Inflation & Alternatives

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2008

Big Bang/Inflationary Picture

big bang

inflationary epoch

radiation epoch

matter epoch

dark energy epoch

"DISJOINT"

Inflation

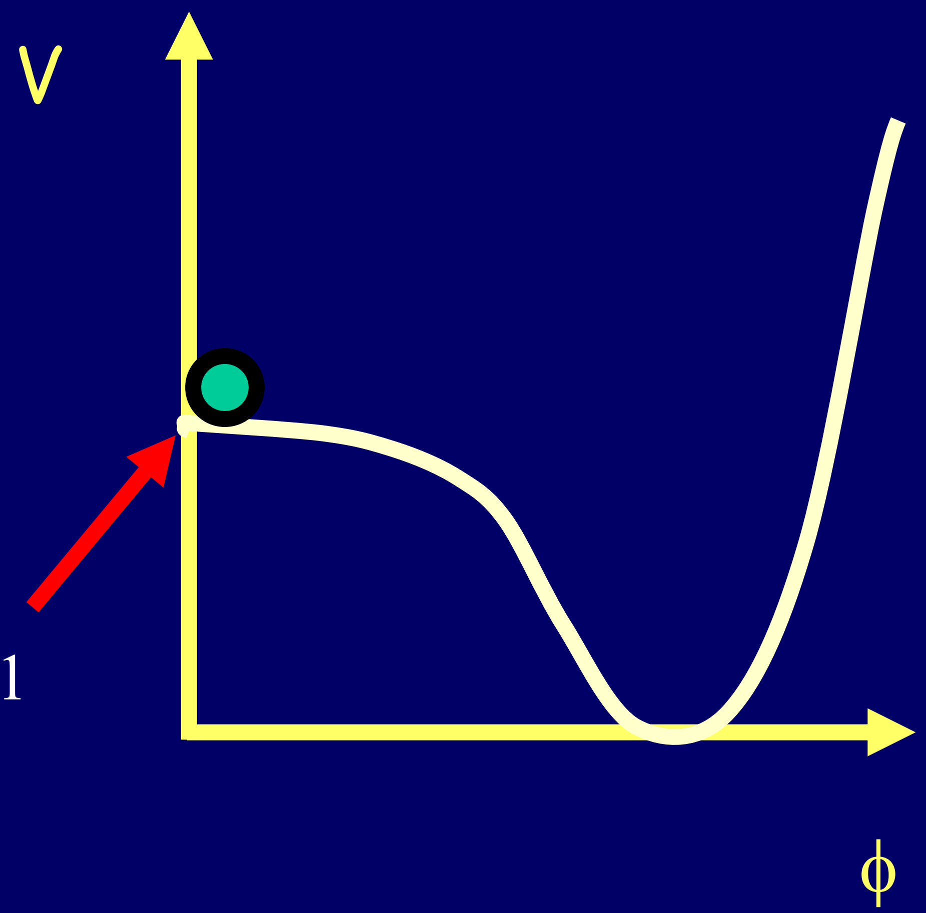
Great explanatory power:

horizon – flatness – monopoles – entropy

Great predictive power:

$\Omega_{total} = 1$
nearly scale-invariant perturbations
slightly red tilt
adiabatic
gaussian
gravitational waves
consistency relations

Inflation: How does it work?



$$w = \frac{\frac{1}{2}\dot{\phi}^2 - V(\phi)}{\frac{1}{2}\dot{\phi}^2 + V(\phi)} \approx -1$$

Analyzing inflation in a simple, model-independent way

How inflation flattens and smoothes the universe

$$H^2 = \frac{8\pi G}{3}\left(\frac{\rho_m^0}{a^3} + \frac{\rho_r^0}{a^4} + \ldots \right) + \frac{\sigma^2}{a^6} - \frac{k}{a^2}$$

"survival of the smallest"

Analyzing inflation in a simple, model-independent way

$$H^2 = \frac{8\pi G}{3}\left(\frac{\rho_m^0}{a^3} + \frac{\rho_r^0}{a^4} + \ldots\right) + \frac{\sigma^2}{a^6} - \frac{k}{a^2} + \frac{8\pi G}{3}\rho_{\text{inflaton}}$$

"survival of the smallest"

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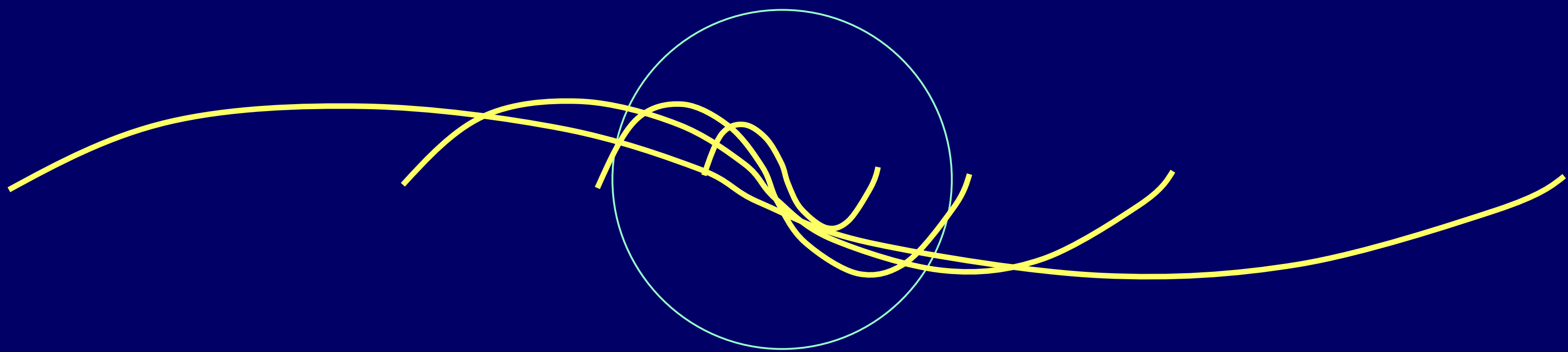
"survival of the smallest"

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho_{\text{inflaton}} \Rightarrow a(t) \sim e^{Ht}$$

How inflation creates a nearly scale-invariant spectrum of density perturbations:

begin with fluctuations in the scalar field
on length scales small compared to H^{-1}



Minkowski fluctuations → scale invariant cosmic perturbations

Analyzing inflation in a model-independent way

$\varepsilon \equiv \frac{3}{2}(1+w)$ = equation of state ($w = p/\rho$)

$$a \sim t^{1/\varepsilon} \quad \text{with} \quad \varepsilon < 1$$

M = mass scale for inflation ($\rho \sim V(\phi) \sim M^4$)

N = number of e-folds of inflation remaining

(Planck units: $8\pi G=1$)

Scalar field fluctuations become fluctuations in time when inflation ends which become temperature fluctuations whose amplitude is determined by M and ε

$$\frac{\delta T}{T} \sim \frac{\delta t / t^{3/2}}{1/t^{1/2}} \sim \frac{\delta t}{t} \sim H\delta t$$

$$\sim H\frac{\delta\phi}{\dot\phi} \sim \frac{H^2}{\dot\phi} \sim \frac{H^2}{\sqrt{p+\rho}} \sim \frac{\rho}{\sqrt{p+\rho}}$$

$$\boxed{\frac{\delta T}{T} \sim \frac{M^2}{\sqrt{\varepsilon}}}$$

gaussian
adiabatic

What do we know about ε ?

Recall: $a \sim t^{1/\varepsilon} \quad and \quad H \equiv \dot{a}/a = 1/\varepsilon t$

And inflation ends after N e-folds. That means:

$$\frac{H_{end}}{H} = \frac{t}{t_{end}} = \left(\frac{a}{a_{end}}\right)^{\varepsilon} = \left(e^{-N}\right)^{\varepsilon} \sim e^{-1}$$

$$or \quad \boxed{\varepsilon \sim 1/N}$$

$$\boxed{\frac{\delta T}{T} \sim \frac{M^2}{\sqrt{\varepsilon}} \sim M^2\sqrt{N} \sim 8M^2}$$

spectral tilt n_s?

from before:

$$\frac{\delta T}{T} \sim \frac{M^2}{\sqrt{\varepsilon}} \sim \sqrt{\frac{\rho}{\varepsilon}} \sim k^{(n_s-1)/2} \qquad \varepsilon \sim 1/N$$

$$\rho \sim 1/a^{2\varepsilon} \qquad k \sim H_I\, a \sim e^{-N}$$

$$n_s - 1 \sim 2\,\frac{d\ln\sqrt{\rho/\varepsilon}}{d\ln k} \sim -2\varepsilon + \frac{d\ln\varepsilon}{dN} \sim -\frac{3}{N}$$

$$\boxed{n_s \sim 0.95}$$

Also predicts the "run"

What does inflation predict about Gravitational wave (tensor) fluctuations

$$A_G^2 \sim H^2 \sim \rho \sim M^4$$

Many authors have claimed:
because the amplitude depends on M^4
it can vary by many orders of magnitude…

so no clear target for experiment

Flaw: The mean square <u>scalar</u> fluctuation amplitude is ALSO proportional to M^4

$$\frac{\delta T}{T} \sim 10^{-5} \sim \frac{M^2}{\sqrt{\varepsilon}} \quad \Rightarrow \; M \sim 10^{-5/2}\,\varepsilon^{1/4} \quad !!$$

So we know the scale of inflation:

For $\varepsilon \sim 1/N$, $M \sim 10^{-3}$

(and making it smaller requires extraordinary fine-tuning)

Flaw: The mean square scalar fluctuation amplitude is ALSO proportional to M^4

$$r \equiv \frac{tensor}{scalar} = \# \frac{M^4}{M^4/\varepsilon} = \# \varepsilon$$

$$= 16\,\varepsilon = \frac{16}{N}$$

$$\boxed{r = 16\,\varepsilon = \frac{16}{N} \sim 27\%}$$

Inflationary Scorecard

Great explanatory power:

horizon – flatness – monopoles – entropy

Great predictive power:

- ✓ $\Omega_{total} = 1$
- ✓ nearly scale-invariant perturbations
- ✓– slightly red tilt ($n_s \sim 0.95$)
- ✓ adiabatic
- ? gaussian
- ? gravitational waves ($r \sim 27\%$)
- ? consistency relations

Common Criticisms

1) Fine-tuning problem?

2) So many models, many with differing predictions!

Two views of inflation:

a) a theory how the universe was made featureless (smooth, flat,) -- namely, by a period of smoothly varying accelerated expansion (with smoothly varying w and H)

b) Anything goes: any kind of accelerated expansion produced by a scalar field and a potential

“the classic perspective”

dominantly a classical process…

an ordering process…

in which quantum physics plays
a small but important perturbative role

“the (true) quantum perspective”

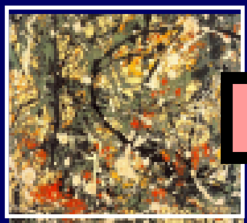
Inflation is dominantly a quantum process…

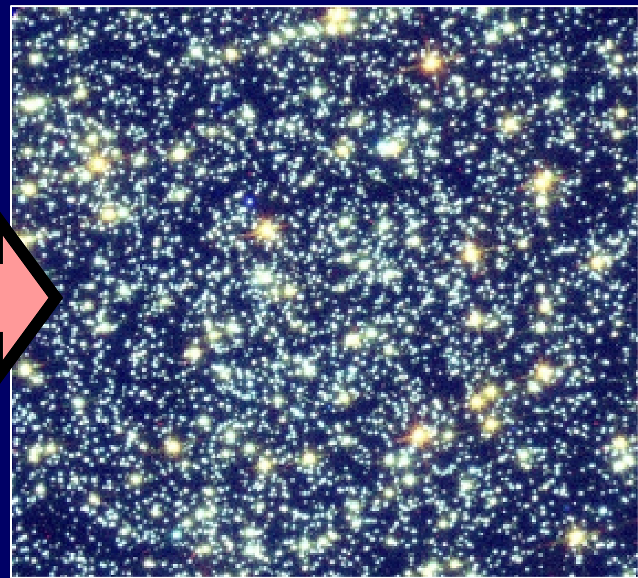
in which (classical) inflation amplifies
rare quantum fluctuations…

resulting in a peculiar kind of disorder

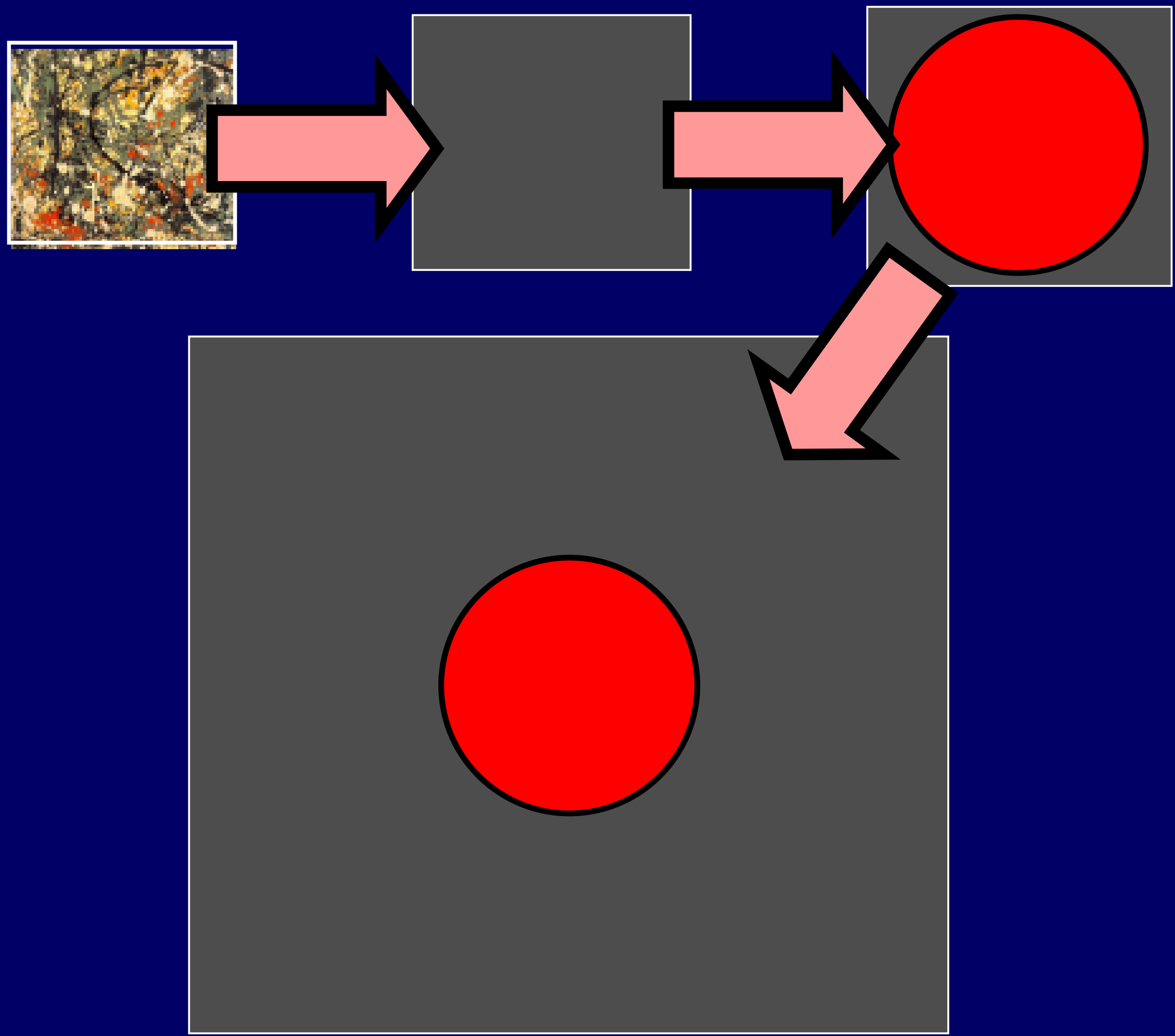
Lecture 2

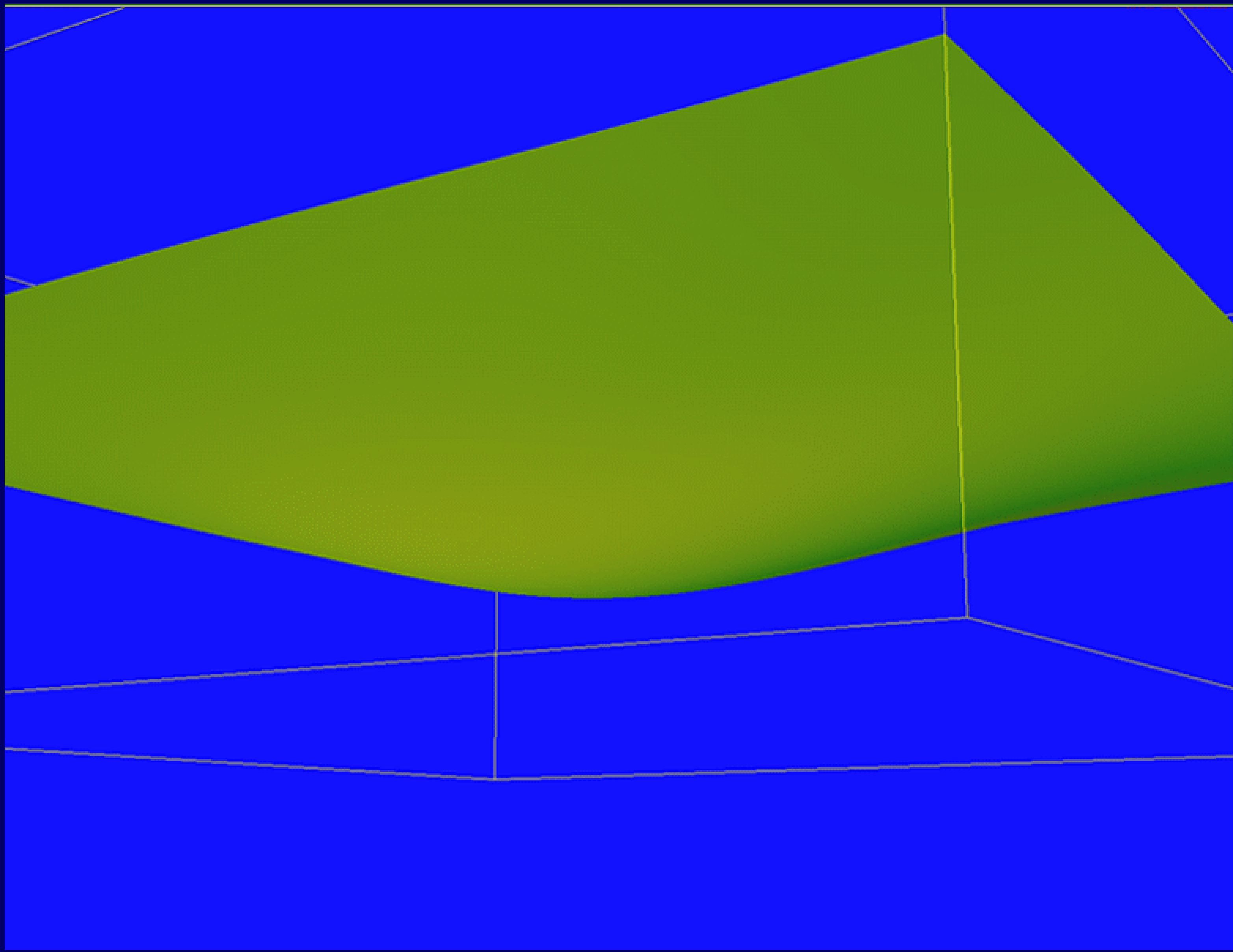
The “classic” picture we present to the public…



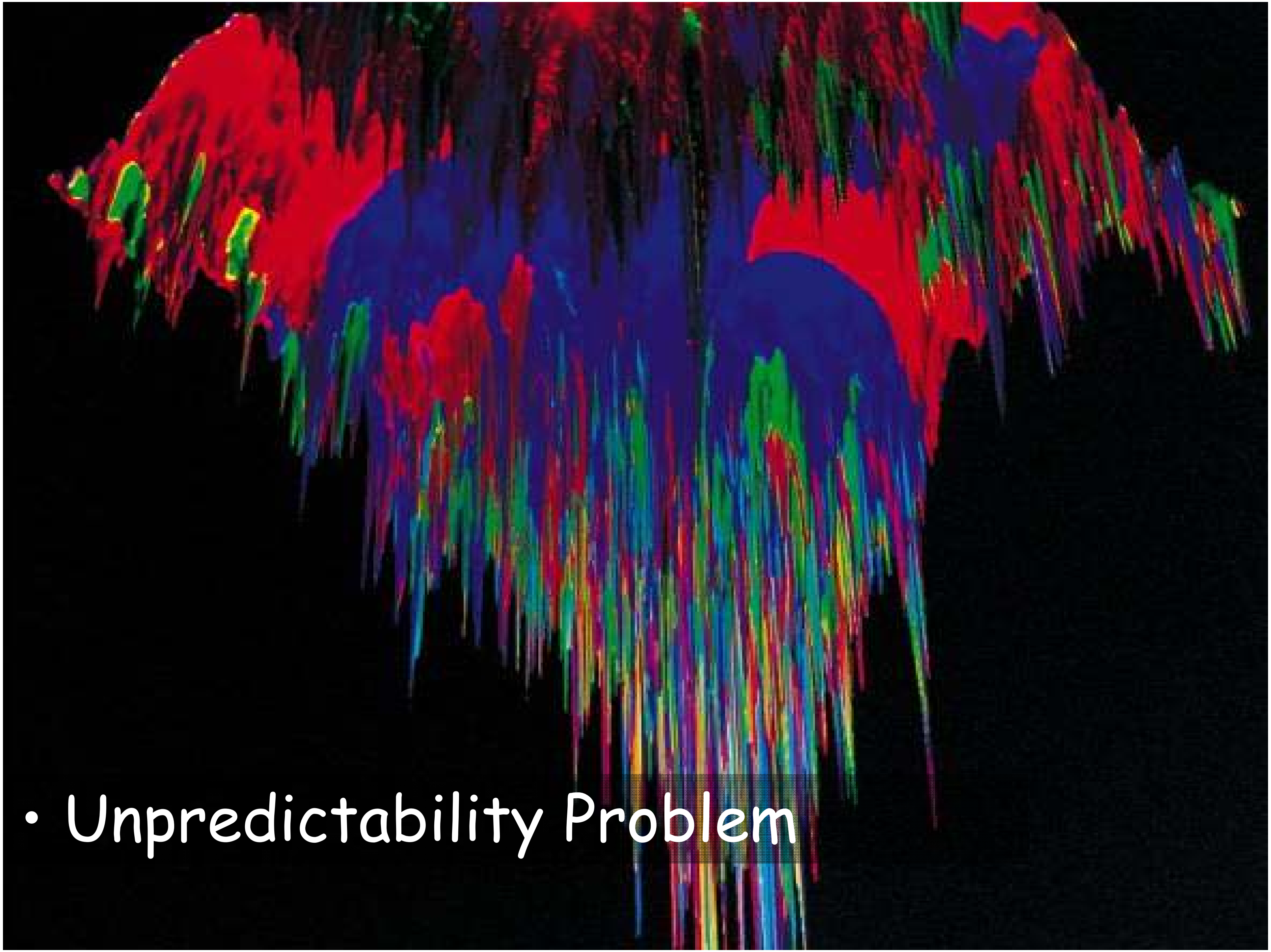


… but the truth is:





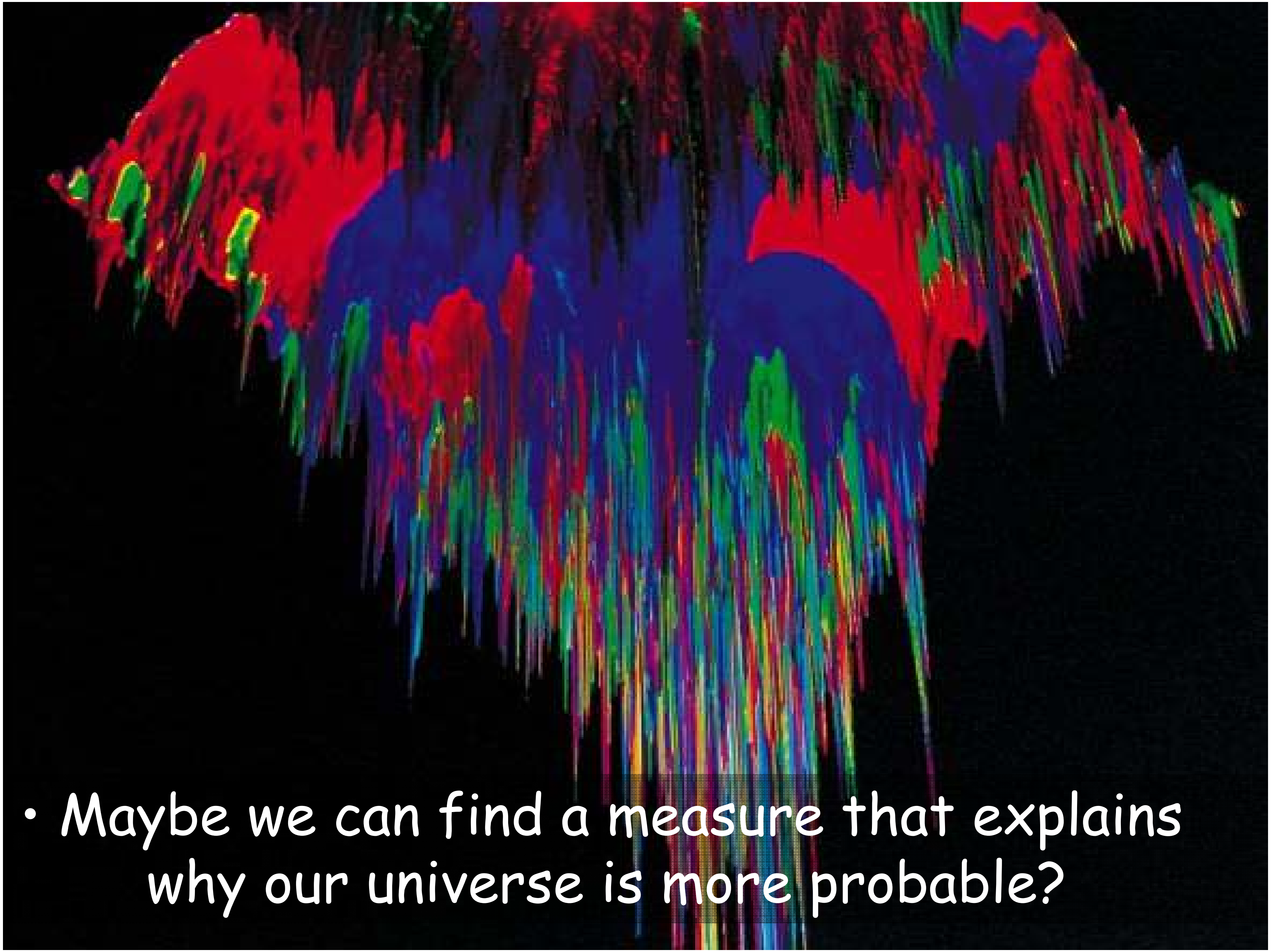
Linde, Linde, Mezhlumian, PRD 50, 2456 (1994)


• Unpredictability Problem

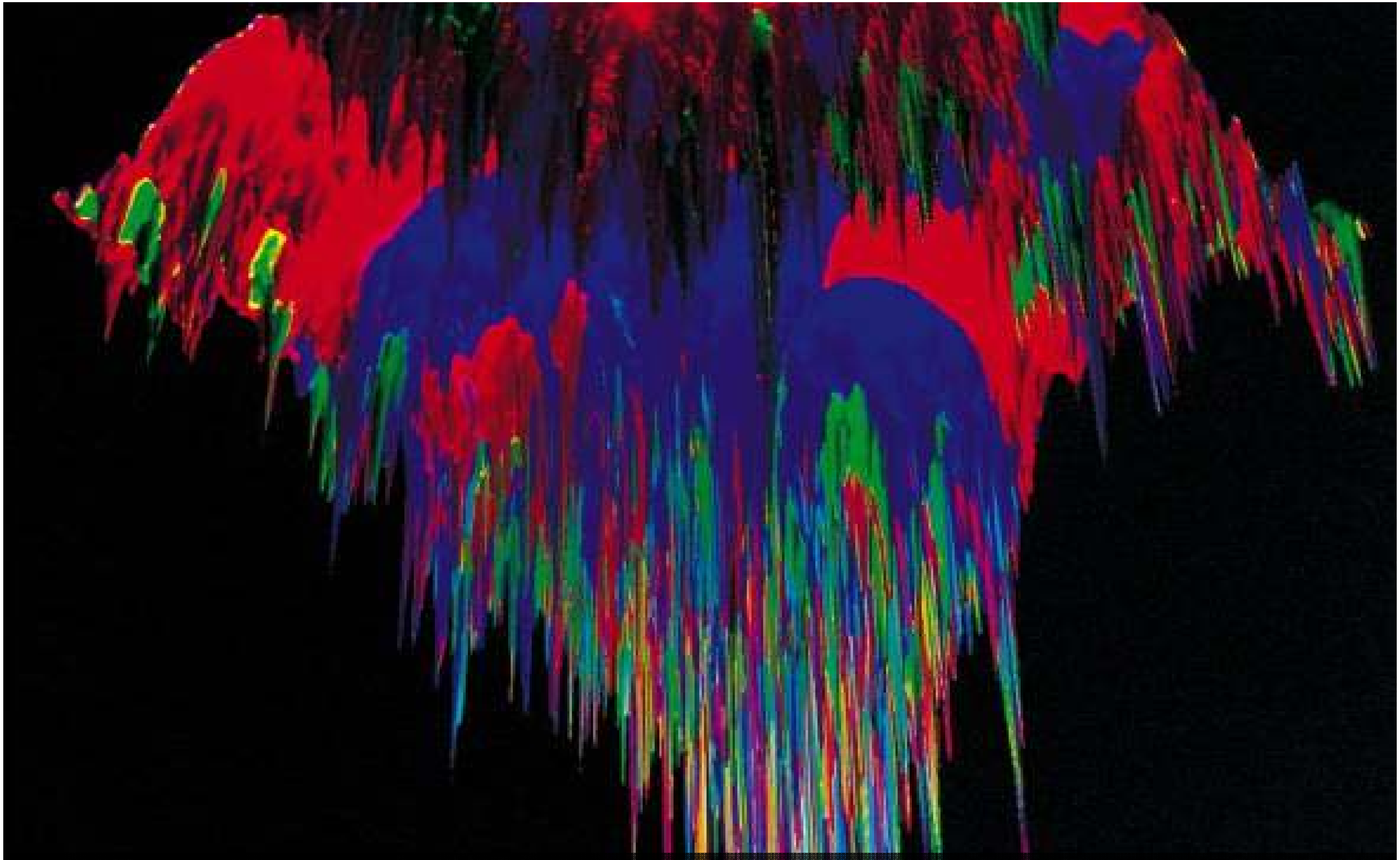
Maybe string theory will save the day?

Energy Landscape – vacua w/different properties

The Anthropic Principle?

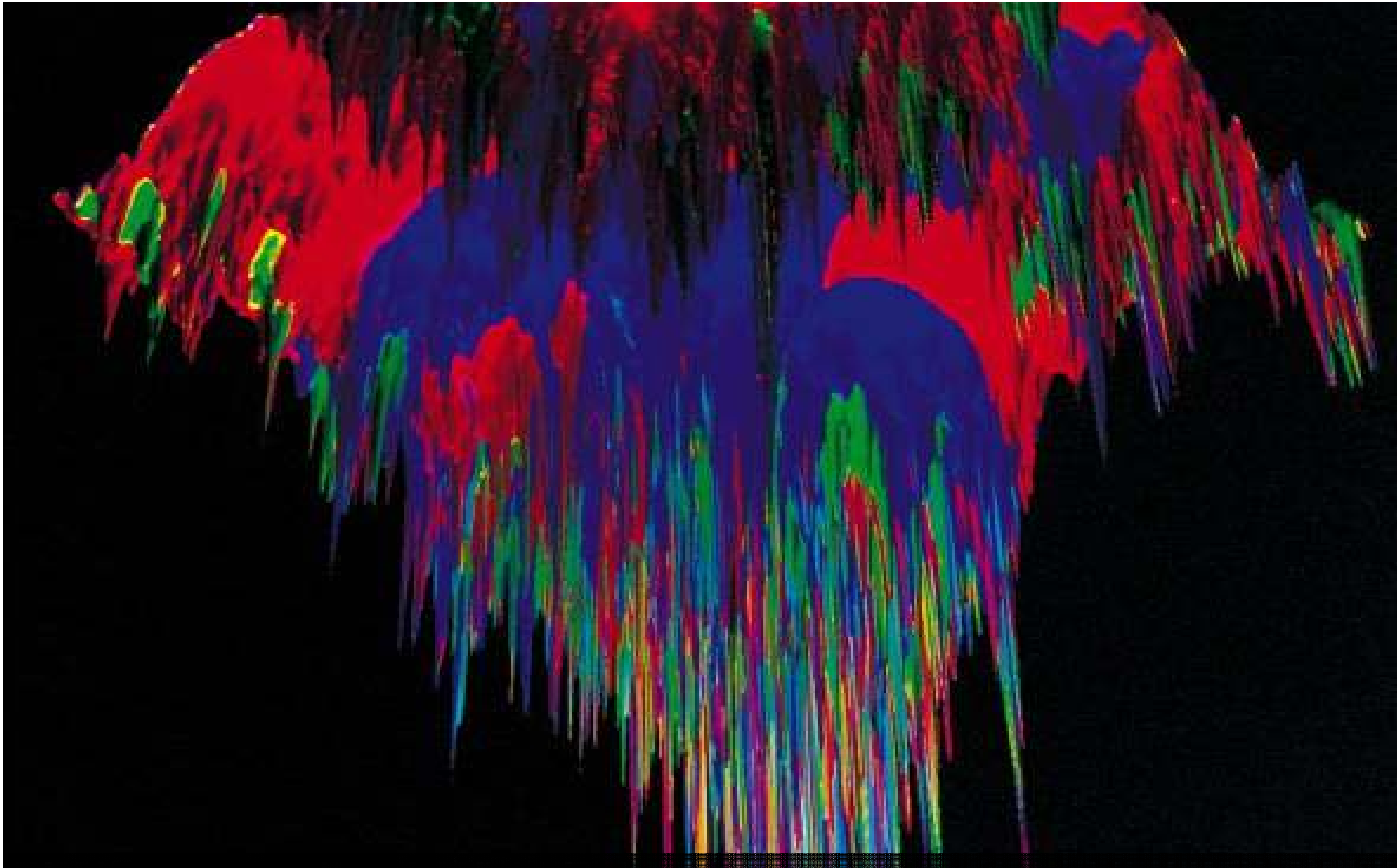


- Maybe we can find a measure that explains why our universe is more probable?



- Source of the Problem:

  Inflation is too powerful for our own good



- Source of the Problem:

$$H_{smoothing} > H_{normal} >> H_{today}$$

- But what if $H_{smoothing} << H_{normal}$?

How do we go from <u>small H</u> to <u>large H</u> ?

$$\dot{H} = -4\pi G(\rho + p) = -H^2\varepsilon$$

H_{smooth} small and contracting!

Of course, then big bang not the beginning!

but then how do we smooth ?!

Recall how inflation worked:

$$H^2 = \frac{8\pi G}{3}\left(\frac{\rho_m^0}{a^3} + \frac{\rho_r^0}{a^4} + \dots\right) + \frac{\sigma^2}{a^6} - \frac{k}{a^2} + \frac{8\pi G}{3}\rho_{\text{inflaton}}$$

Expanding universe: "survival of the smallest"

What if the universe is contracting?

$$H^2 = \frac{8\pi G}{3}\left(\frac{\rho_m^0}{a^3} + \frac{\rho_r^0}{a^4} + \dots\right) + \frac{\sigma^2}{a^6} - \frac{k}{a^2} + \frac{8\pi G}{3}\rho_{\text{inflaton}}$$

“survival of the largest”

What if the universe is contracting?

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$$+ \frac{8\pi G}{3}\frac{\rho_\phi^0}{a^{3(1+w)}}$$

"survival of the largest"

What if the universe is contracting?

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$$+ \frac{8\pi G}{3}\frac{\rho_\phi^0}{a^{3(1+\mathrm{w})}} \longleftarrow \quad \mathrm{w} \gg 1$$

"survival of the largest"

What if the universe is contracting?

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$$+\frac{8\pi G}{3}\frac{\rho_\phi^0}{a^{3(1+w)}} \longleftarrow \quad w \gg 1$$

A NEW NON-INFLATIONARY SMOOTHING MECHANISM:

no acceleration; not after the big bang; not superluminal; not nearly de Sitter; …

What if the universe is contracting?

$$H^2 = \frac{8\pi G}{3}\left(\frac{\rho_m^0}{a^3} + \frac{\rho_r^0}{a^4} + \ldots\right) + \frac{\sigma^2}{a^6} - \frac{k}{a^2} + \frac{8\pi G}{3}\rho_{\text{inflaton}}$$

$$+ \frac{8\pi G}{3}\frac{\rho_\phi^0}{a^{3(1+w)}} \quad \Longleftarrow \quad w \gg 1$$

and no chaotic mixmaster behavior …

What if the universe is contracting?

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$$+ \frac{8\pi G}{3}\frac{\rho_\phi^0}{a^{3(1+w)}} \longleftarrow \quad w \gg 1$$

and $H_{smooth} \ll H_{normal}$ **…**

What if the universe is contracting?

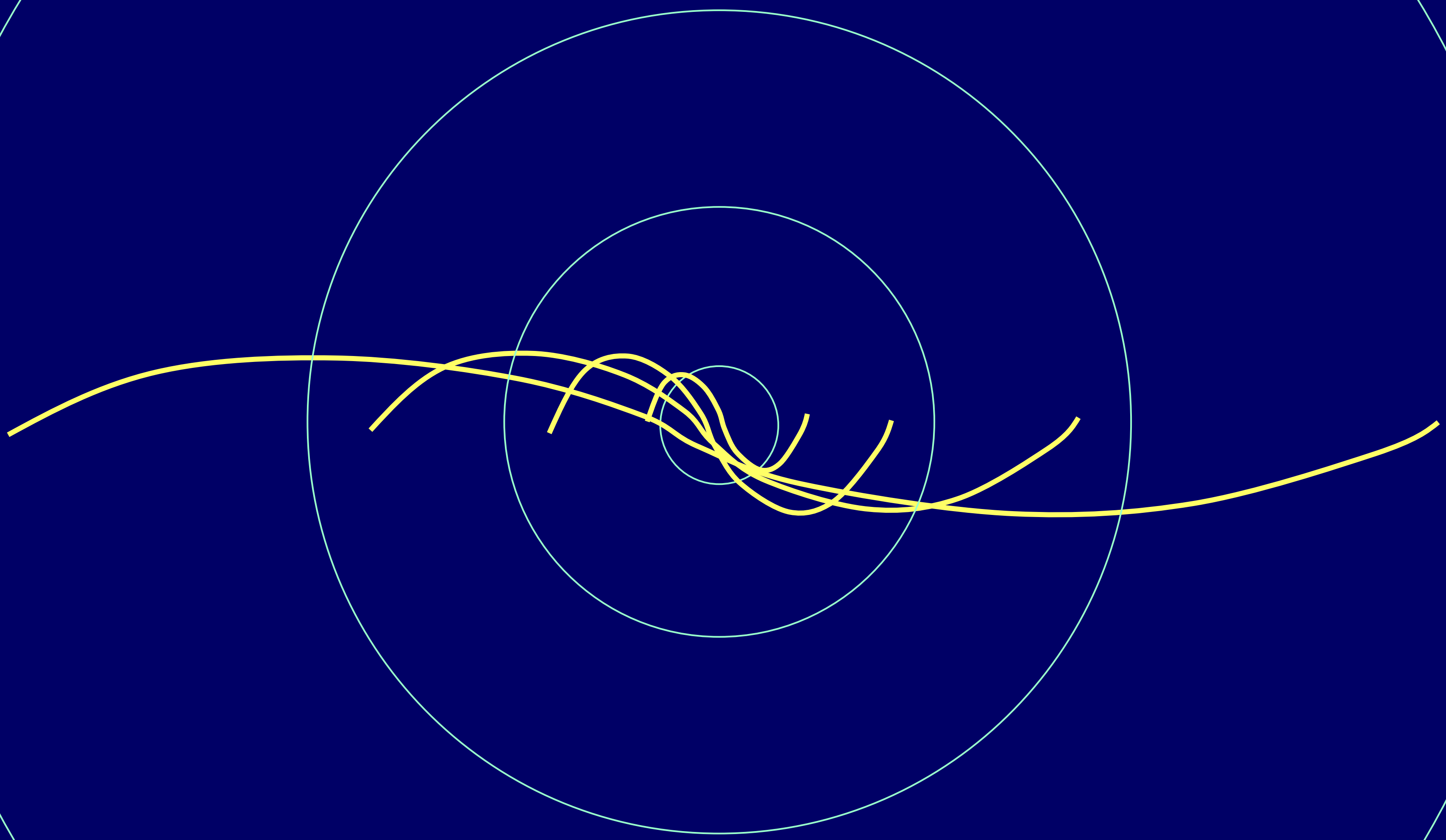
$$H^2 = \frac{8\pi G}{3}\left(\frac{\rho_m^0}{a^3} + \frac{\rho_r^0}{a^4} + \ldots\right) + \frac{\sigma^2}{a^6} - \frac{k}{a^2} + \frac{8\pi G}{3}\rho_{\text{inflaton}}$$

$$+\frac{8\pi G}{3}\frac{\rho_\phi^0}{a^{3(1+w)}}$$

$w \gg 1$

and makes scale-invariant fluctuations!!

Nearly scale-invariant density perturbations?
the reverse of inflation



$H^{-1} \sim t$ and $a(t) \sim t^{1/\varepsilon}$ with $\varepsilon \gg 1$ and $t \to 0^-$

scale-invariant fluctuations?

approach: quantum fluct. exit horizon & re-enter later

$$\varepsilon \equiv \tfrac{3}{2}(1+w)$$

$$a(t) \sim t^{\frac{1}{\varepsilon}} \sim (H^{-1})^{\frac{1}{\varepsilon}}$$

	expanding	contracting
	$\varepsilon < 1$	$\varepsilon > 1$
$n_s \approx 1$:	$\varepsilon \ll 1$	$\varepsilon \gg 1$ (or $w \gg 1$)
	$\boxed{n_s - 1 = -2\varepsilon + \frac{d \ln \varepsilon}{dN}}$	$\boxed{n_s - 1 = -\frac{2}{\varepsilon} - \frac{d \ln \varepsilon}{d N}}$

scale-invariant fluctuations?

approach: quantum fluct. exit horizon & re-enter later

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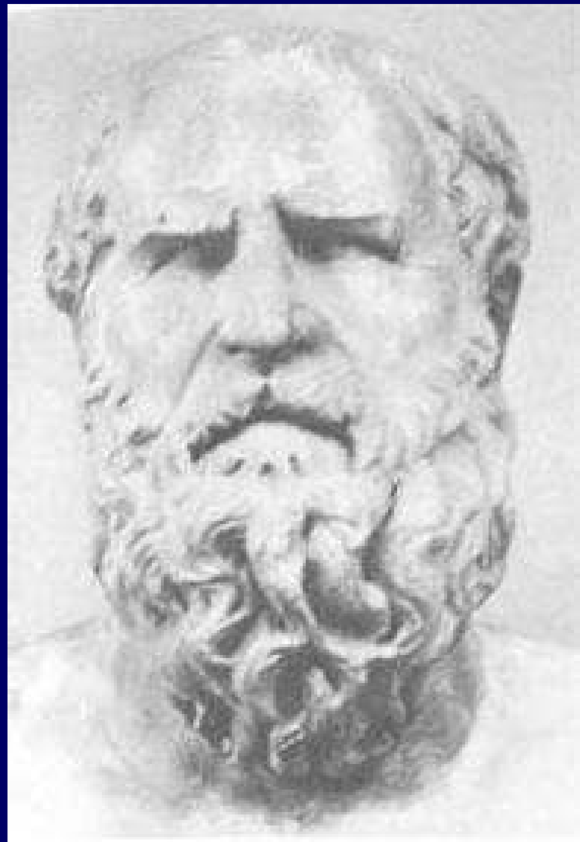
$$\boxed{n_s - 1 = -2\varepsilon + \frac{d \ln \varepsilon}{dN}}$$

⇦ "dual" ⇨

$\varepsilon \rightarrow 1/\varepsilon$

$$\boxed{n_s - 1 = -\frac{2}{\varepsilon} - \frac{d \ln \varepsilon}{d N}}$$

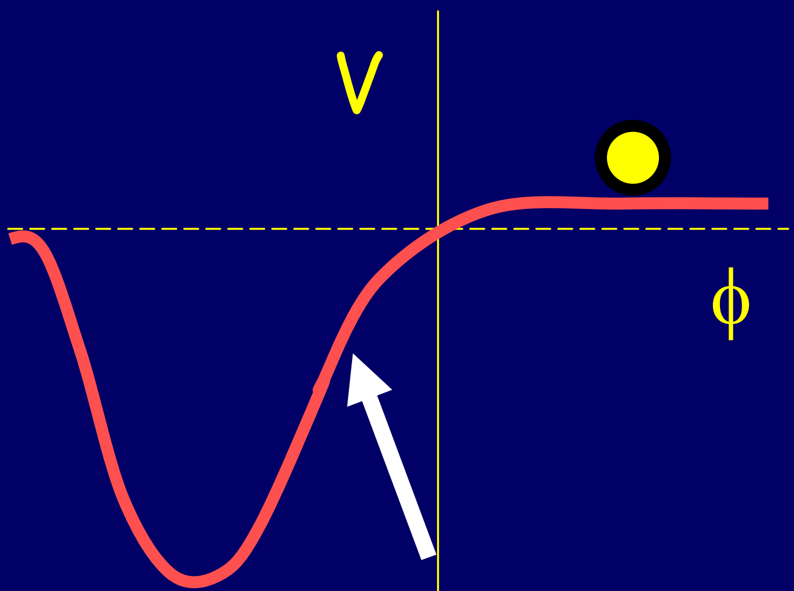
Ekpyrotic contraction



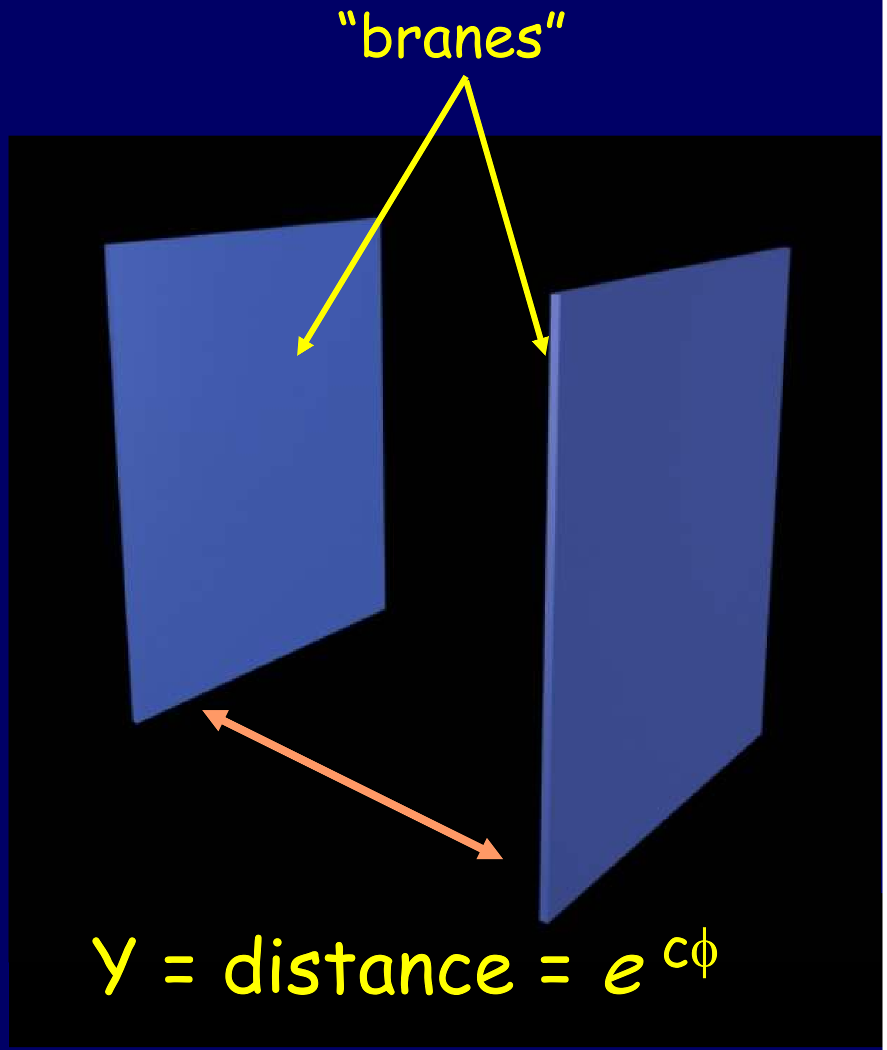
Heraclitus

ek-pyr-o-sis:
(*Gr.*) conflagration

How to get w >> 1 ?



$$w = \frac{\frac{1}{2}\dot{\phi}^2 - V(\phi)}{\frac{1}{2}\dot{\phi}^2 + V(\phi)} \gg {}^{+}1$$



The Cyclic Model of the Universe

"bang"

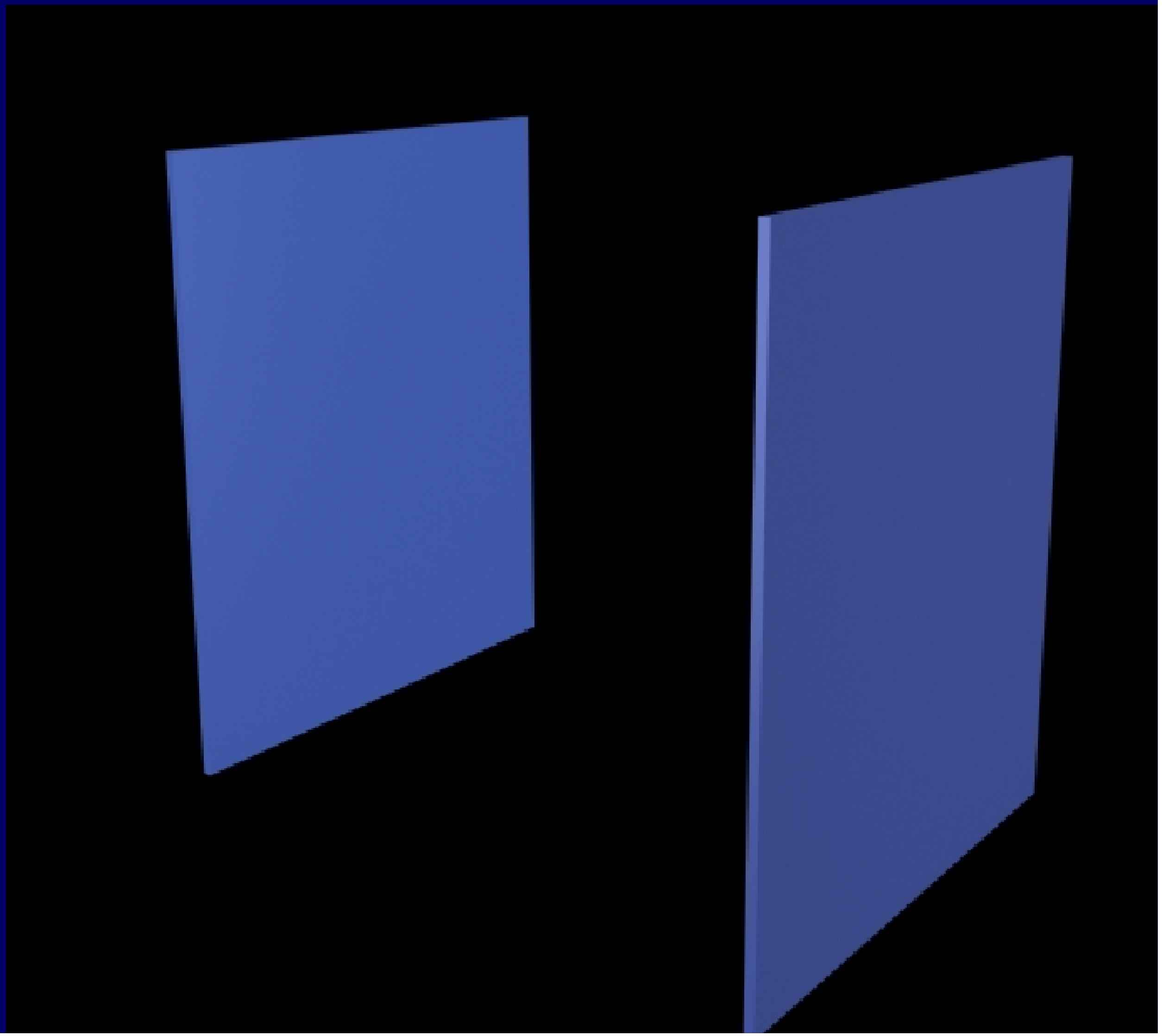
radiation

matter

dark energy

"ekpyrotic contraction"

"crunch"



Big Bang/Inflationary Picture

big bang

inflationary epoch

radiation epoch

matter epoch

dark energy epoch

"DISJOINT"

What about the Tolman Entropy Problem? Or violating the laws of thermodynamics?

Richard Tolman

How can we distinguish which model is right?

Curiously, *precision* tests can distinguish the two key *qualitative* differences between inflation and ekpyrotic/cyclic models

- $H_{smoothing}$ is exponentially different

  gravitational waves

- w is orders of magnitude different

  local non-gaussianity

"non-gaussianity generated when modes are outside the horizon ("local" NG)

$$\zeta \sim (\delta\rho/\rho)^2 = \zeta_L + \frac{3}{5} f_{NL}\, \zeta_L^{\,2}$$

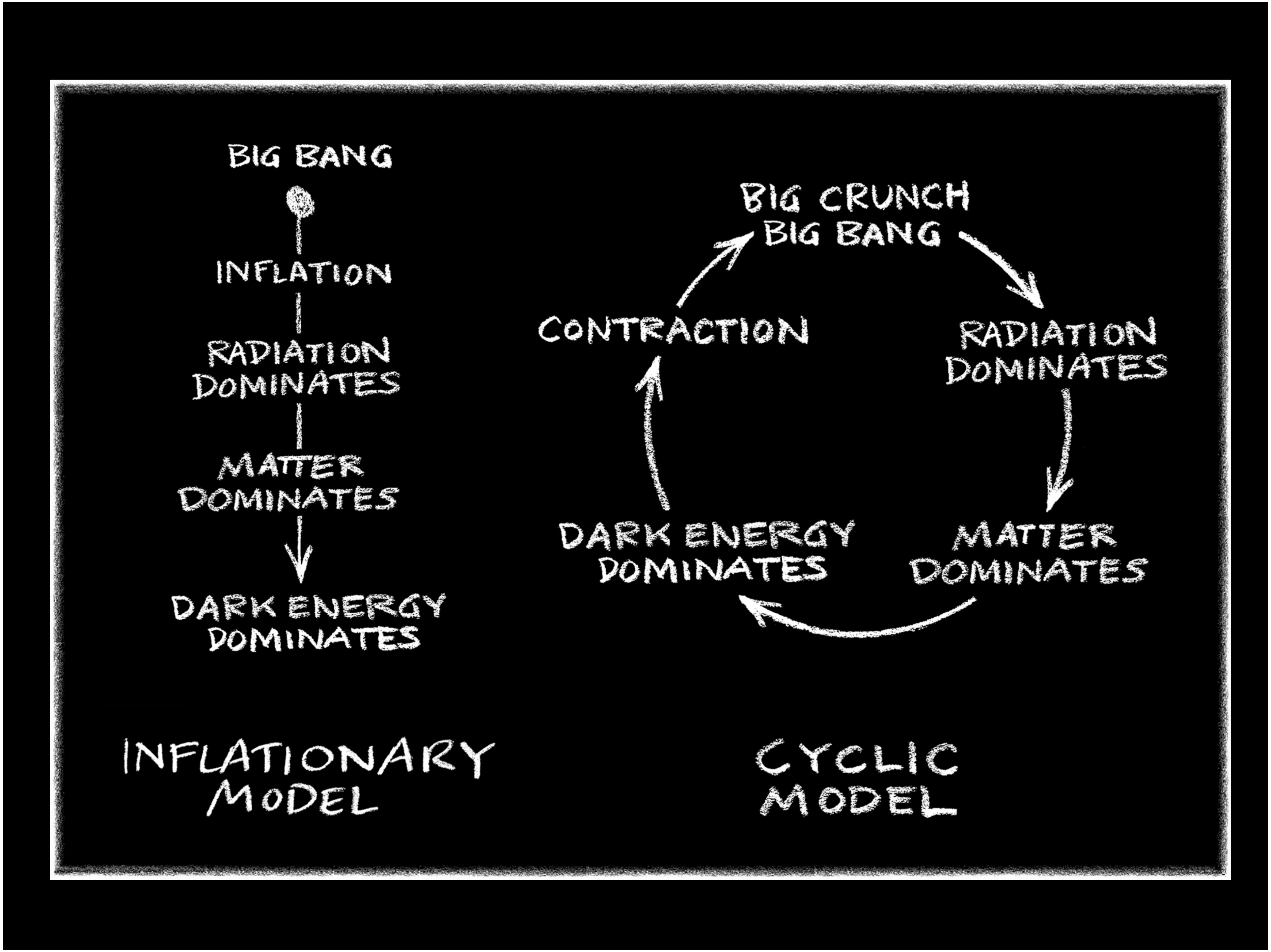
Maldacena
Komatsu & Spergel

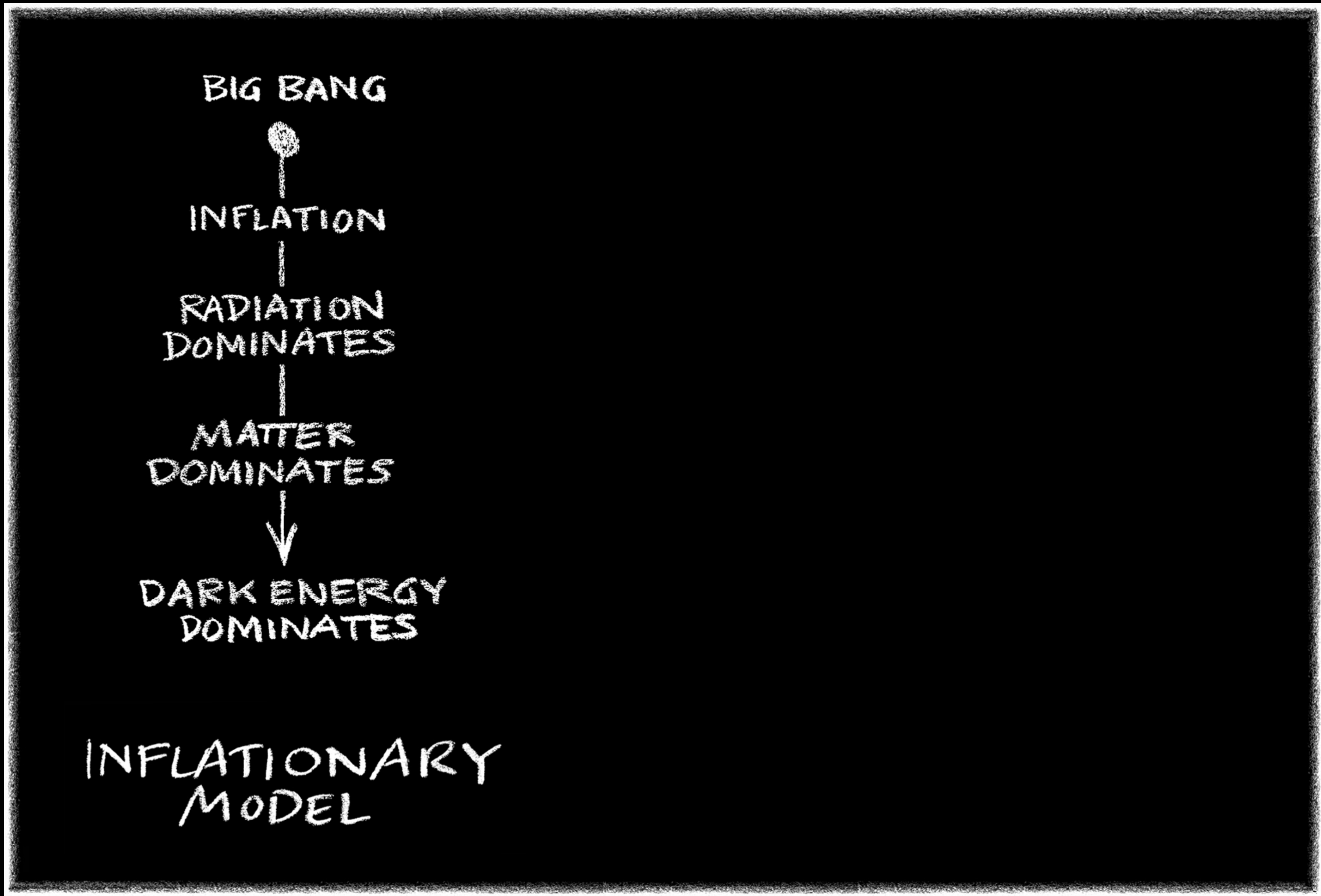
$$f_{NL} \sim \sqrt{\varepsilon}$$

$$+110 > f_{NL}^{\,observed} > -9$$ (WMAP5 team)

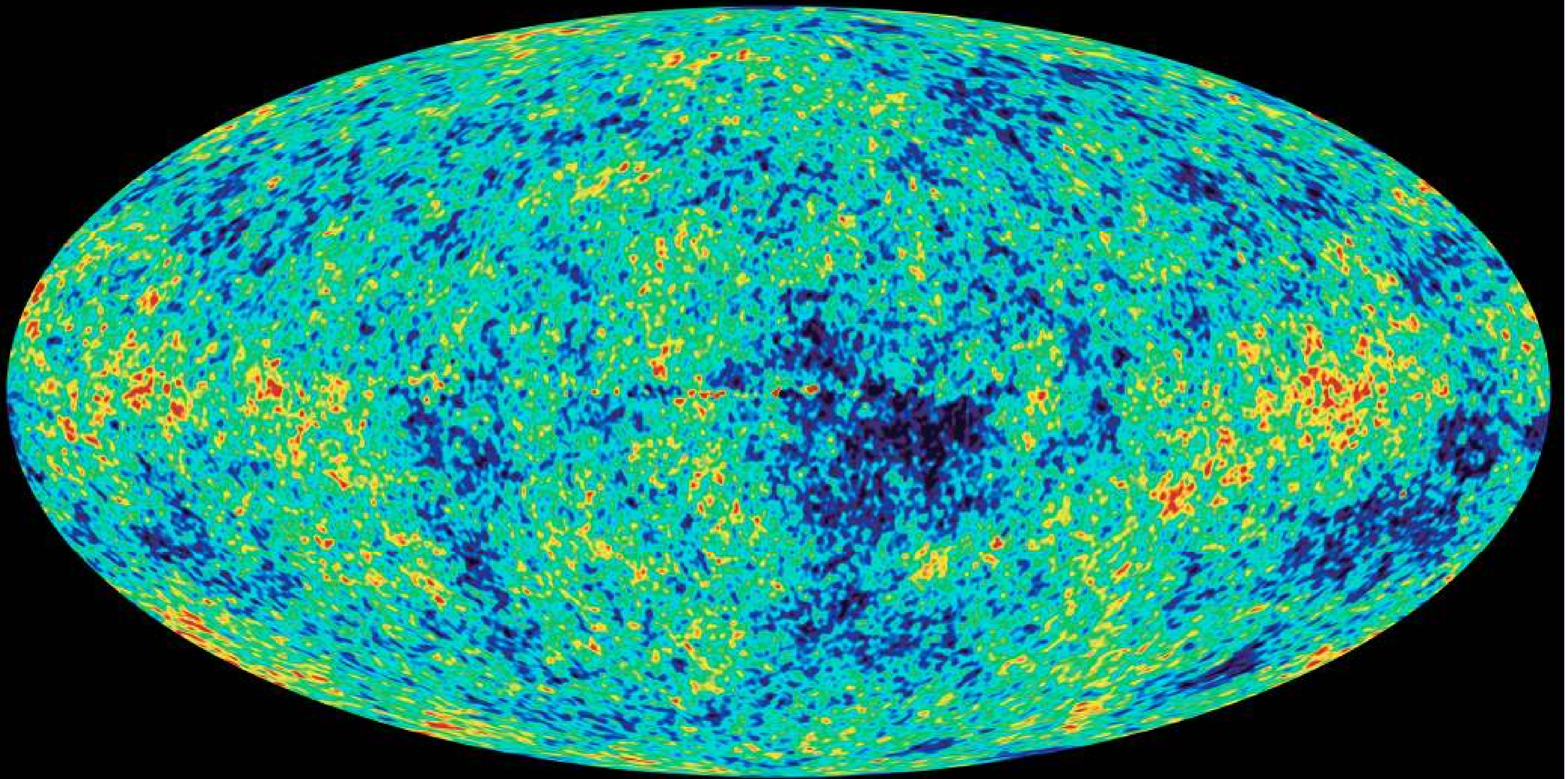
$$+147 > f_{NL}^{\,observed} > +27$$ (Yadav & Wandelt)

What is at stake


BIG BANG
INFLATION
RADIATION DOMINATES
MATTER DOMINATES
DARK ENERGY DOMINATES
INFLATIONARY MODEL
BIG CRUNCH
BIG BANG
RADIATION DOMINATES
MATTER DOMINATES
DARK ENERGY DOMINATES
CONTRACTION
CYCLIC MODEL


BIG BANG
INFLATION
RADIATION DOMINATES
MATTER DOMINATES
DARK ENERGY DOMINATES
INFLATIONARY MODEL

our vision is limited:

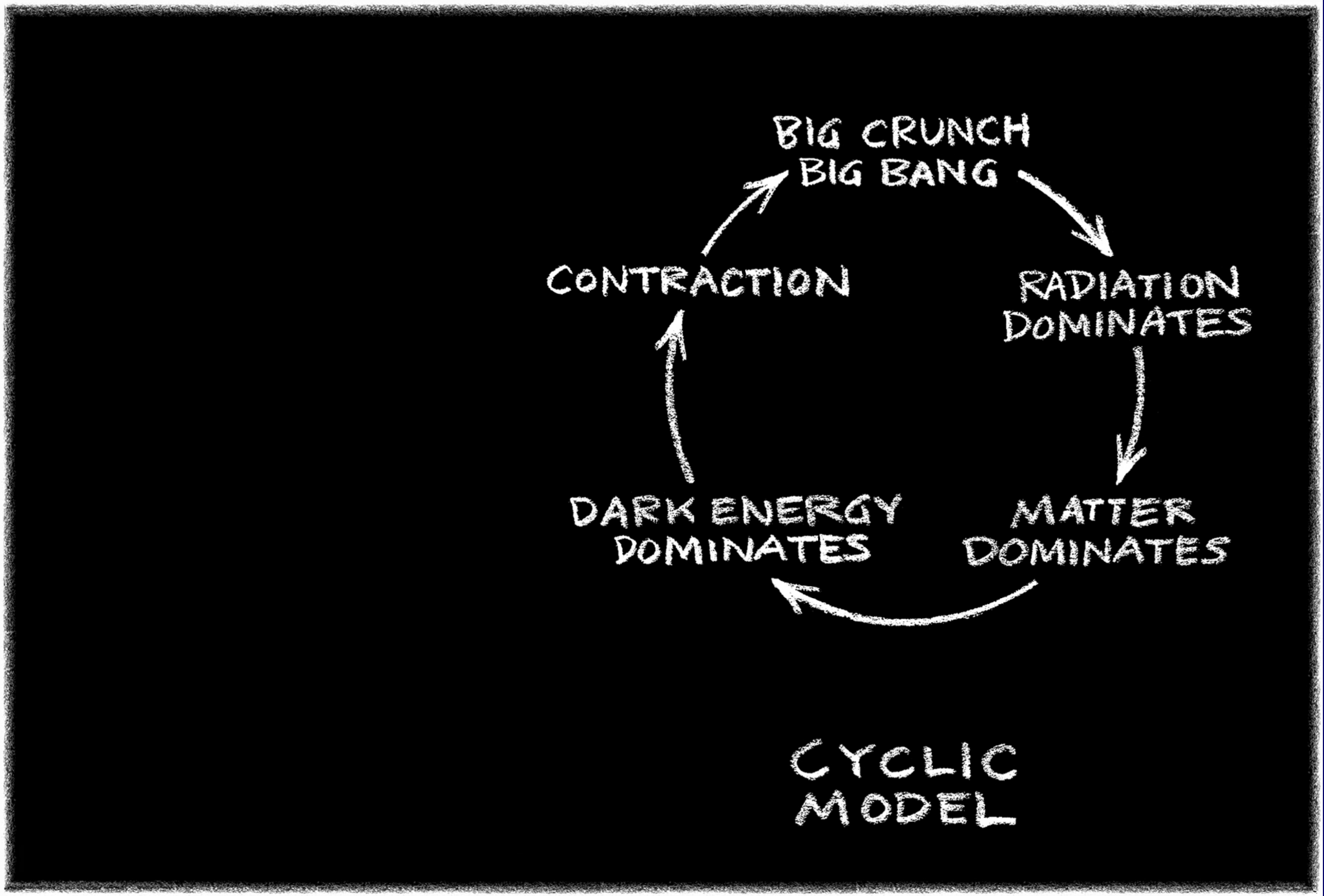


the future is bleak

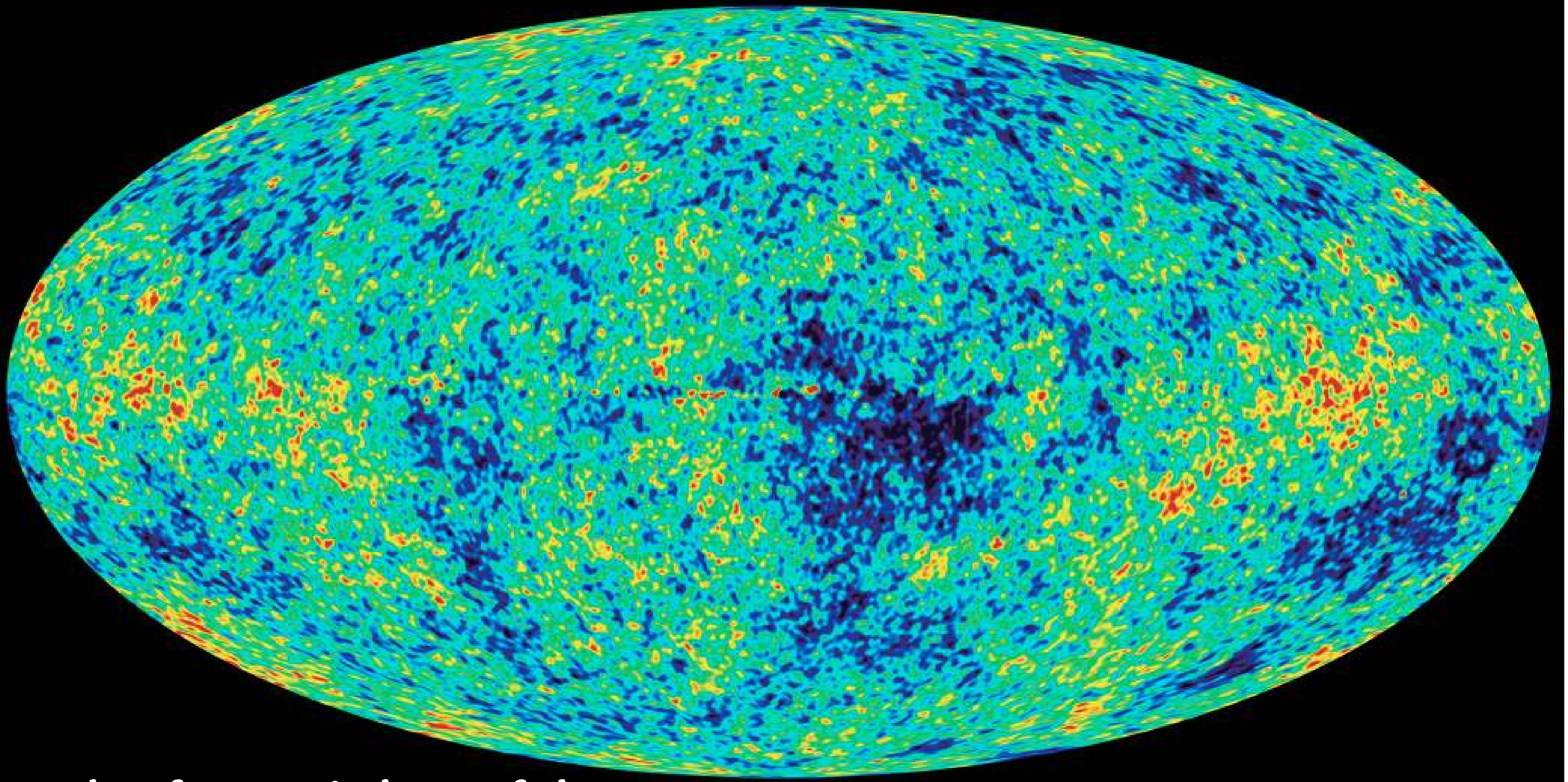

what we see is atypical

Landscape of Possibilities ?

. . . or the End of Science ?


BIG CRUNCH
BIG BANG
RADIATION DOMINATES
MATTER DOMINATES
DARK ENERGY DOMINATES
CONTRACTION
CYCLIC MODEL

our vision extends beyond the big bang:



the future is hopeful

what we see is typical

Appendix I
The discussion of the cyclic model was only the tip of the iceberg: Some interesting topics we did not discuss:

Getting through the bounce
    (non-singular vs. singular bounces)
How cycling might address the
    cosmological constant problem
The axion problem and how cycling might address it
Conversion of scalar fluctuations to temperature
    fluctuations from 4d and 5d point of view
    (entropic mechanism)
Generation of scalar-induced gravitational waves

Appendix II
Some References

More efficient than going to the arXiv:
Check my website
www.physics.princeton.edu/~steinh
to find to a collection of articles
ranging from popular to advanced:
I might recommend: "The Cyclic Model Simplified"

See also recent review article by Jean-Luc Lehner
(bit more technical, but more up-to-date:
a lot of progress has been made in just the last year)

For popular discussion of both inflation & cyclic:
Endless Universe by Neil Turok and myself
(almost all the ideas but no equations)

An interesting variant by Khoury, Ovrut & Buchbinder is
Called the "new ekpyrotic model"
(look on the arxiv)