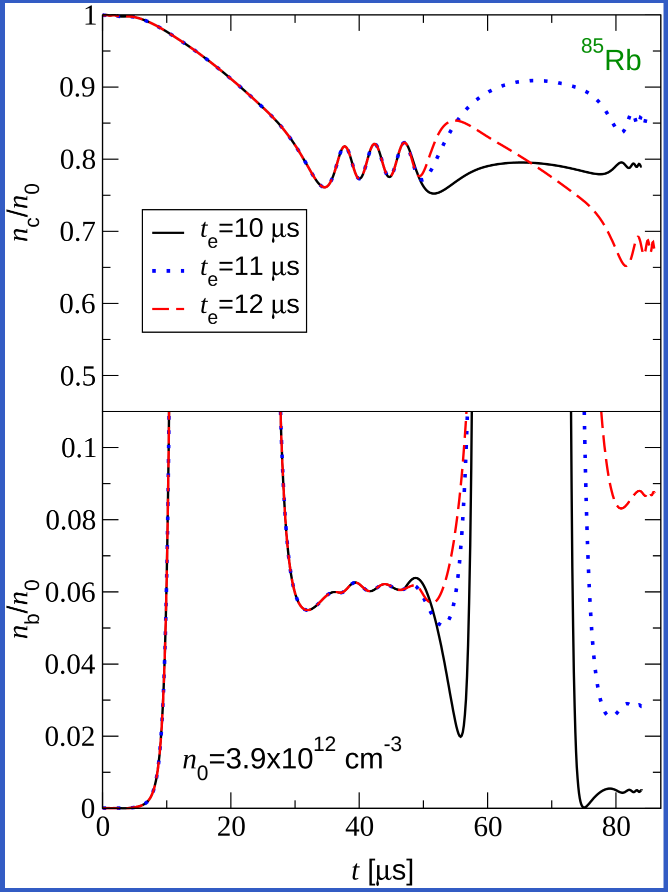
$$\mathrm{N_b} = \frac{1}{2} \sum_{\substack{i,j=1 \\ i \neq j}}^{N} |\phi^{\mathrm{b}}_{ij}\rangle\langle\phi^{\mathrm{b}}_{ij}|$$

- Two-body correlation function:

$$G^{(2)}(\mathbf{x}', \mathbf{y}'; \mathbf{x}, \mathbf{y}) = \langle \psi^\dagger(\mathbf{x}')\psi^\dagger(\mathbf{y}')\psi(\mathbf{y})\psi(\mathbf{x})\rangle$$

- Number of diatomic molecules:

$$N_{\mathrm{b}} = \frac{1}{2} \int d\mathbf{r}' \int d\mathbf{r} \int d\mathbf{R}\, \phi_{\mathrm{b}}(\mathbf{r}')\phi^*_{\mathrm{b}}(\mathbf{r}) \\ \times G^{(2)}\left(\mathbf{R} + \mathbf{r}'/2, \mathbf{R} - \mathbf{r}'/2; \mathbf{R} + \mathbf{r}/2, \mathbf{R} - \mathbf{r}/2\right)$$



Atom-molecule coherence

Number of diatomic molecules in a gas

- Number operator:

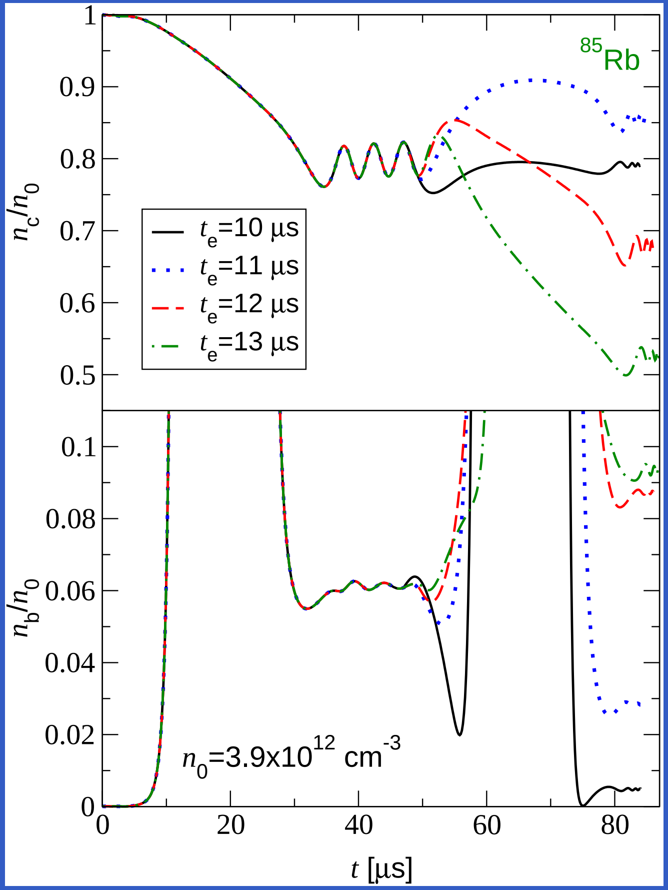
$$\mathrm{N_b} = \frac{1}{2} \sum_{\substack{i,j=1 \\ i\neq j}}^{N} |\phi^{\mathrm{b}}_{ij}\rangle\langle\phi^{\mathrm{b}}_{ij}|$$

- Two-body correlation function:

$$G^{(2)}(\mathbf{x}',\mathbf{y}';\mathbf{x},\mathbf{y}) = \langle\psi^\dagger(\mathbf{x}')\psi^\dagger(\mathbf{y}')\psi(\mathbf{y})\psi(\mathbf{x})\rangle$$

- Number of diatomic molecules:

$$N_{\mathrm{b}} = \frac{1}{2}\int d\mathbf{r}' \int d\mathbf{r} \int d\mathbf{R}\, \phi_{\mathrm{b}}(\mathbf{r}')\phi^*_{\mathrm{b}}(\mathbf{r}) \times G^{(2)}\left(\mathbf{R}+\mathbf{r}'/2, \mathbf{R}-\mathbf{r}'/2; \mathbf{R}+\mathbf{r}/2, \mathbf{R}-\mathbf{r}/2\right)$$

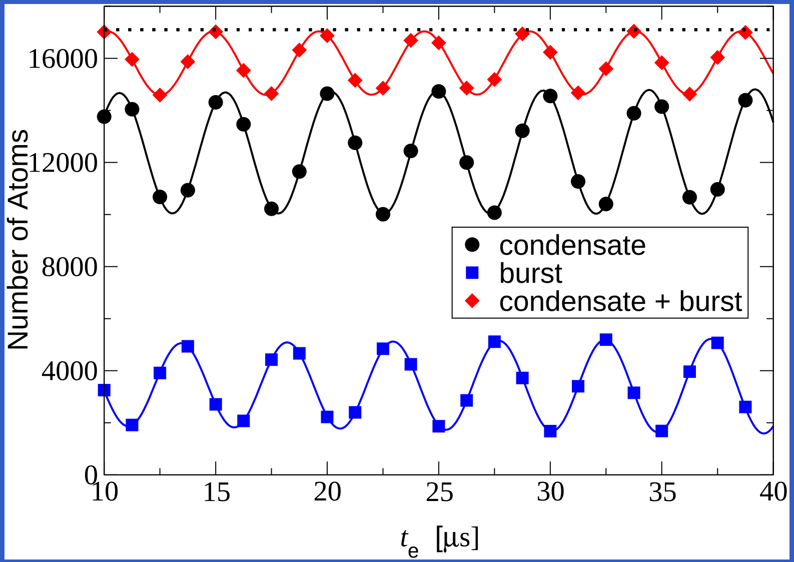


Atom-molecule coherence

Burst of atoms

- Number of non-condensed atoms:

$$
\begin{aligned}
N_{\rm nc}(t) &= \int d\mathbf{x}\,\Gamma(\mathbf{x},\mathbf{x},t) \\
&= \int d\mathbf{x} \int d\mathbf{y}\, |\Phi(\mathbf{x},\mathbf{y},t)|^2
\end{aligned}
$$



Atom-molecule coherence

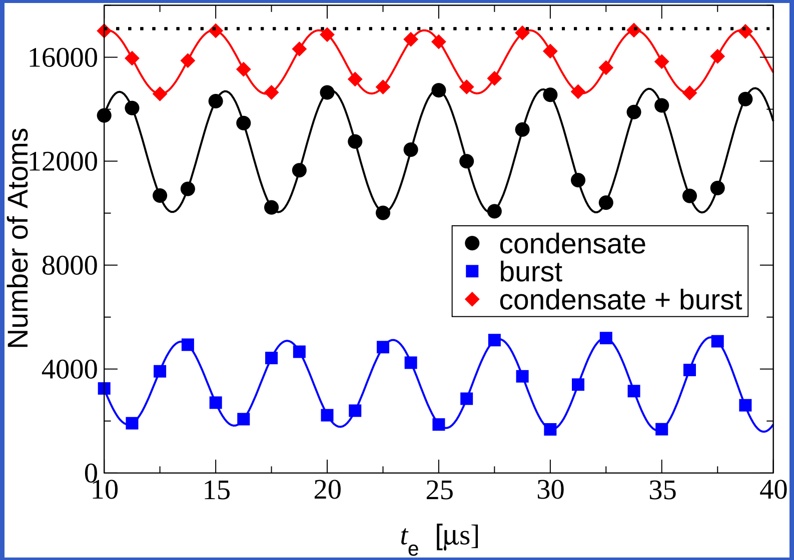
Burst of atoms

- Number of non-condensed atoms:

$$N_{\mathrm{nc}}(t) = \int d\mathbf{x}\,\Gamma(\mathbf{x},\mathbf{x},t)$$
$$= \int d\mathbf{x} \int d\mathbf{y}\,|\Phi(\mathbf{x},\mathbf{y},t)|^2$$

- Total number of atoms:

$$\int d\mathbf{x}\,\left|\Psi(\mathbf{x},t_{\mathrm{f}})\right|^2 + \int d\mathbf{R}\left[\left|\langle \mathbf{R},\phi_{\mathrm{b}}^{\mathrm{f}}|\Phi(t_{\mathrm{f}})\rangle\right|^2 + \int d\mathbf{p}\,\left|\langle \mathbf{R},\phi_{\mathbf{p}}^{\mathrm{f}}|\Phi(t_{\mathrm{f}})\rangle\right|^2\right] = N$$

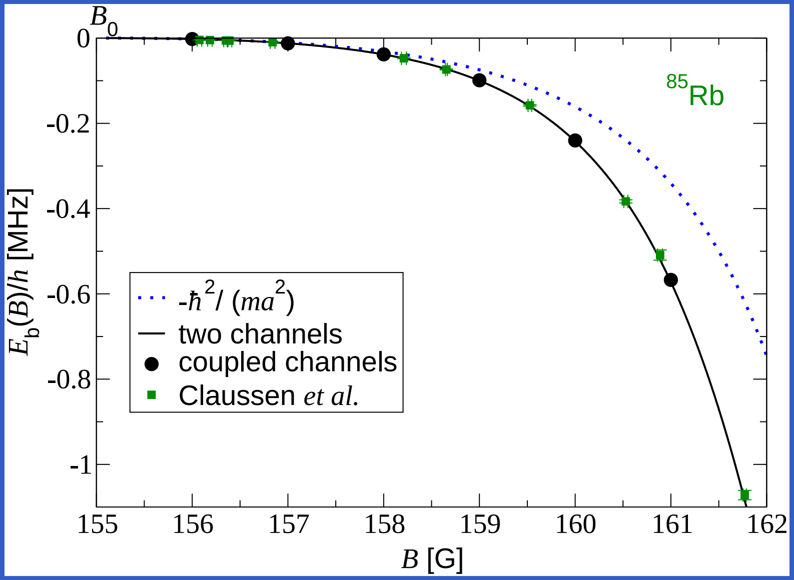


Atom-molecule coherence

Oscillation frequencies determine bound-state energies!

- Number of non-condensed atoms:

$$N_{\mathrm{nc}}(t) = \int d\mathbf{x}\, \Gamma(\mathbf{x},\mathbf{x},t) = \int d\mathbf{x} \int d\mathbf{y}\, |\Phi(\mathbf{x},\mathbf{y},t)|^2$$



- Total number of atoms:

$$\int d\mathbf{x}\, |\Psi(\mathbf{x},t_{\mathrm{f}})|^2 + \int d\mathbf{R} \left[\left|\langle \mathbf{R}, \phi_{\mathrm{b}}^{\mathrm{f}} | \Phi(t_{\mathrm{f}})\rangle\right|^2 + \int d\mathbf{p}\, \left|\langle \mathbf{R}, \phi_{\mathbf{p}}^{\mathrm{f}} | \Phi(t_{\mathrm{f}})\rangle\right|^2 \right] = N$$

N.R. Claussen, S.J.J.M.F. Kokkelmans, S.T. Thompson, E.A. Donley, E. Hodby, and C.E. Wieman, PRA **67**, 060701(R) (2003)

Atom-molecule coherence

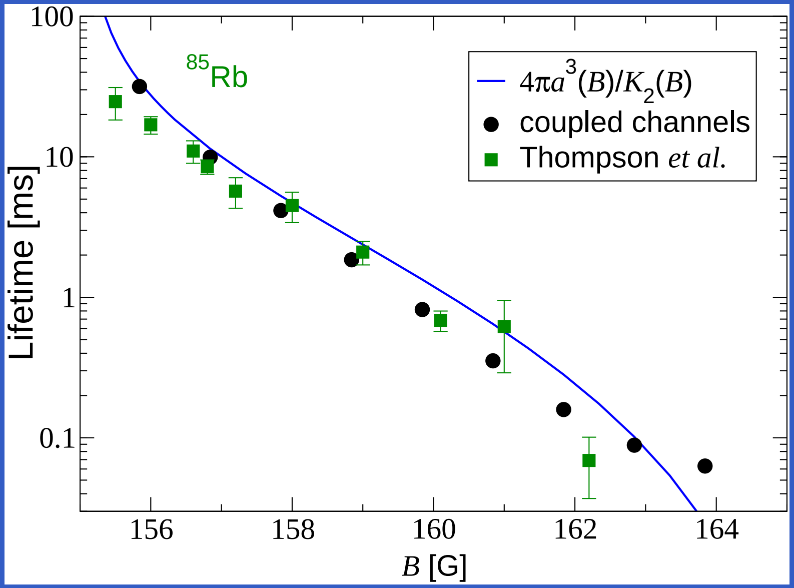
Why are the Feshbach molecules undetectable?

- Number of non-condensed atoms:

$$N_{\mathrm{nc}}(t) = \int d\mathbf{x}\, \Gamma(\mathbf{x}, \mathbf{x}, t) = \int d\mathbf{x} \int d\mathbf{y}\, |\Phi(\mathbf{x}, \mathbf{y}, t)|^2$$

- Total number of atoms:

$$\int d\mathbf{x}\, |\Psi(\mathbf{x}, t_{\mathrm{f}})|^2 + \int d\mathbf{R} \left[\left|\langle \mathbf{R}, \phi_{\mathrm{b}}^{\mathrm{f}} | \Phi(t_{\mathrm{f}})\rangle\right|^2 + \int d\mathbf{p}\, \left|\langle \mathbf{R}, \phi_{\mathbf{p}}^{\mathrm{f}} | \Phi(t_{\mathrm{f}})\rangle\right|^2 \right] = N$$

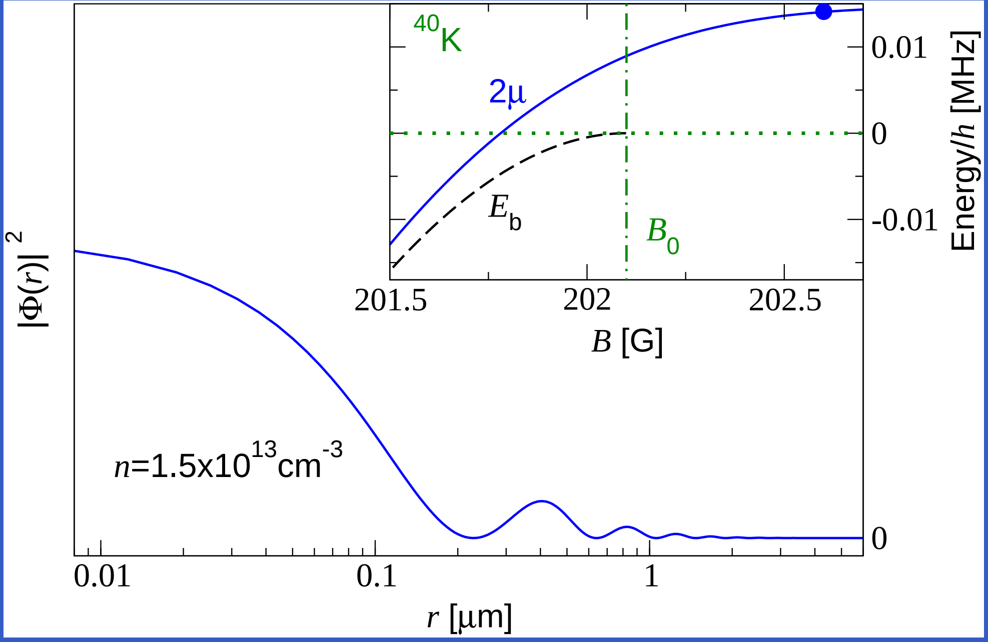


Experiment: S.T. Thompson, E. Hodby, and C.E. Wieman, PRL **94**, 020401 (2005)
Theory: TK, E. Tiesinga, and P.S. Julienne, PRL **94**, 020402 (2005)

Outlook: Pairing in correlated gases

Pairing in two-spin-component Fermi gases

- Twice the chemical potential, 2μ, approaches $E_{\rm b}$, as the magnetic-field strength B crosses the resonance position, B_0.
- The pair function is transferred smoothly into the Feshbach molecule.

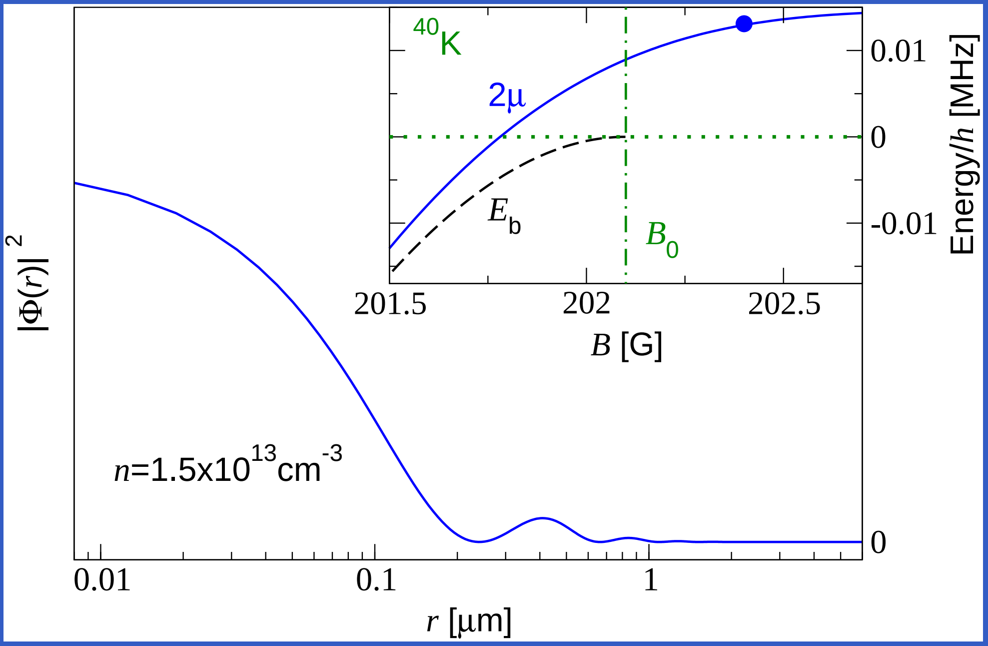


This calculation: M.H. Szymańska, K. Góral, TK, and K. Burnett, PRA **72**, 013610 (2005)
See also: M.H. Szymańska, B.D. Simons, and K. Burnett, PRL **94**, 170402 (2005)

Outlook: Pairing in correlated gases

Pairing in two-spin-component Fermi gases

- Twice the chemical potential, 2μ, approaches E_b, as the magnetic-field strength B crosses the resonance position, B_0.
- The pair function is transferred smoothly into the Feshbach molecule.

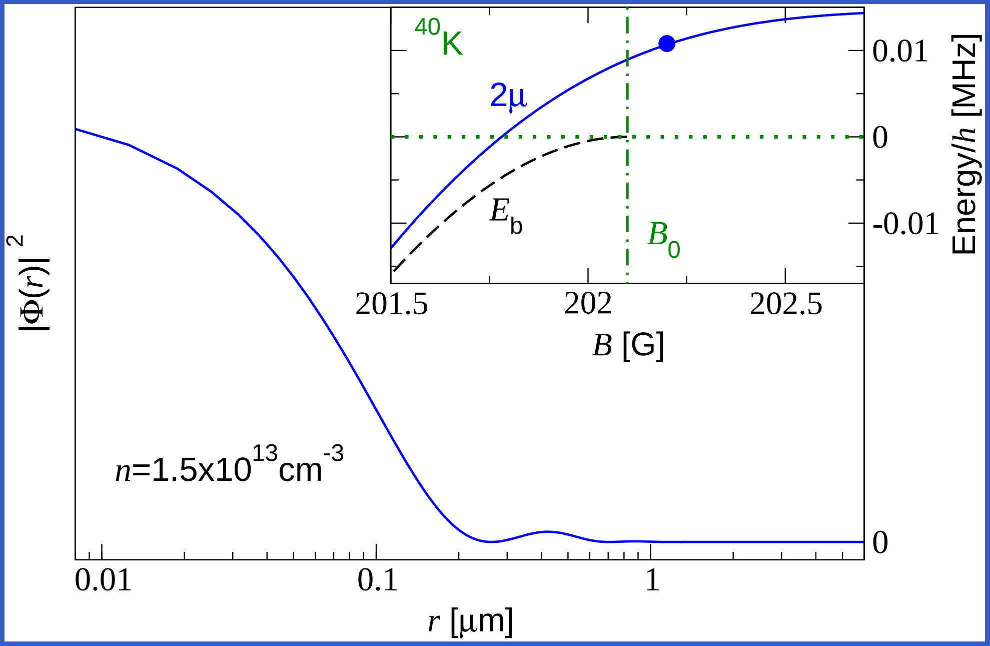


This calculation: M.H. Szymańska, K. Góral, TK, and K. Burnett, PRA **72**, 013610 (2005)
See also: M.H. Szymańska, B.D. Simons, and K. Burnett, PRL **94**, 170402 (2005)

Outlook: Pairing in correlated gases

Pairing in two-spin-component Fermi gases

- Twice the chemical potential, 2μ, approaches E_b, as the magnetic-field strength B crosses the resonance position, B_0.
- The pair function is transferred smoothly into the Feshbach molecule.

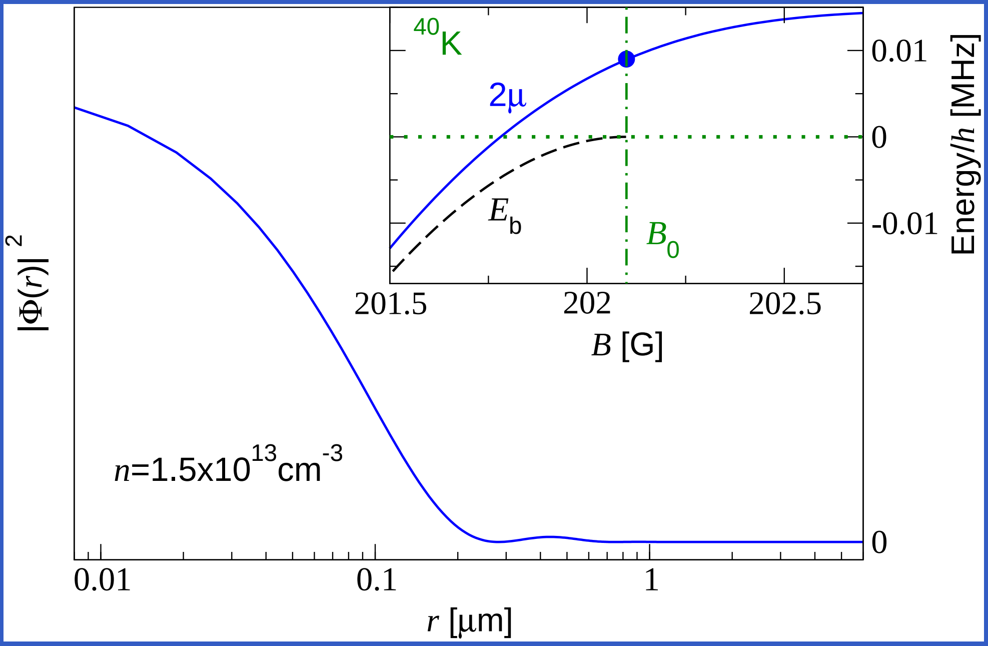


This calculation: M.H. Szymańska, K. Góral, TK, and K. Burnett, PRA **72**, 013610 (2005)
See also: M.H. Szymańska, B.D. Simons, and K. Burnett, PRL **94**, 170402 (2005)

Outlook: Pairing in correlated gases

Pairing in two-spin-component Fermi gases

- Twice the chemical potential, 2μ, approaches $E_{\rm b}$, as the magnetic-field strength B crosses the resonance position, B_0.
- The pair function is transferred smoothly into the Feshbach molecule.

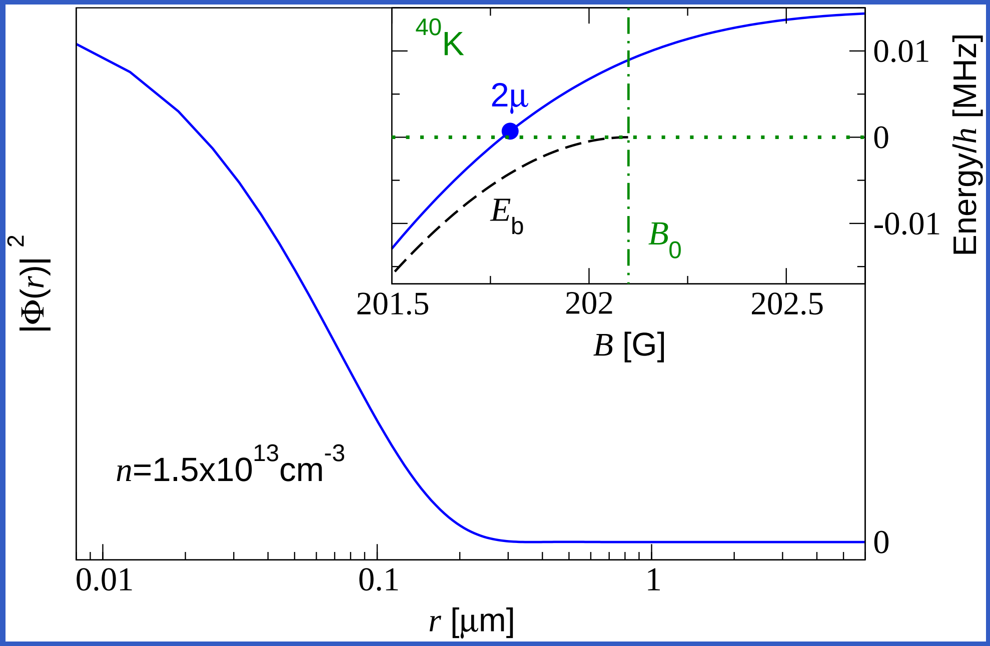


This calculation: M.H. Szymańska, K. Góral, TK, and K. Burnett, PRA **72**, 013610 (2005)
See also: M.H. Szymańska, B.D. Simons, and K. Burnett, PRL **94**, 170402 (2005)

Outlook: Pairing in correlated gases

Pairing in two-spin-component Fermi gases

- Twice the chemical potential, 2μ, approaches E_b, as the magnetic-field strength B crosses the resonance position, B_0.
- The pair function is transferred smoothly into the Feshbach molecule.

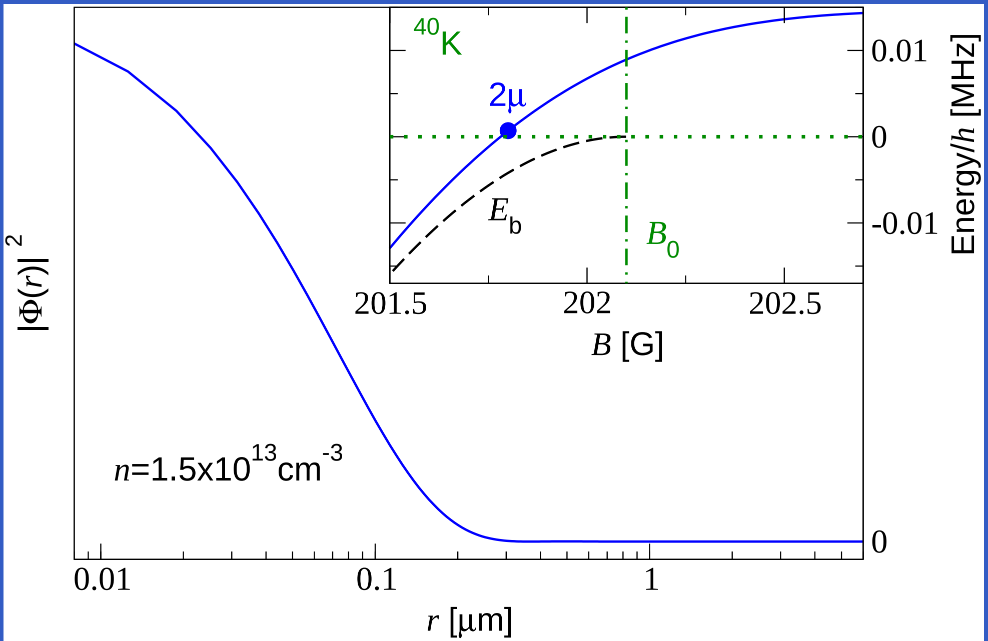


This calculation: M.H. Szymańska, K. Góral, TK, and K. Burnett, PRA **72**, 013610 (2005)
See also: M.H. Szymańska, B.D. Simons, and K. Burnett, PRL **94**, 170402 (2005)

Outlook: Pairing in correlated gases

Pairing in two-spin-component Fermi gases

- Twice the chemical potential, 2μ, approaches E_b, as the magnetic-field strength B crosses the resonance position, B_0.
- The pair function is transferred smoothly into the Feshbach molecule.
- In the mean-field approximation at zero temperature an atomic gas is converted adiabatically into a molecular Bose-Einstein condensate.

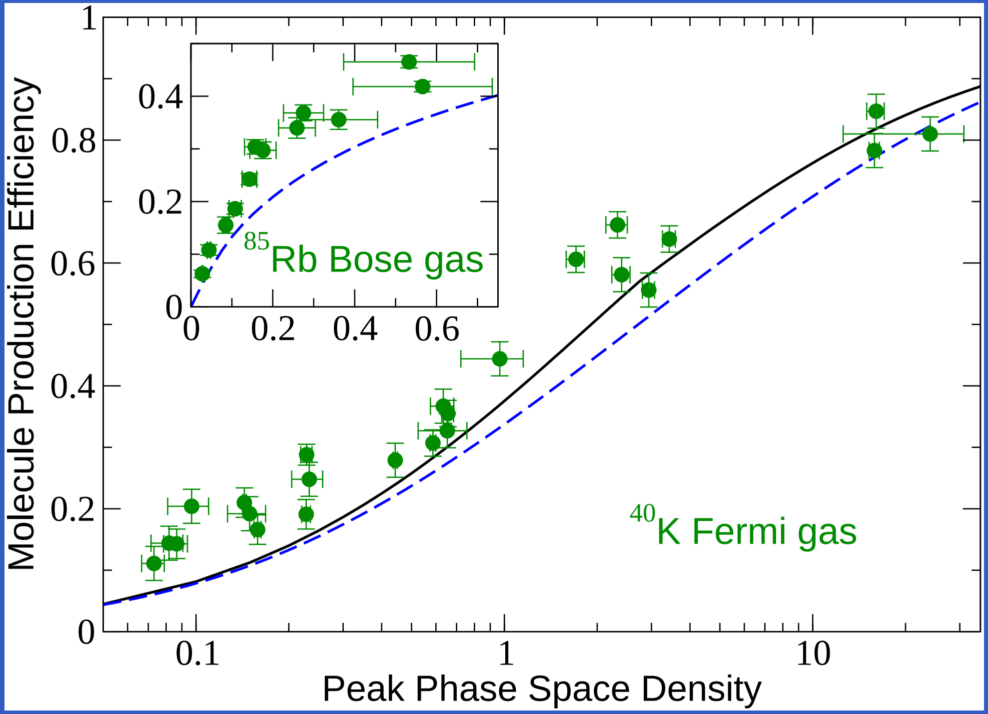


This calculation: M.H. Szymańska, K. Góral, TK, and K. Burnett, PRA **72**, 013610 (2005)
See also: M.H. Szymańska, B.D. Simons, and K. Burnett, PRL **94**, 170402 (2005)

Outlook: Pairing in correlated gases

Pairing in two-spin-component Fermi gases

- Twice the chemical potential, 2μ, approaches E_{b}, as the magnetic-field strength B crosses the resonance position, B_0.
- The pair function is transferred smoothly into the Feshbach molecule.
- In the mean-field approximation at zero temperature an atomic gas is converted adiabatically into a molecular Bose-Einstein condensate.



This experiment: E. Hodby *et al.*, PRL **94**, 120402 (2005)
This theory: J.E. Williams, N. Nygaard, and C.W. Clark, New J. Phys. **8**, 150 (2006)

Thanks to

- DPhil students: Thomas Hanna and Hugo Martay
- Collaborators: Marzena Szymańska, Mark Lee, and Martin Stoll
- Group leader: Keith Burnett
- Coupled-channels calculations: Paul Julienne (NIST)
- Experimental groups at: Oxford, JILA, Innsbruck, MIT, Rice, and Munich

THE ROYAL SOCIETY

EPSRC