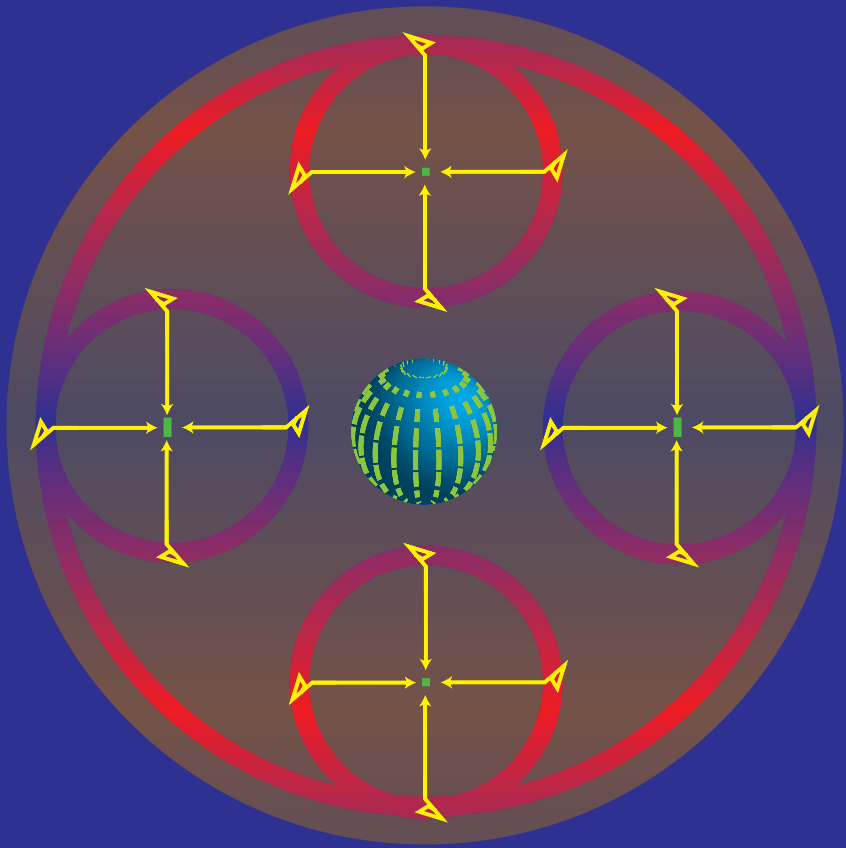
Lecture II



Polarization Anisotropy Spectrum

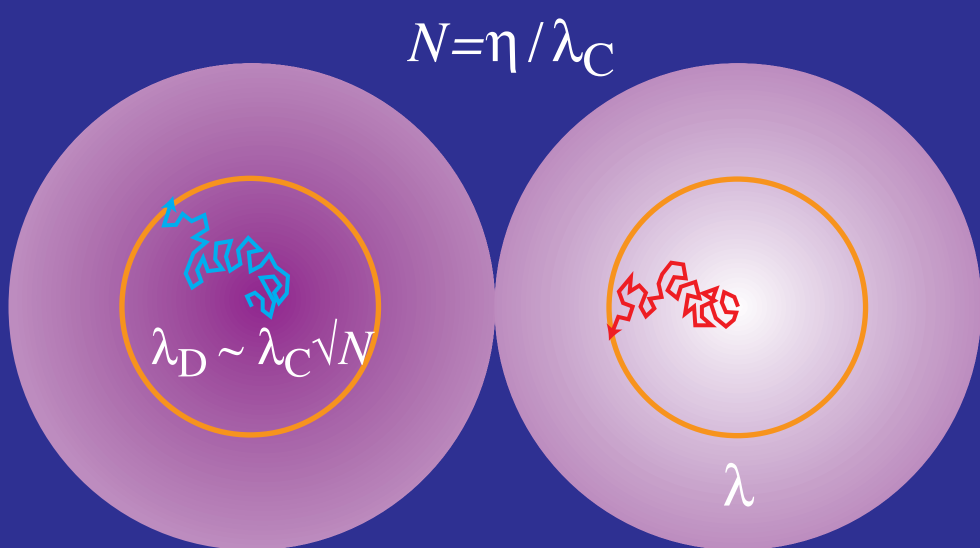
Wayne Hu

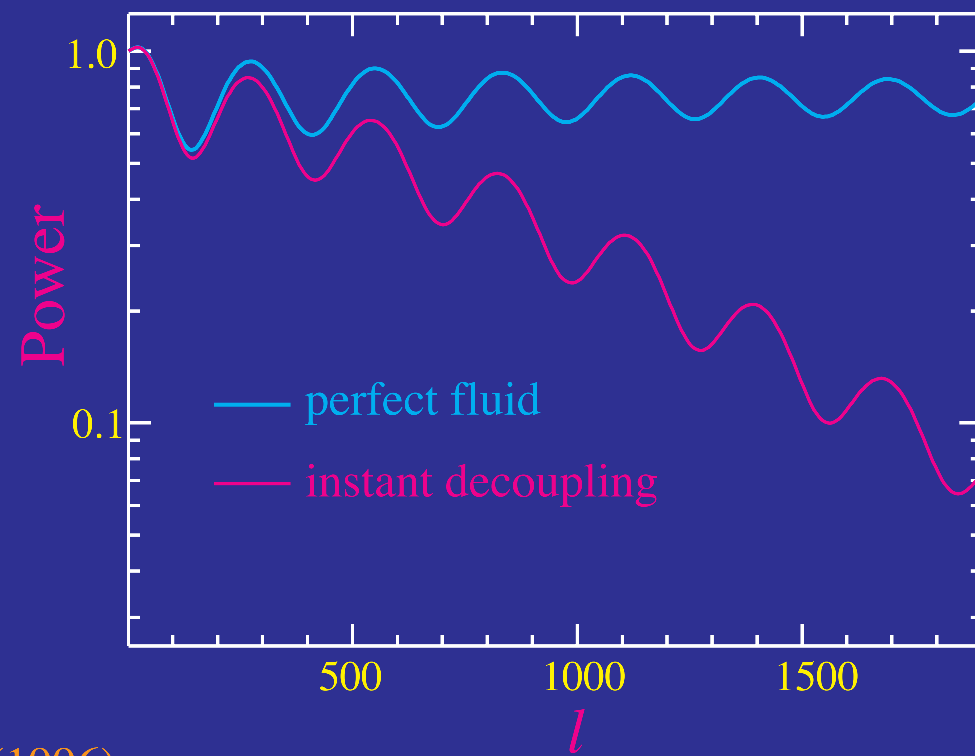
Crete, July 2008

Damping Tail

Dissipation / Diffusion Damping

- Imperfections in the coupled fluid → mean free path λ_C in the baryons
- Random walk over diffusion scale: geometric mean of mfp & horizon
  $\lambda_D \sim \lambda_C \sqrt{N} \sim \sqrt{\lambda_C \eta} \gg \lambda_C$
- Overtake wavelength: $\lambda_D \sim \lambda$; second order in λ_C/λ
- Viscous damping for $R<1$; heat conduction damping for $R>1$

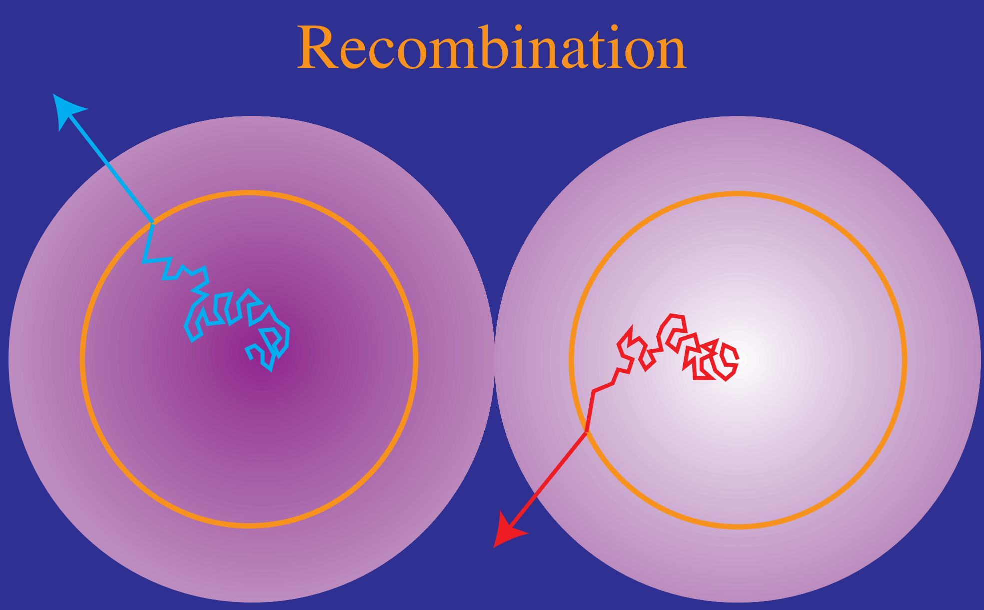


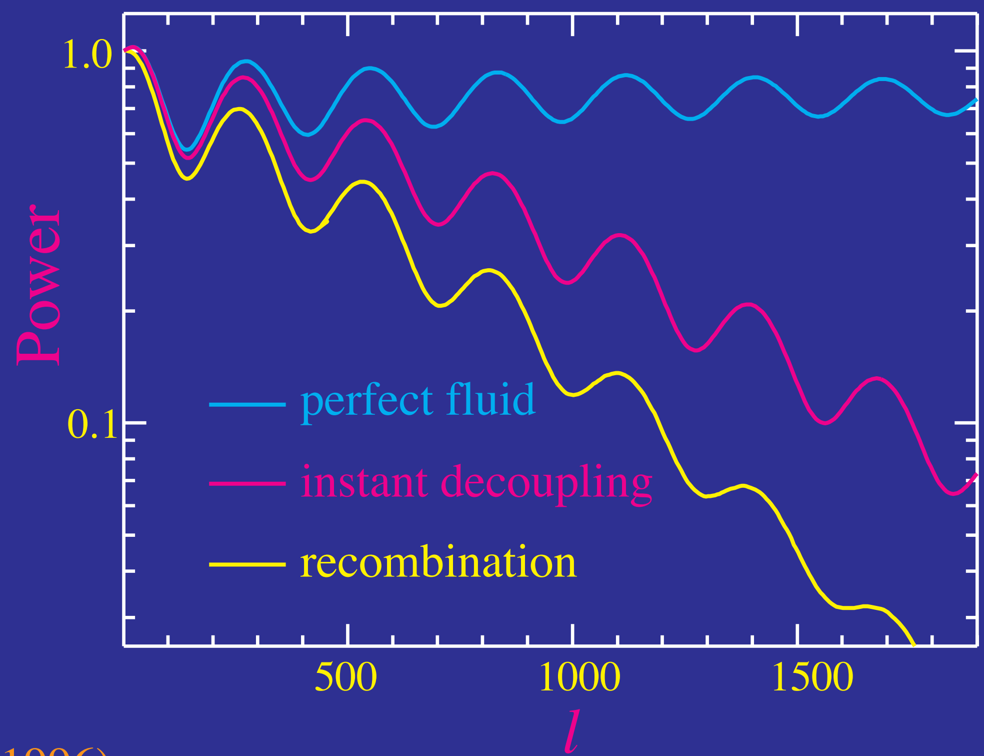


Silk (1968); Hu & Sugiyama (1995); Hu & White (1996)

Dissipation / Diffusion Damping

- Rapid increase at recombination as mfp ↑
- Independent of (robust to changes in) perturbation spectrum
- Robust physical scale for angular diameter distance test (Ω_K, Ω_Λ)





Silk (1968); Hu & Sugiyama (1995); Hu & White (1996)

Damping

- Tight coupling equations assume a perfect fluid: no viscosity, no heat conduction
- Fluid imperfections are related to the mean free path of the photons in the baryons

$$\lambda_C = \dot\tau^{-1} \quad \text{where} \quad \dot\tau = n_e \sigma_T a$$

  is the conformal opacity to Thomson scattering
- Dissipation is related to the diffusion length: random walk approximation

$$\lambda_D = \sqrt{N}\lambda_C = \sqrt{\eta/\lambda_C}\,\lambda_C = \sqrt{\eta\lambda_C}$$

  the geometric mean between the horizon and mean free path
- $\lambda_D/\eta_* \sim$ few %, so expect the peaks $:> 3$ to be affected by dissipation

Equations of Motion

- Continuity

$$\dot{\Theta} = -\frac{k}{3}v_\gamma - \dot{\Phi}\,, \qquad \dot{\delta}_b = -kv_b - 3\dot{\Phi}$$

where the photon equation remains unchanged and the baryons follow number conservation with $\rho_b = m_b n_b$

- Euler

$$\begin{aligned} \dot{v}_\gamma &= k(\Theta + \Psi) - \frac{k}{6}\pi_\gamma - \dot{\tau}(v_\gamma - v_b) \\ \dot{v}_b &= -\frac{\dot{a}}{a}v_b + k\Psi + \dot{\tau}(v_\gamma - v_b)/R \end{aligned}$$

where the photons gain an anisotropic stress term π_γ from radiation viscosity and a momentum exchange term with the baryons and are compensated by the opposite term in the baryon Euler equation

Viscosity

- Viscosity is generated from radiation streaming from hot to cold regions
- Expect

$$\pi_\gamma \sim v_\gamma \frac{k}{\dot\tau}$$

generated by streaming, suppressed by scattering in a wavelength of the fluctuation. Radiative transfer says

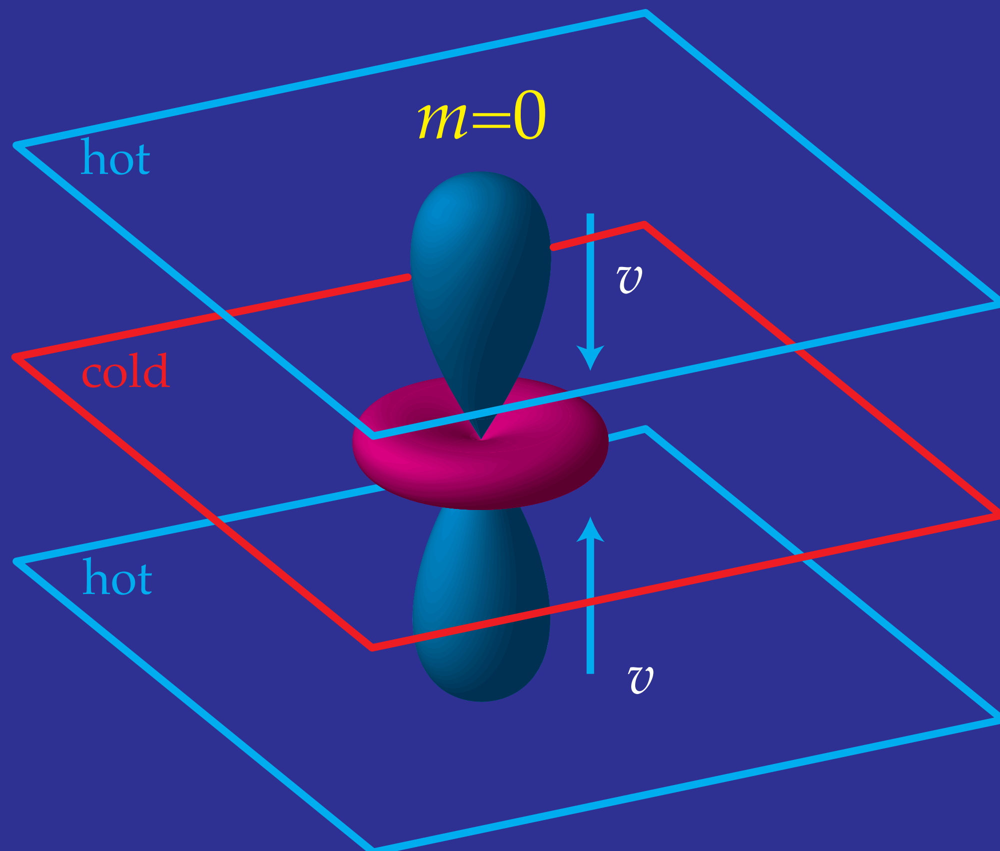
$$\pi_\gamma \approx 2 A_v v_\gamma \frac{k}{\dot\tau}$$

where $A_v = 16/15$

$$\dot v_\gamma = k(\Theta + \Psi) - \frac{k}{3} A_v \frac{k}{\dot\tau} v_\gamma$$

Viscosity & Heat Conduction

- Both fluid imperfections are related to the gradient of the velocity kv_γ by opacity $\dot{\tau}$: slippage of fluids $v_\gamma - v_b$.
- Viscosity is an anisotropic stress or quadrupole moment formed by radiation streaming from hot to cold regions

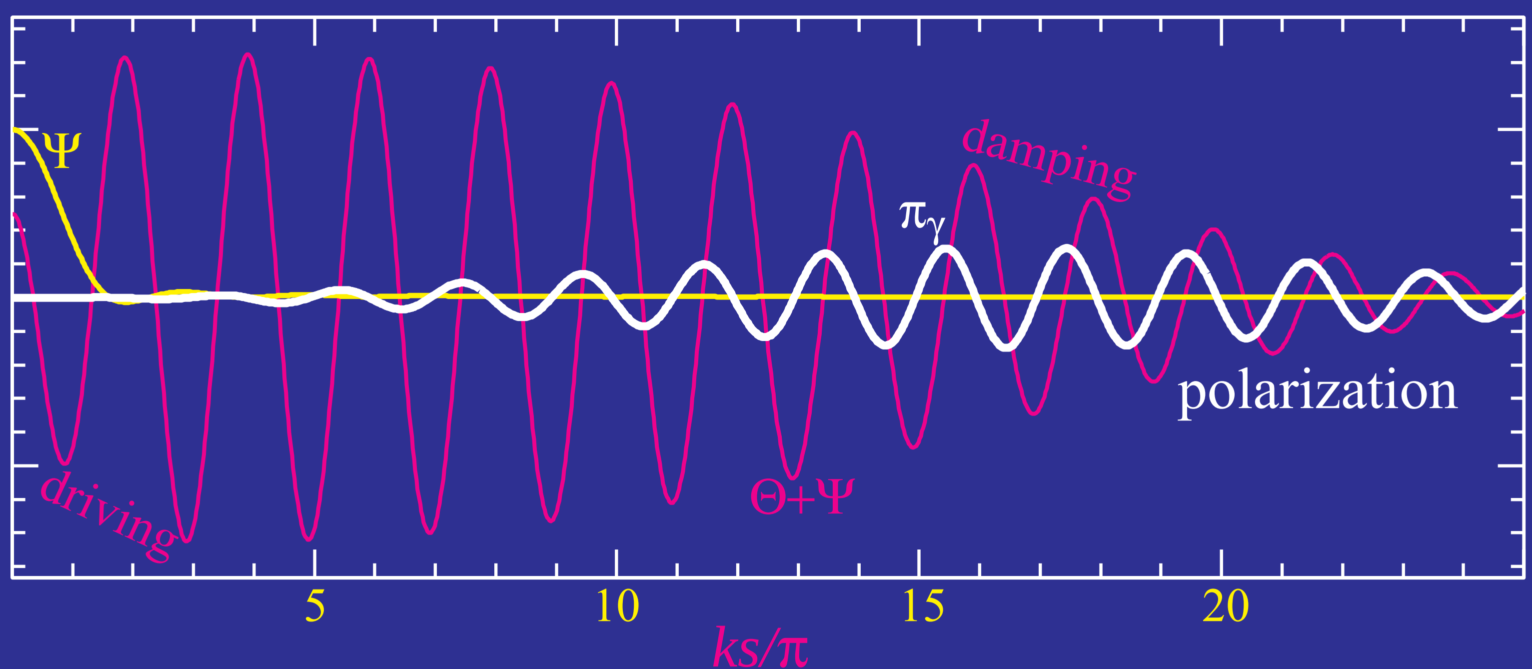


Damping & Viscosity

- Quadrupole moments:

  damp acoustic oscillations from fluid viscosity
  generates polarization from scattering (next lecture)

- Rise in polarization power coincides with fall in temperature power – $l \sim 1000$



Oscillator: Penultimate Take

- Adiabatic approximation ($\omega \gg \dot{a}/a$)

$$\dot{\Theta} \approx -\frac{k}{3} v_\gamma$$

- Oscillator equation contains a $\dot{\Theta}$ damping term

$$c_s^2 \frac{d}{d\eta}(c_s^{-2}\dot{\Theta}) + \frac{k^2 c_s^2}{\dot{\tau}} A_v \dot{\Theta} + k^2 c_s^2 \Theta = -\frac{k^2}{3}\Psi - c_s^2 \frac{d}{d\eta}(c_s^{-2}\dot{\Phi})$$

- Heat conduction term similar in that it is proportional to v_γ and is suppressed by scattering $k/\dot{\tau}$. Expansion of Euler equations to leading order in $k/\dot{\tau}$ gives

$$A_h = \frac{R^2}{1+R}$$

since the effects are only significant if the baryons are dynamically important

Oscillator: Final Take

- Final oscillator equation

$$c_s^2 \frac{d}{d\eta}(c_s^{-2}\dot{\Theta}) + \frac{k^2 c_s^2}{\dot{\tau}}[A_v + A_h]\dot{\Theta} + k^2 c_s^2 \Theta = -\frac{k^2}{3}\Psi - c_s^2 \frac{d}{d\eta}(c_s^{-2}\dot{\Phi})$$

- Solve in the adiabatic approximation

$$\Theta \propto \exp(i \int \omega d\eta)$$

$$-\omega^2 + \frac{k^2 c_s^2}{\dot{\tau}}(A_v + A_h) i\omega + k^2 c_s^2 = 0$$

Dispersion Relation

- Solve

$$\omega^2 = k^2 c_s^2 \left[1 + i\frac{\omega}{\dot\tau}(A_v + A_h)\right]$$

$$\omega = \pm k c_s \left[1 + \frac{i}{2}\frac{\omega}{\dot\tau}(A_v + A_h)\right]$$

$$= \pm k c_s \left[1 \pm \frac{i}{2}\frac{k c_s}{\dot\tau}(A_v + A_h)\right]$$

- Exponentiate

$$\exp(i\int \omega d\eta) = e^{\pm iks} \exp[-k^2 \int d\eta \frac{1}{2}\frac{c_s^2}{\dot\tau}(A_v + A_h)]$$

$$= e^{\pm iks} \exp[-(k/k_D)^2]$$

- Damping is exponential under the scale k_D

Diffusion Scale

- Diffusion wavenumber

$$k_D^{-2} = \int d\eta \frac{1}{\dot\tau} \frac{1}{6(1+R)} \left(\frac{16}{15} + \frac{R^2}{(1+R)} \right)$$

- Limiting forms

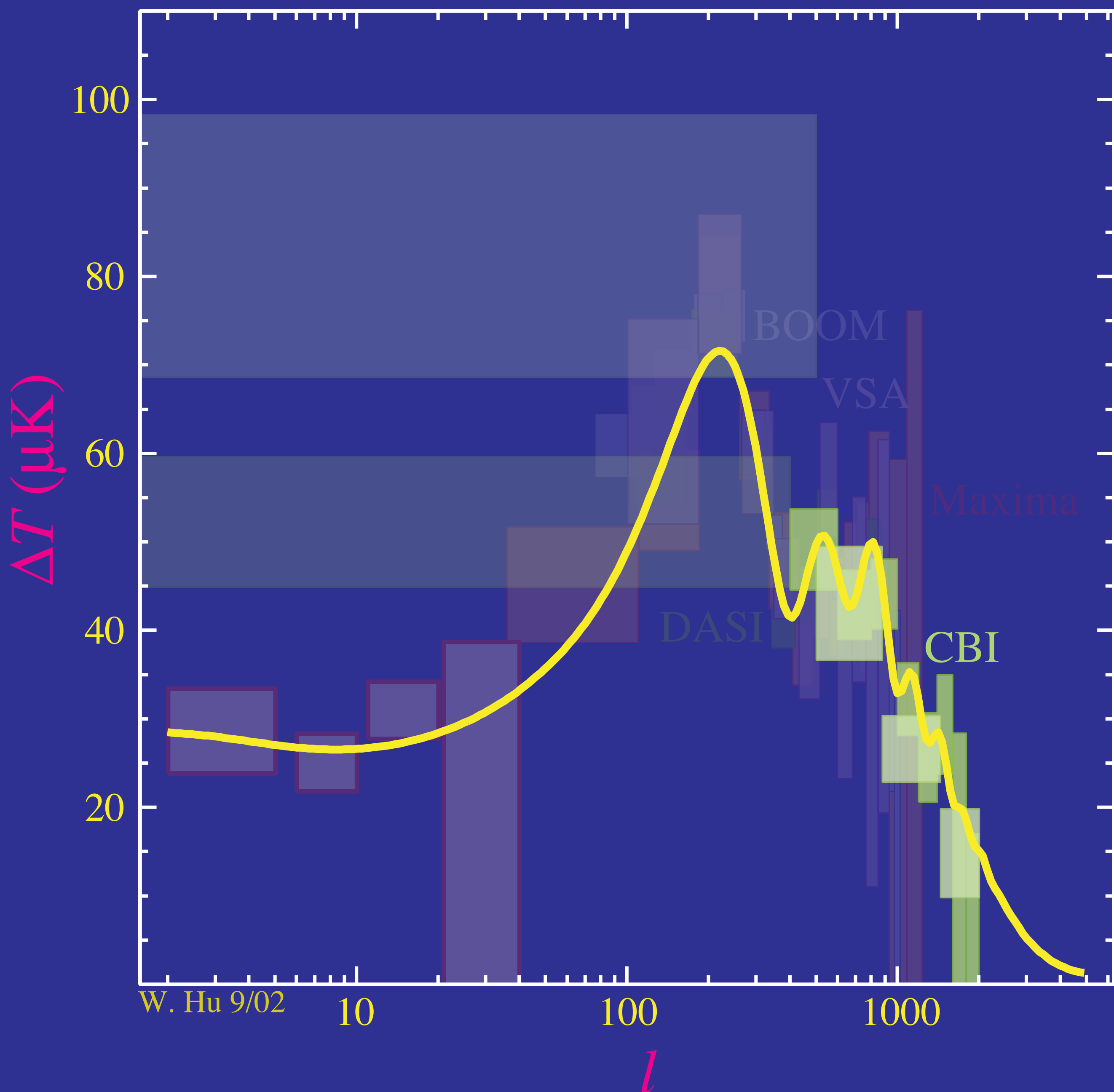
$$\lim_{R\to 0} k_D^{-2} = \frac{1}{6}\frac{16}{15} \int d\eta \frac{1}{\dot\tau}$$

$$\lim_{R\to \infty} k_D^{-2} = \frac{1}{6} \int d\eta \frac{1}{\dot\tau}$$

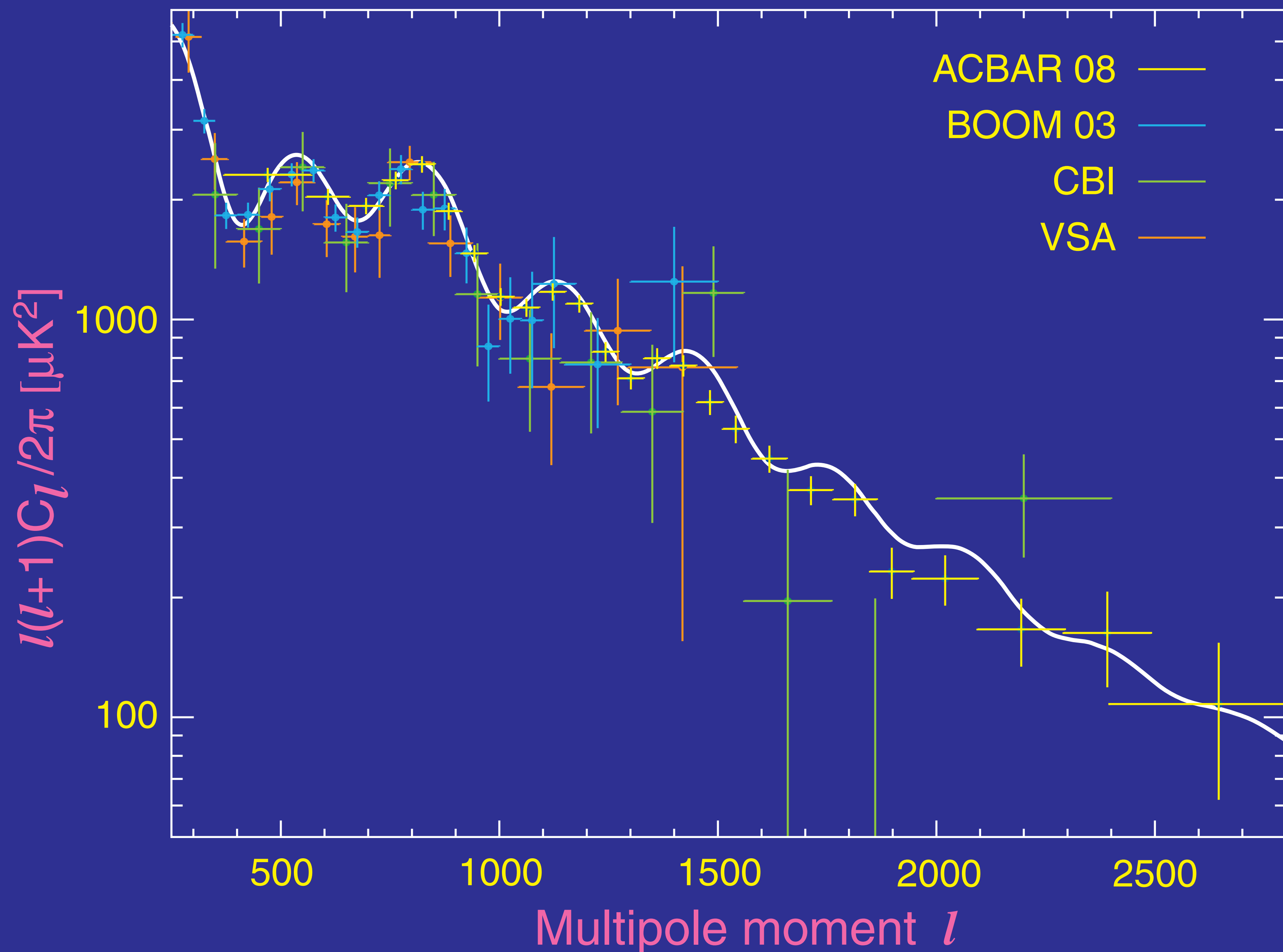
- Geometric mean between horizon and mean free path as expected from a random walk

$$\lambda_D = \frac{2\pi}{k_D} \sim \frac{2\pi}{\sqrt{6}} (\eta \dot\tau^{-1})^{1/2}$$

Damping Tail Measured

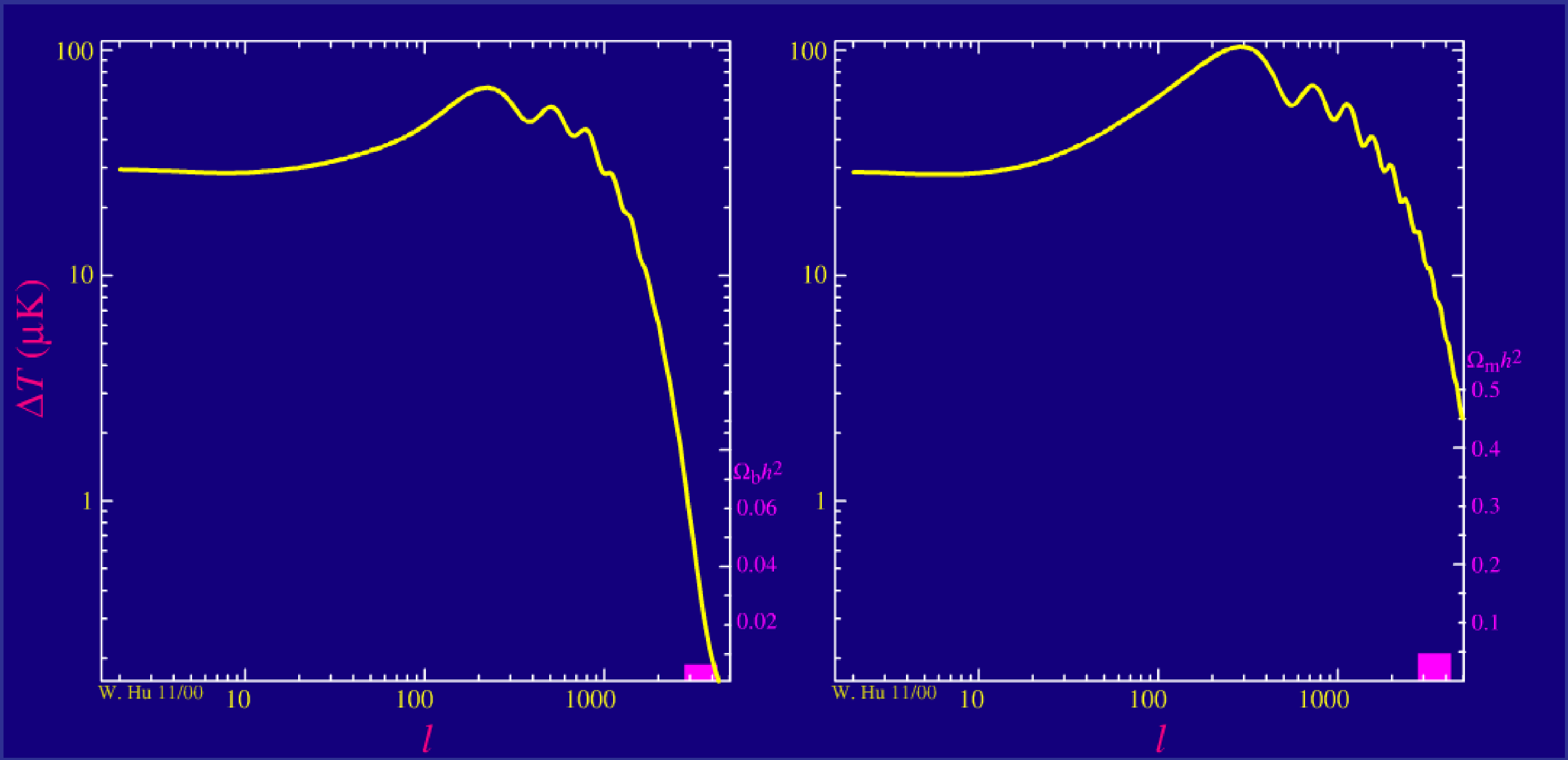


Power Spectrum Present



Standard Ruler

- Damping length is a fixed physical scale given properties at recombination
- Gemoetric mean of mean free path and horizon: depends on baryon-photon ratio and matter-radiation ratio



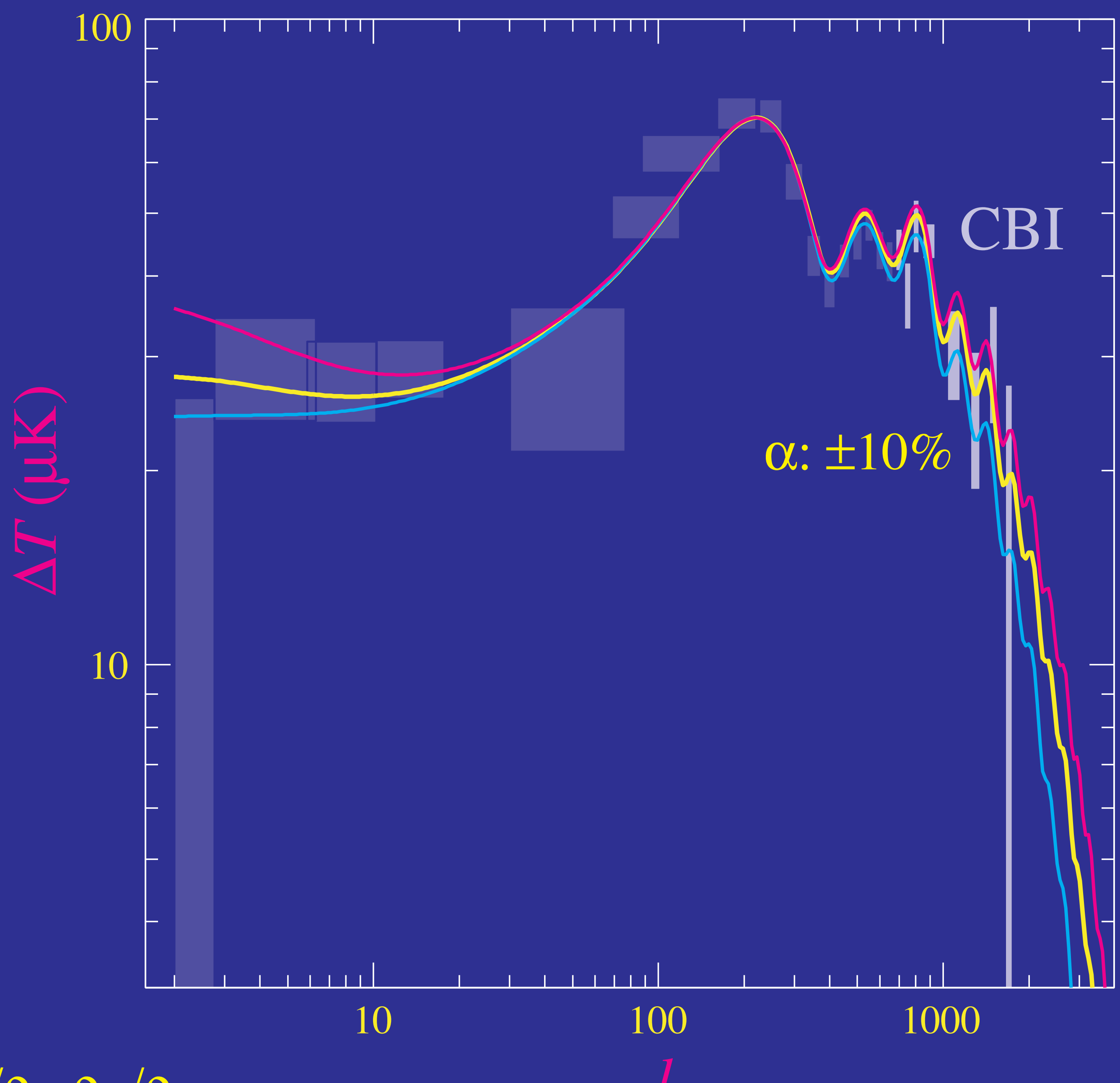
Standard Rulers

- Calibrating the Standard Rulers

- Sound Horizon

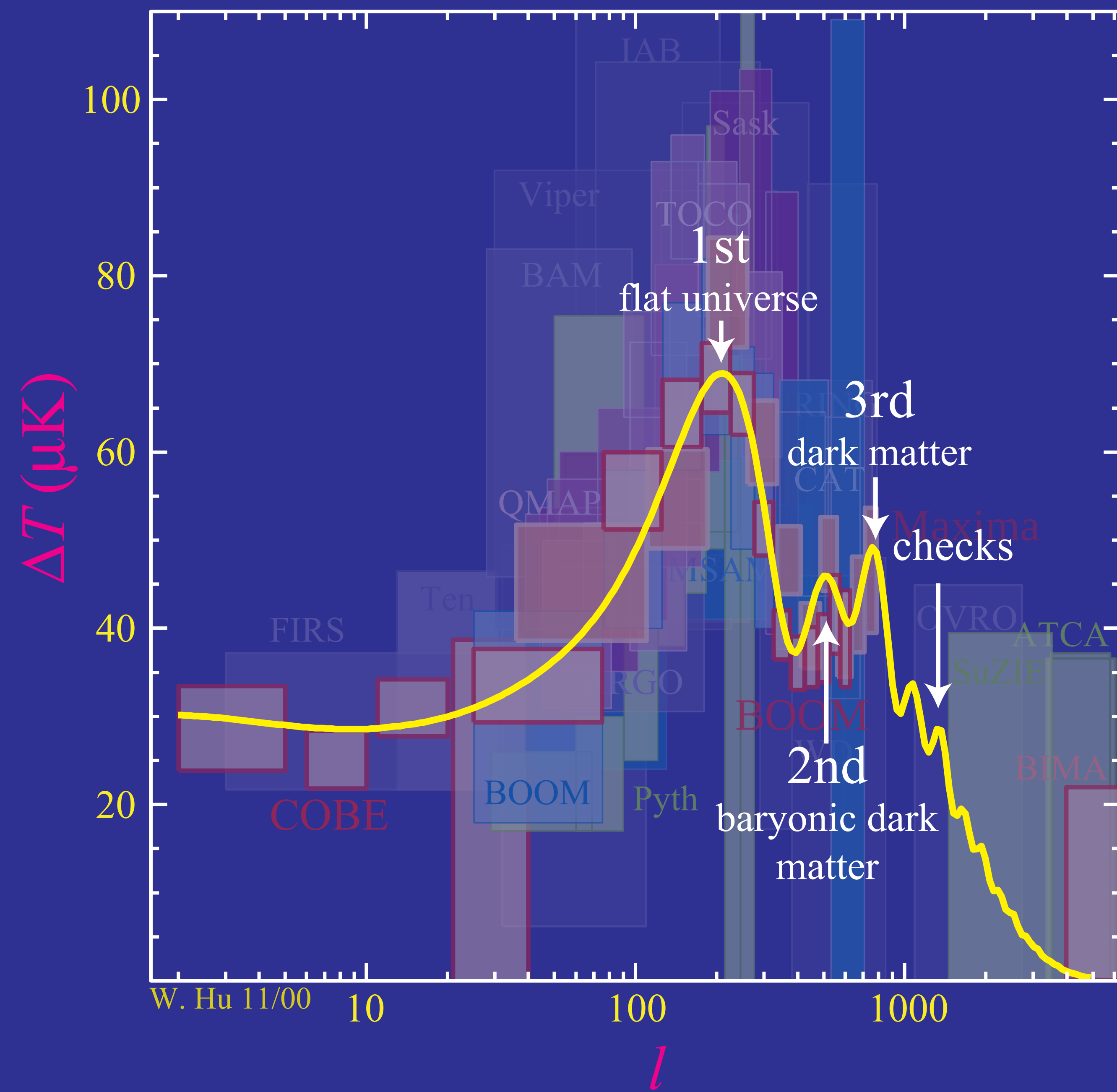
- Damping Scale

Consistency Check on Recombinaton



fixed l_A, ρ_b/ρ_γ, ρ_m/ρ_r

The Peaks



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ΔT (μK)
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MSAM
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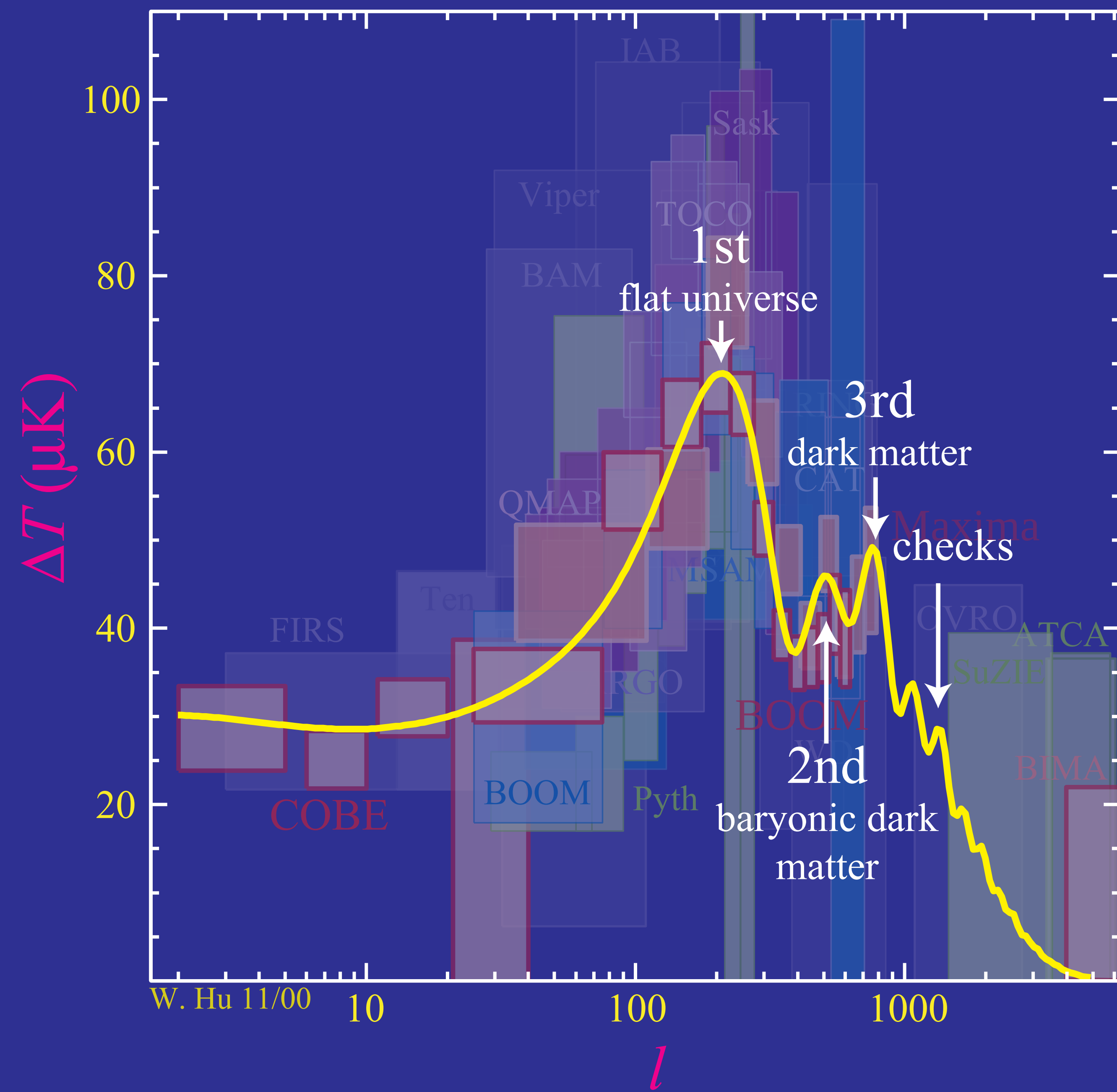
The Peaks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1st
flat universe
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dark matter
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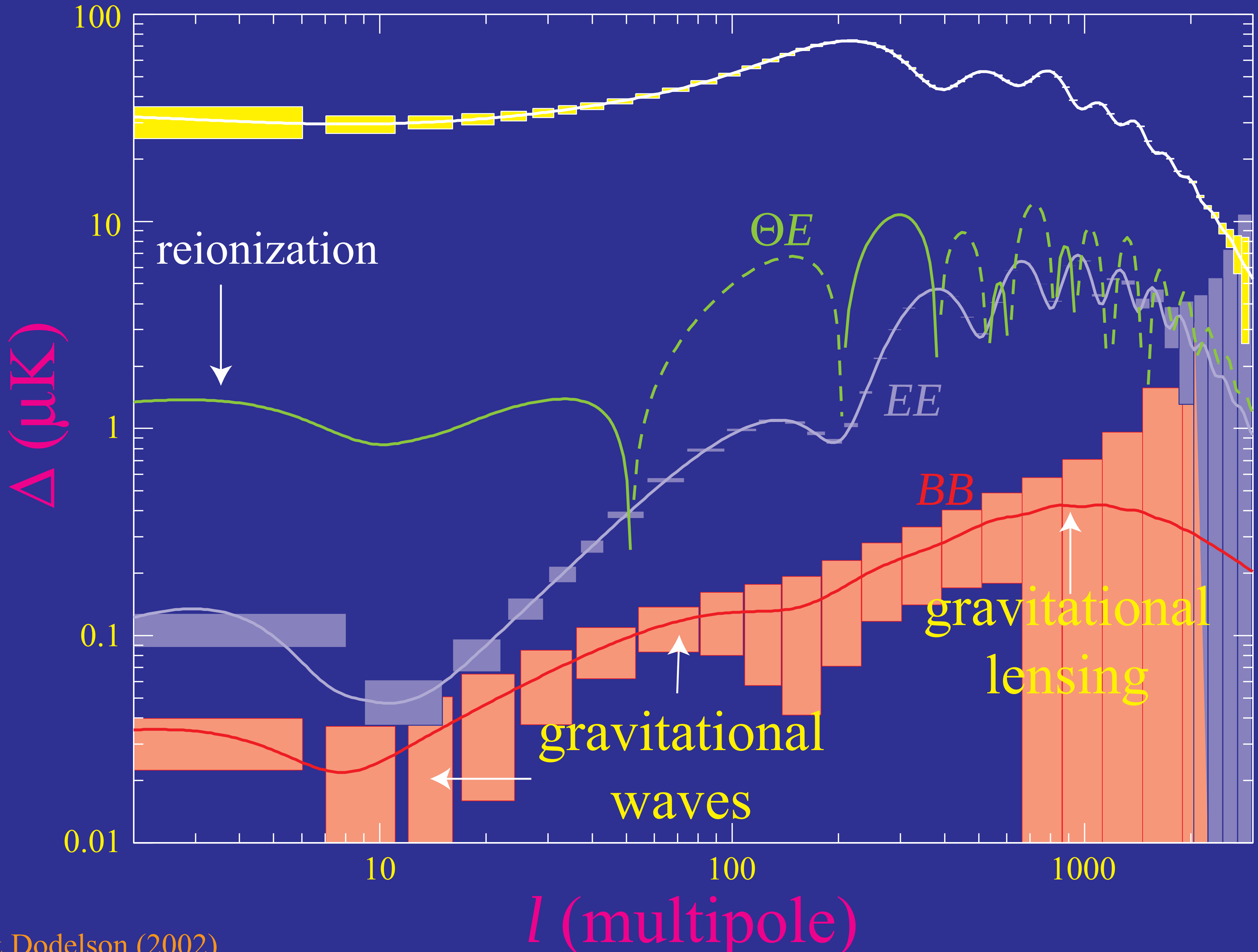
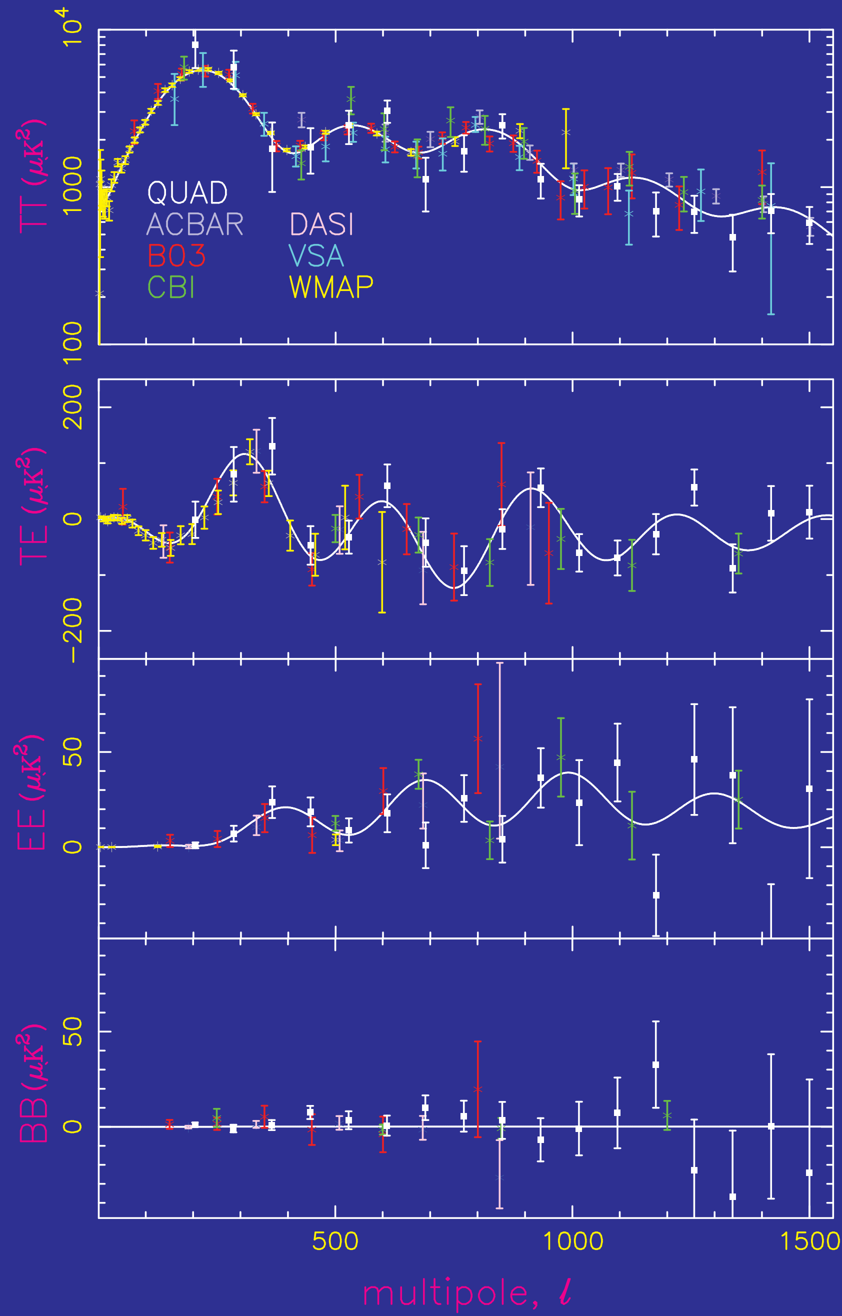
matter

Polarized Landscape



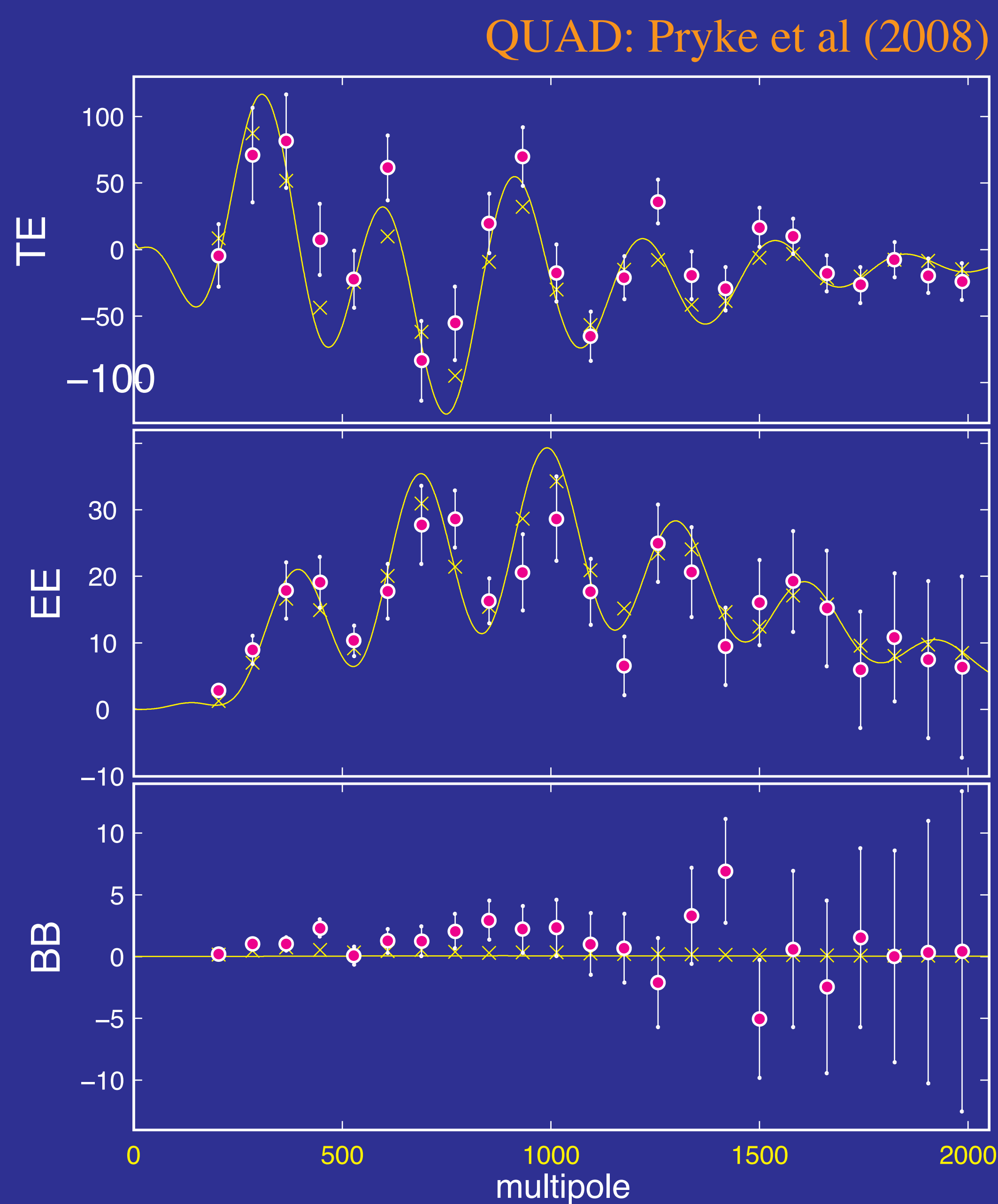
Hu & Dodelson (2002)

Recent Data



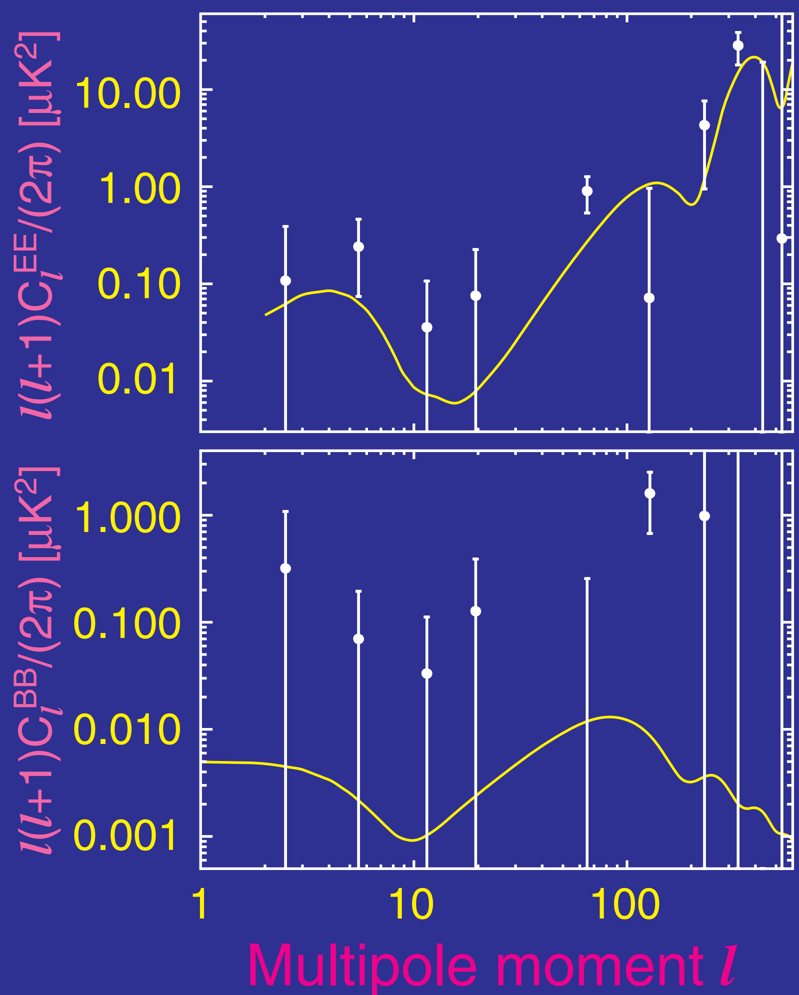
Ade et al (QUAD, 2007)

Power Spectrum Present



Instantaneous Reionization

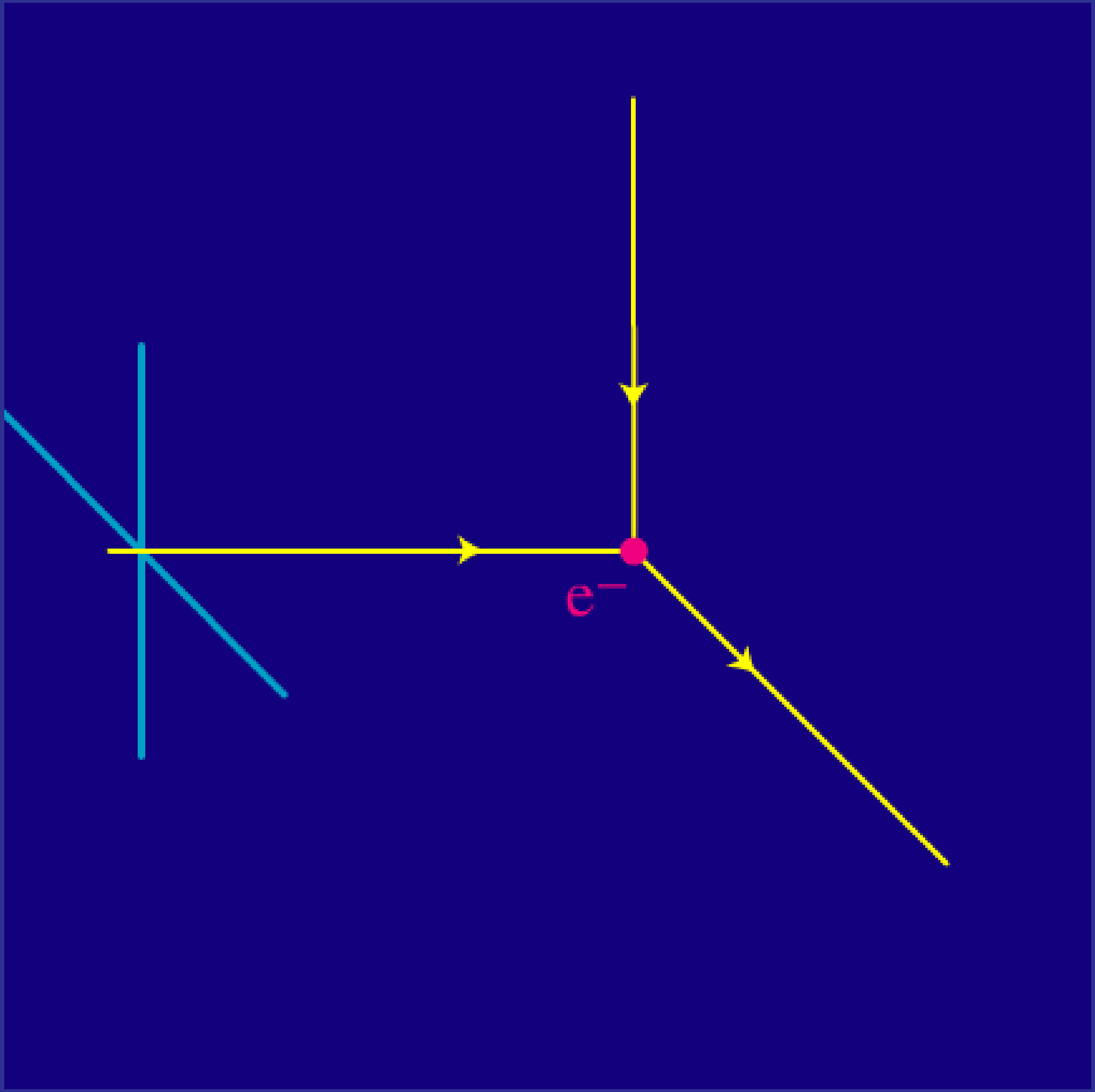
- WMAP data constrains optical depth for instantaneous models of $\tau=0.087\pm0.017$
- Upper limit on gravitational waves weaker than from temperature



Why is the CMB polarized?

Polarization from Thomson Scattering

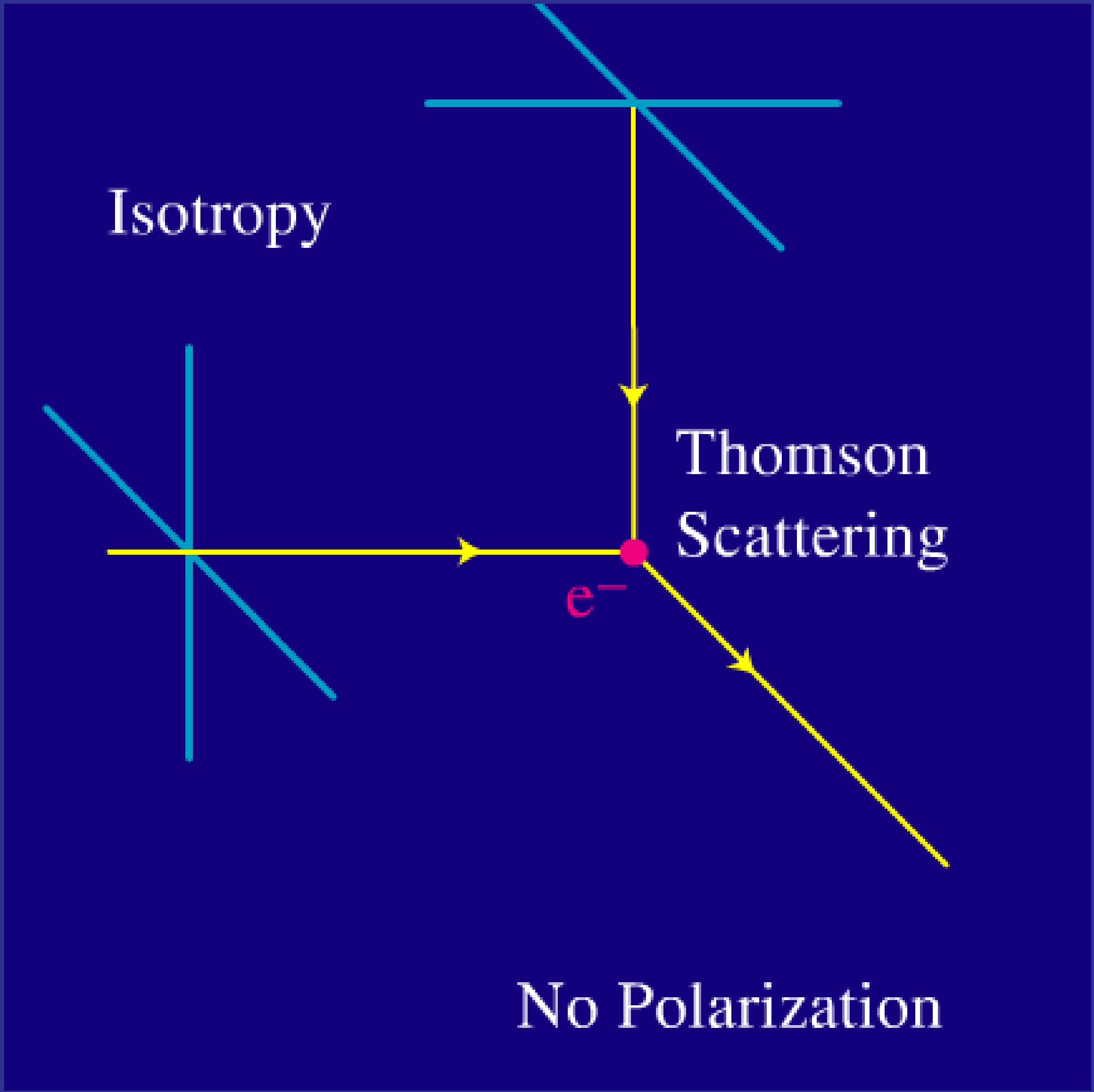
- Differential cross section depends on polarization and angle



$$\frac{d\sigma}{d\Omega} = \frac{3}{8\pi}|\hat{\varepsilon}' \cdot \hat{\varepsilon}|^2 \sigma_T$$

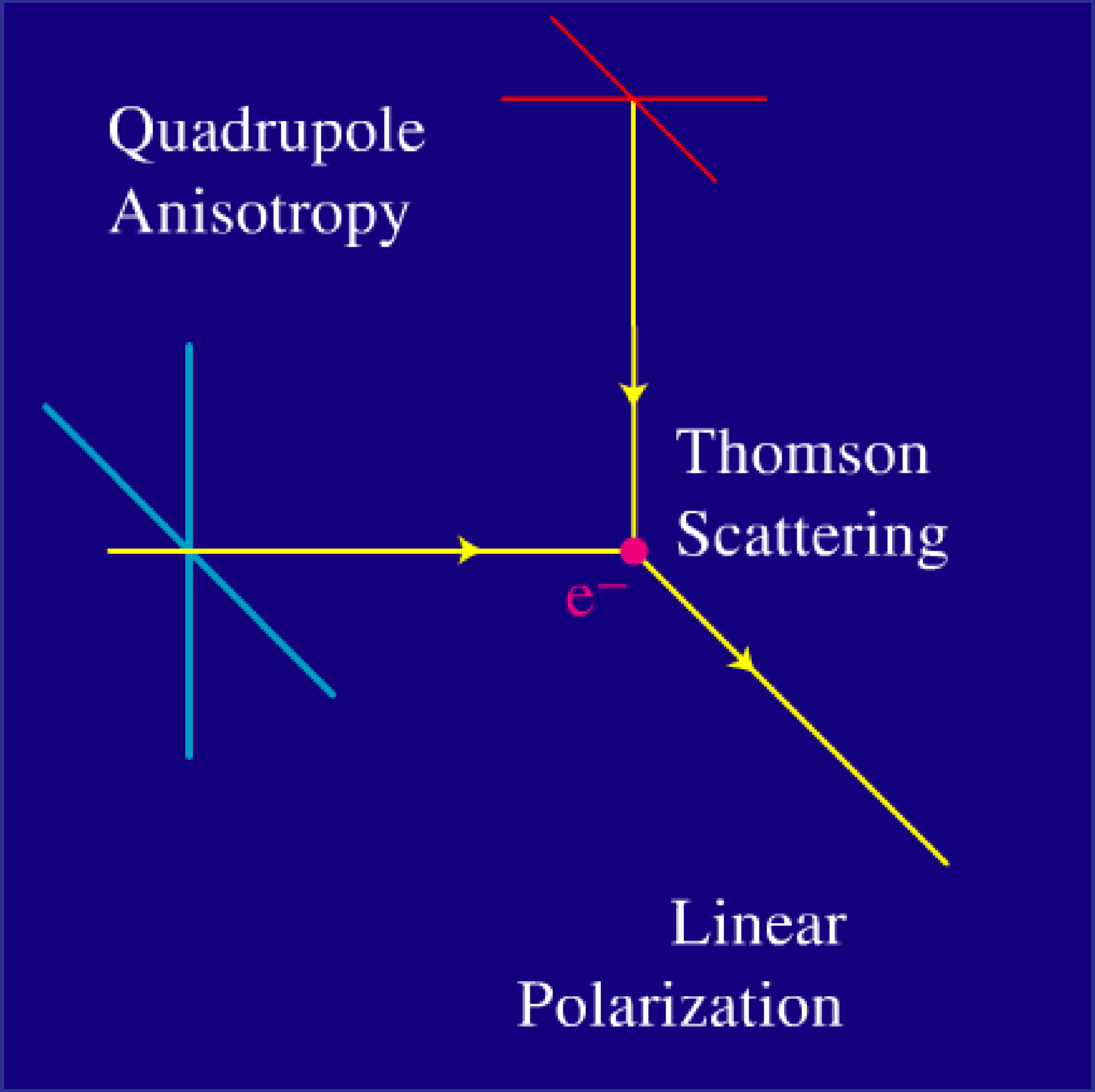
Polarization from Thomson Scattering

- Isotropic radiation scatters into unpolarized radiation



Polarization from Thomson Scattering

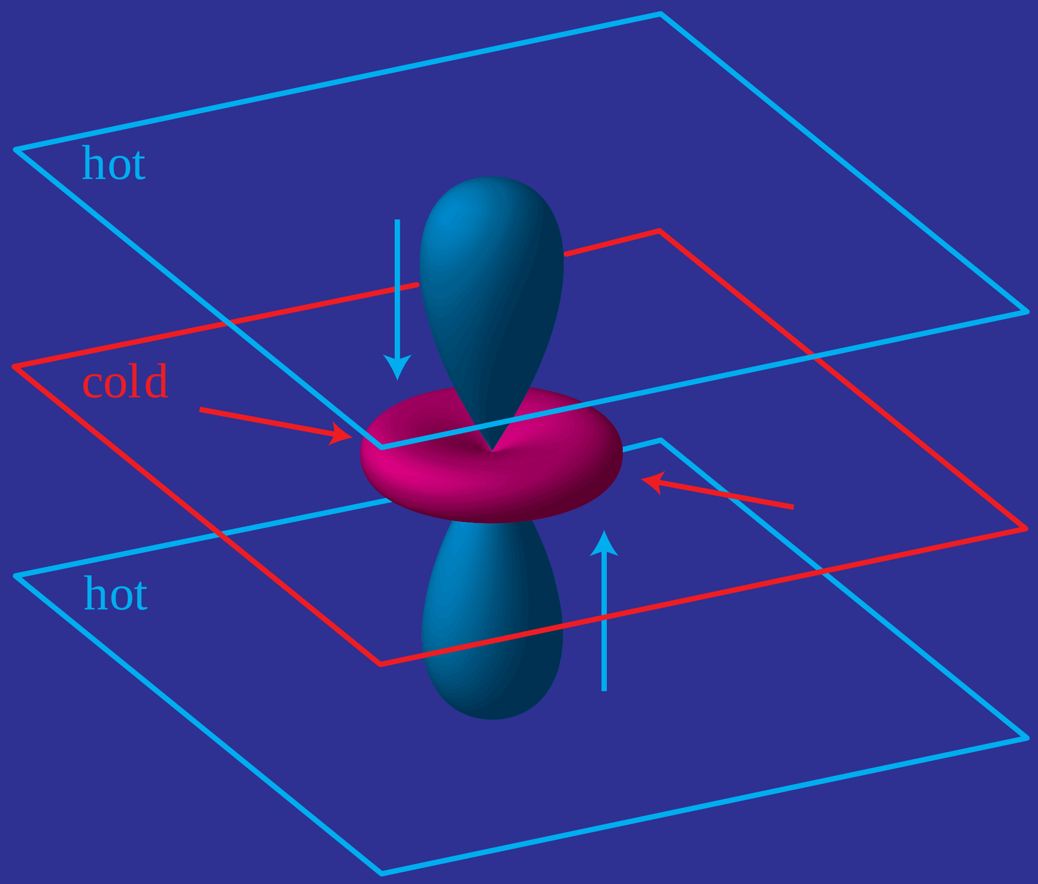
- Quadrupole anisotropies scatter into linear polarization



aligned with
cold lobe

Whence Quadrupoles?

- Temperature inhomogeneities in a medium
- Photons arrive from different regions producing an anisotropy

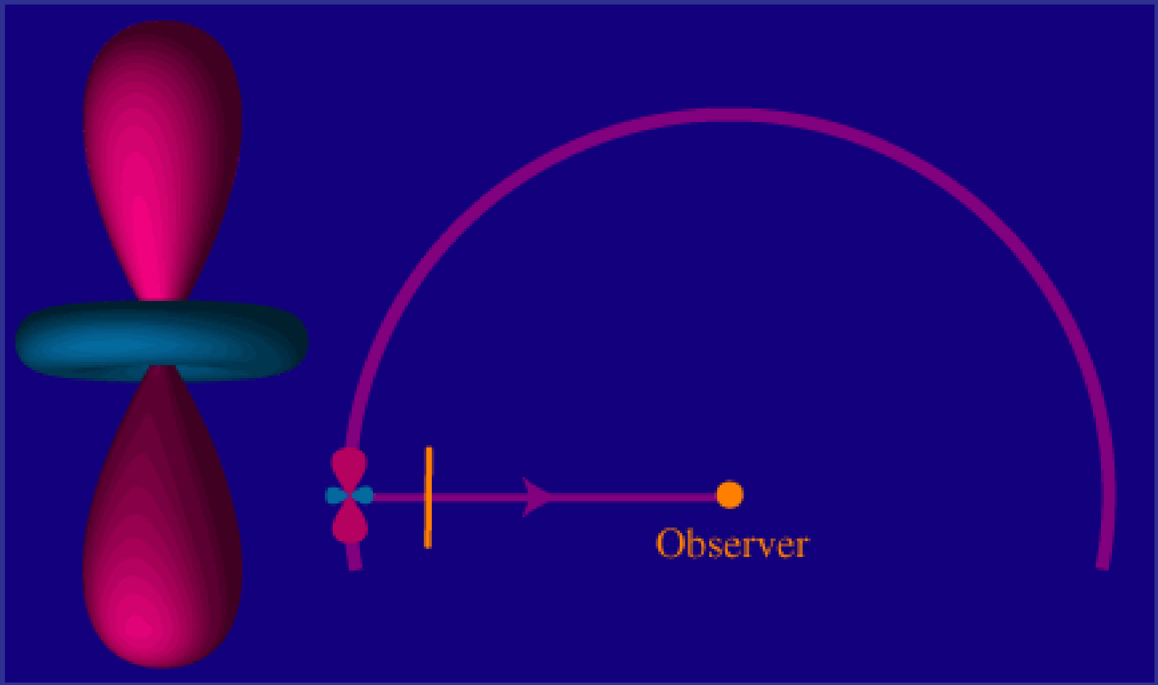


(Scalar) Temperature Inhomogeneity

Hu & White (1997)

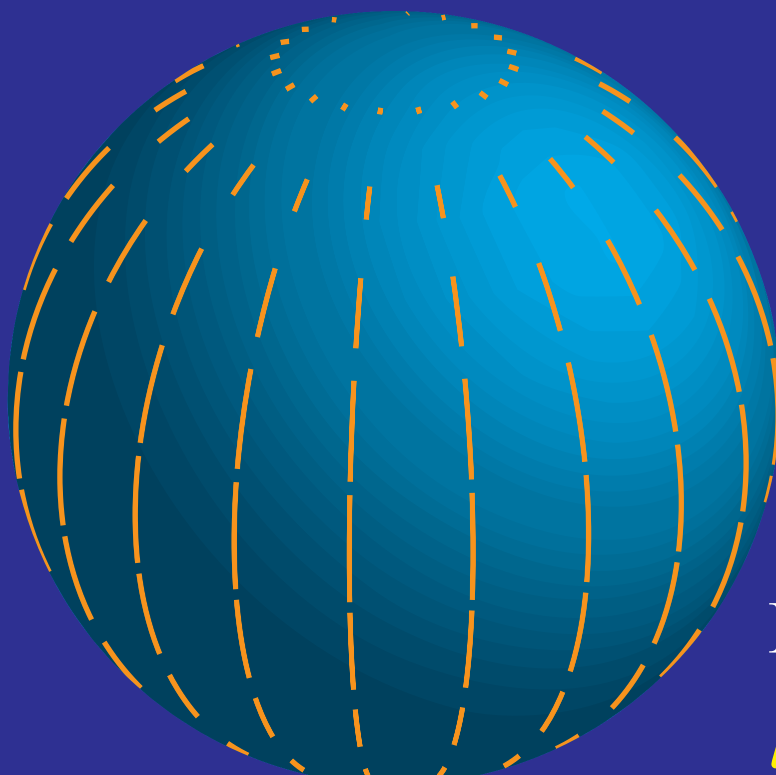
Whence Polarization Anisotropy?

- Observed photons scatter into the line of sight
- Polarization arises from the projection of the quadrupole on the transverse plane



Polarization Multipoles

- Mathematically pattern is described by the tensor (spin-2) spherical harmonics [eigenfunctions of Laplacian on trace-free 2 tensor]
- Correspondence with scalar spherical harmonics established via Clebsch-Gordan coefficients (spin x orbital)
- Amplitude of the coefficients in the spherical harmonic expansion are the multipole moments; averaged square is the power



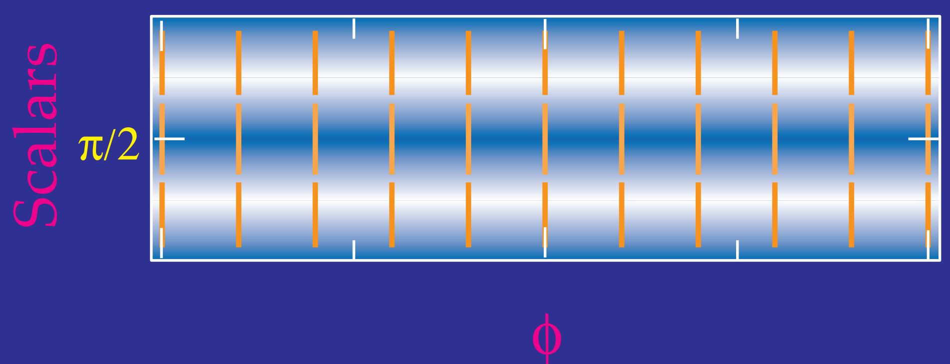
E-tensor harmonic

$l=2,\ m=0$

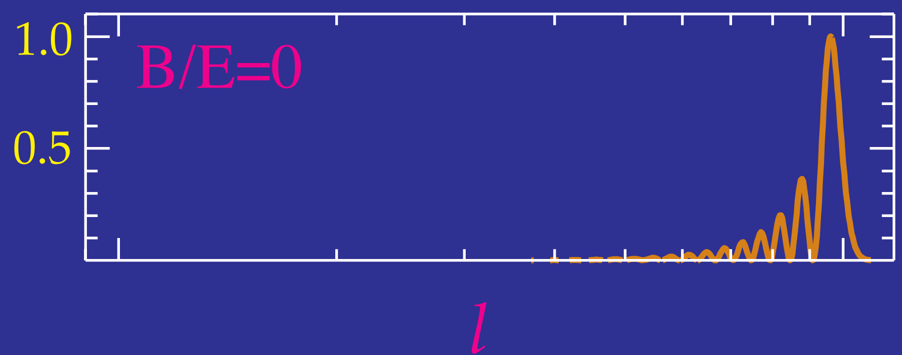
Modulation by Plane Wave

- Amplitude modulated by plane wave → higher multipole moments
- Direction detemined by perturbation type → E-modes

Polarization Pattern

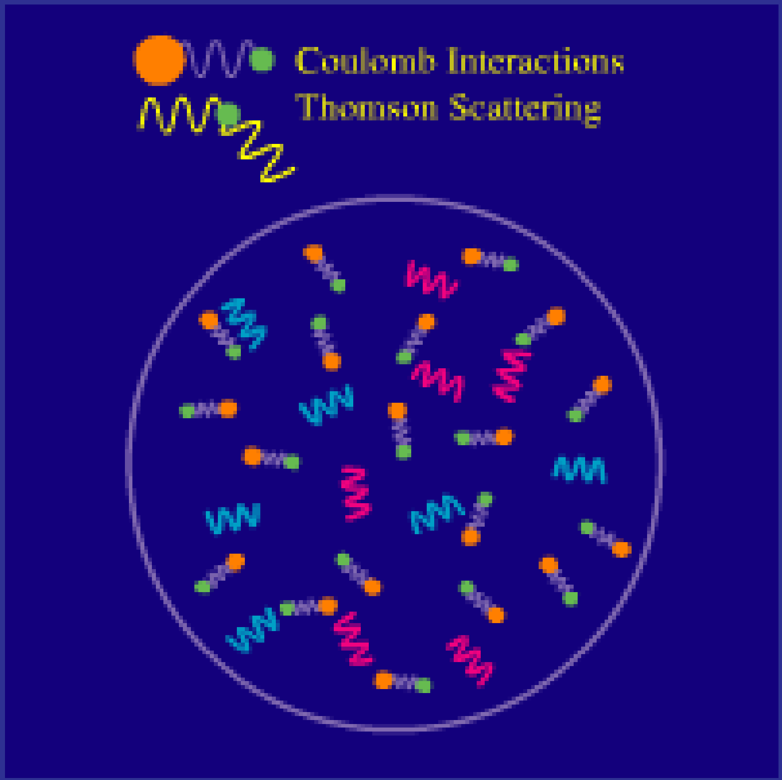


Multipole Power



A Catch-22

- Polarization is generated by scattering of anisotropic radiation
- Scattering isotropizes radiation
- Polarization only arises in optically thin conditions: reionization and end of recombination
- Polarization fraction is at best a small fraction of the 10^{-5} anisotropy: $\sim 10^{-6}$ or μK in amplitude



Polarization Peaks

Fluid Imperfections

- Perfect fluid: no anisotropic stresses due to scattering isotropization; baryons and photons move as single fluid
- Fluid imperfections are related to the mean free path of the photons in the baryons

$$\lambda_C = \dot{\tau}^{-1} \quad \text{where} \quad \dot{\tau} = n_e \sigma_T a$$

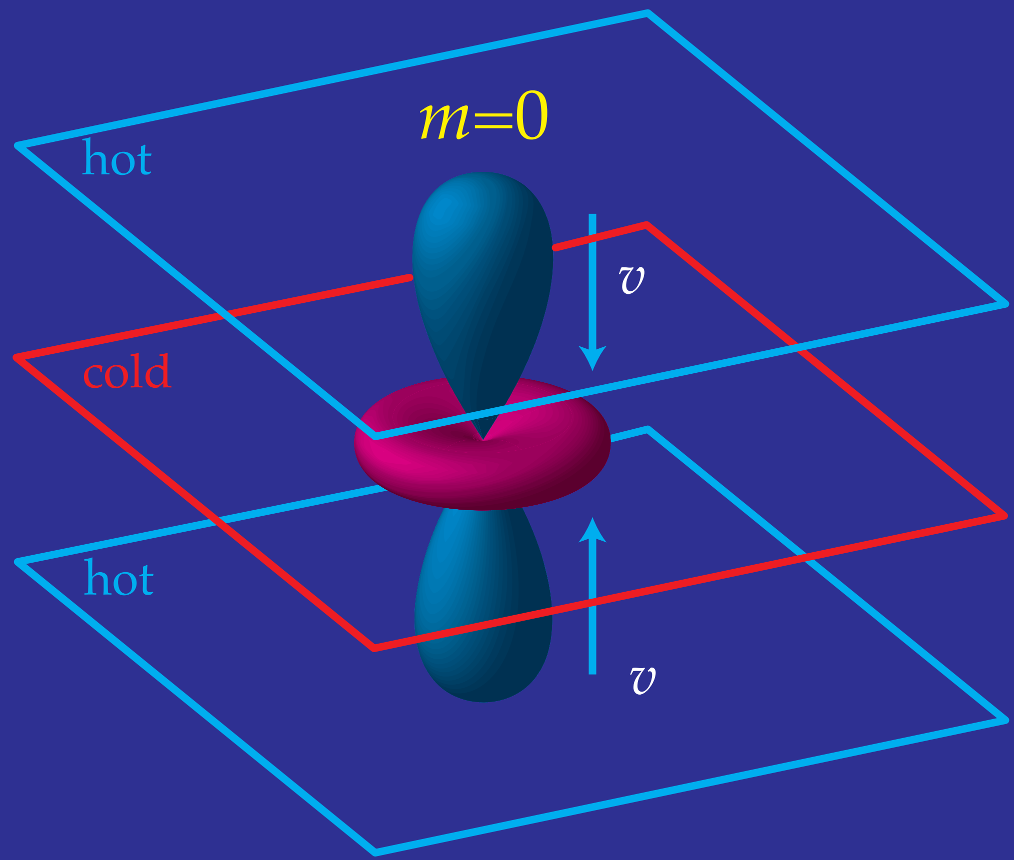
  is the conformal opacity to Thomson scattering
- Dissipation is related to the diffusion length: random walk approximation

$$\lambda_D = \sqrt{N} \lambda_C = \sqrt{\eta / \lambda_C}\, \lambda_C = \sqrt{\eta \lambda_C}$$

  the geometric mean between the horizon and mean free path
- $\lambda_D / \eta_* \sim$ few %, so expect the peaks >3 to be affected by dissipation

Viscosity & Heat Conduction

- Both fluid imperfections are related to the gradient of the velocity kv_γ by opacity $\dot{\tau}$: slippage of fluids $v_\gamma - v_b$.
- Viscosity is an anisotropic stress or quadrupole moment formed by radiation streaming from hot to cold regions



Dimensional Analysis

- Viscosity= quadrupole anisotropy that follows the fluid velocity

$$\pi_\gamma \approx \frac{k}{\dot\tau} v_\gamma$$

- Mean free path related to the damping scale via the random walk $k_D = (\dot\tau/\eta_*)^{1/2} \rightarrow \dot\tau = k_D^2 \eta_*$
- Damping scale at $\ell \sim 1000$ vs horizon scale at $\ell \sim 100$ so $k_D \eta_* \approx 10$
- Polarization amplitude rises to the damping scale to be $\sim 10\%$ of anisotropy

$$\pi_\gamma \approx \frac{k}{k_D}\frac{1}{10} v_\gamma \qquad \Delta_P \approx \frac{\ell}{\ell_D}\frac{1}{10}\Delta_T$$

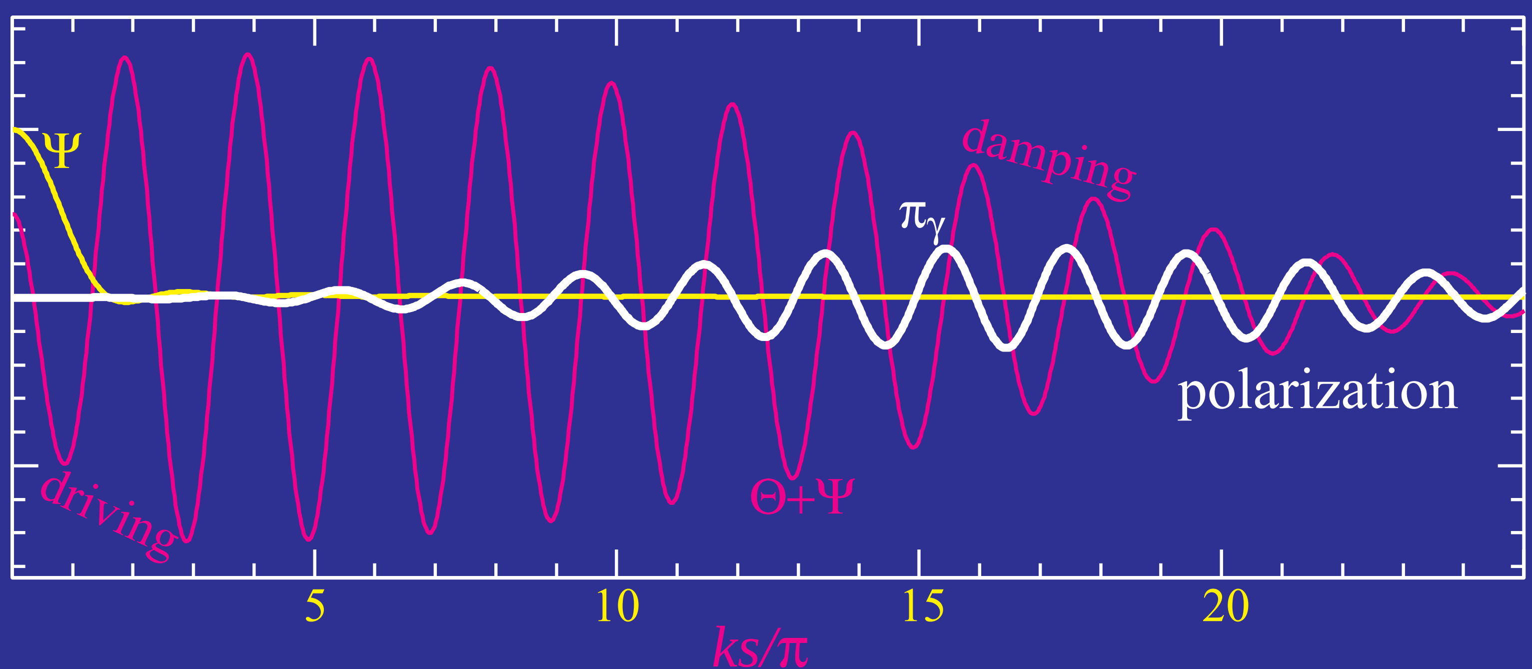
- Polarization phase follows fluid velocity

Damping & Polarization

- Quadrupole moments:

  damp acoustic oscillations from fluid viscosity
  generates polarization from scattering

- Rise in polarization power coincides with fall in temperature power – $l \sim 1000$



Acoustic Polarization

- Gradient of velocity is along direction of wavevector, so polarization is pure E-mode
- Velocity is 90° out of phase with temperature – turning points of oscillator are zero points of velocity:

$$\Theta + \Psi \propto \cos(ks); \quad v_\gamma \propto \sin(ks)$$

- Polarization peaks are at troughs of temperature power

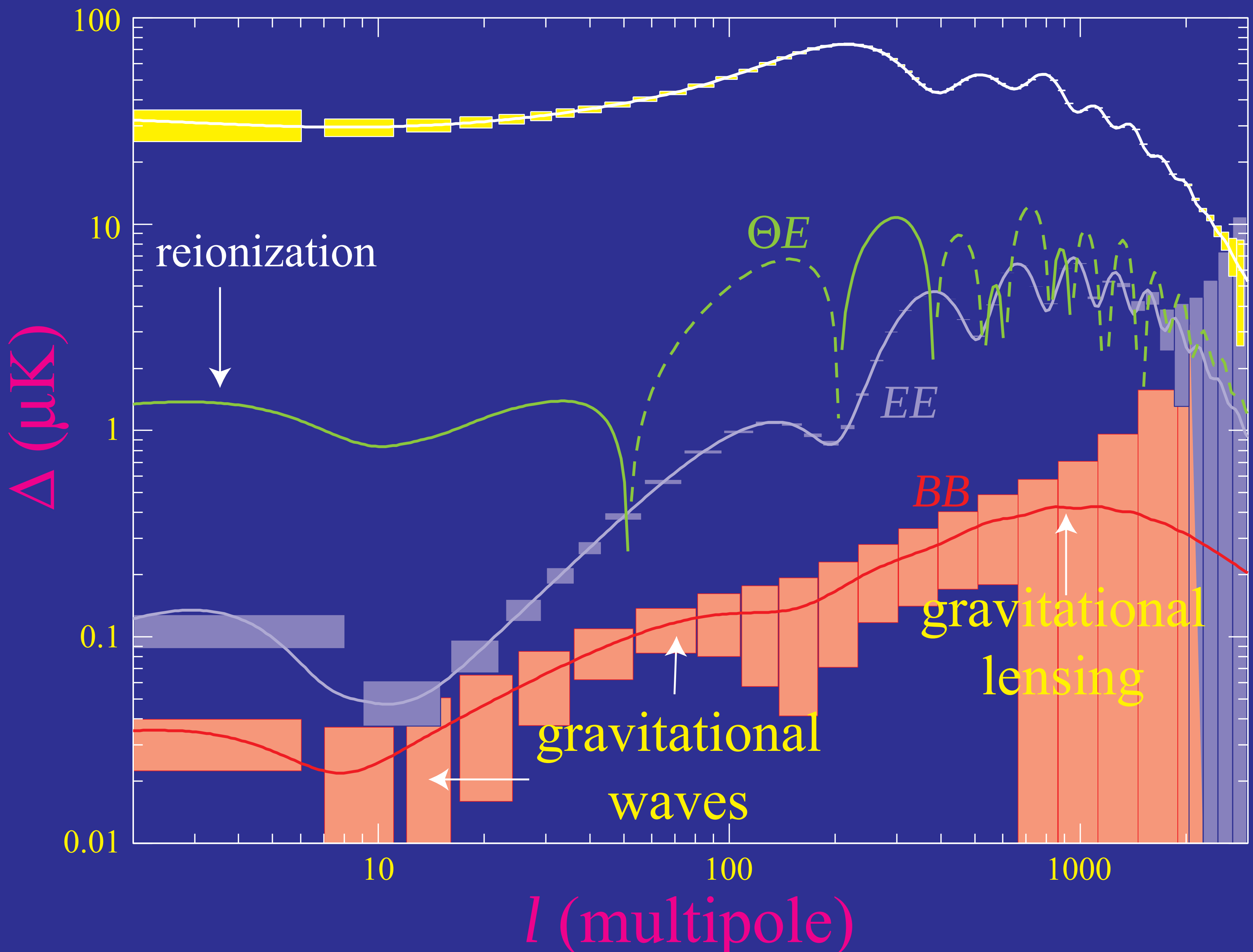
Cross Correlation

- Cross correlation of temperature and polarization

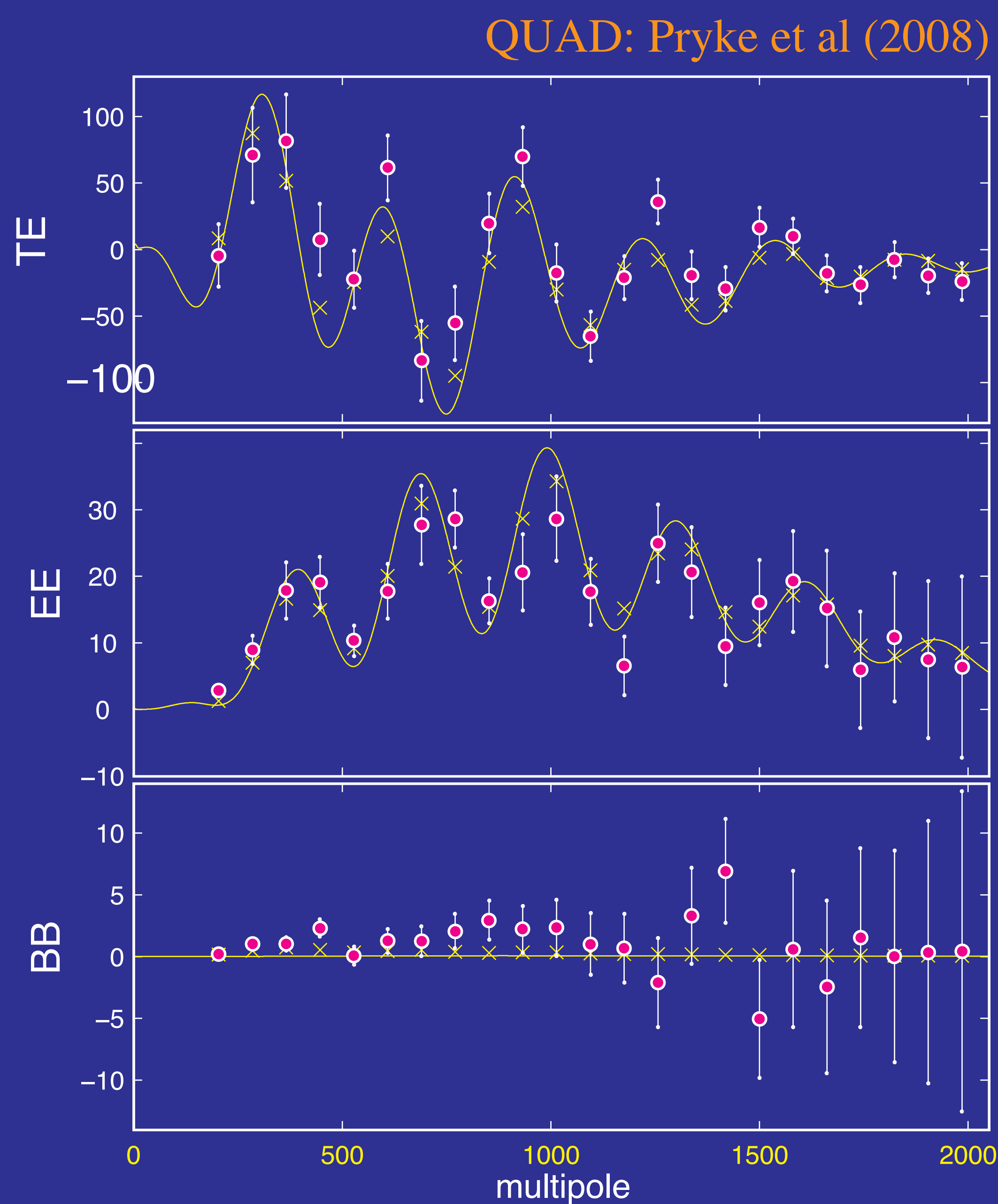
$$(\Theta + \Psi)(v_\gamma) \propto \cos(ks)\sin(ks) \propto \sin(2ks)$$

- Oscillation at twice the frequency
- Correlation: radial or tangential around hot spots
- Partial correlation: easier to measure if polarization data is noisy, harder to measure if polarization data is high S/N or if bands do not resolve oscillations
- Good check for systematics and foregrounds
- Comparison of temperature and polarization is proof against features in initial conditions mimicking acoustic features

Temperature and Polarization Spectra



Power Spectrum Present



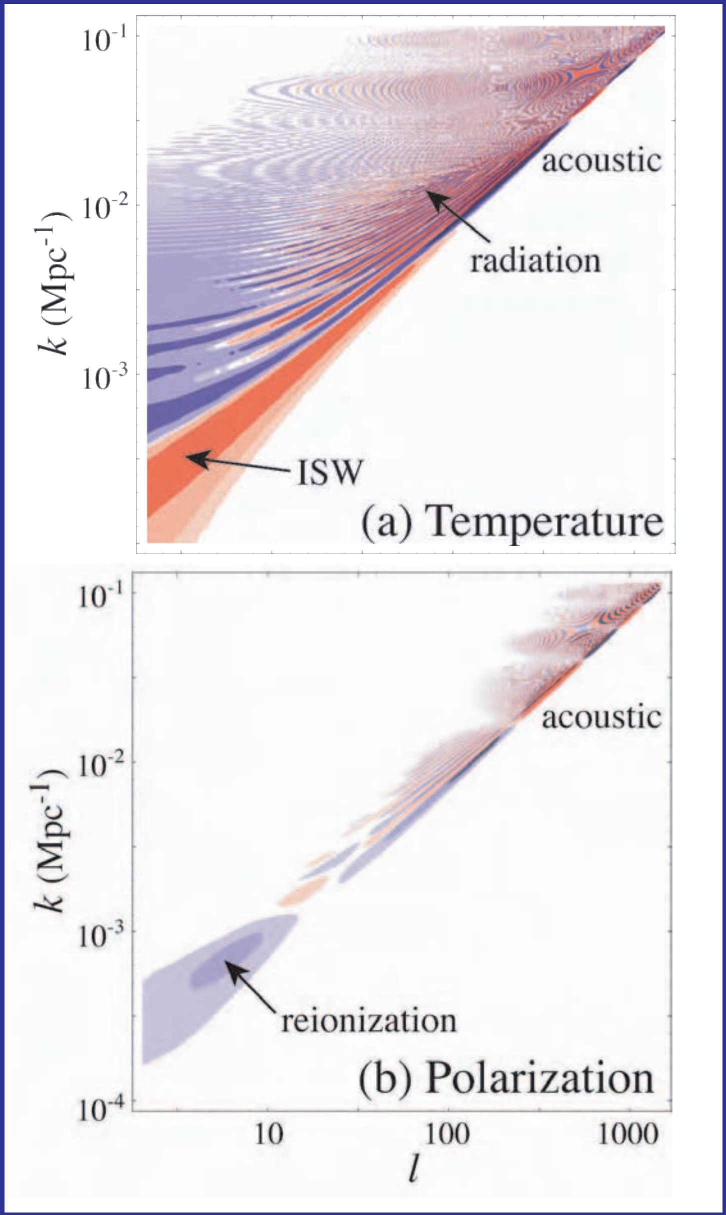
Why Care?

- In the standard model, acoustic polarization spectra uniquely predicted by same parameters that control temperature spectra
- Validation of standard model
- Improved statistics on cosmological parameters controlling peaks

- Polarization is a complementary and intrinsically more incisive probe of the initial power spectrum and hence inflationary (or alternate) models

- Acoustic polarization is lensed by the large scale structure into B-modes
- Lensing B-modes sensitive to the growth of structure and hence neutrino mass and dark energy
- Contaminate the gravitational wave B-mode signature

Transfer of Initial Power

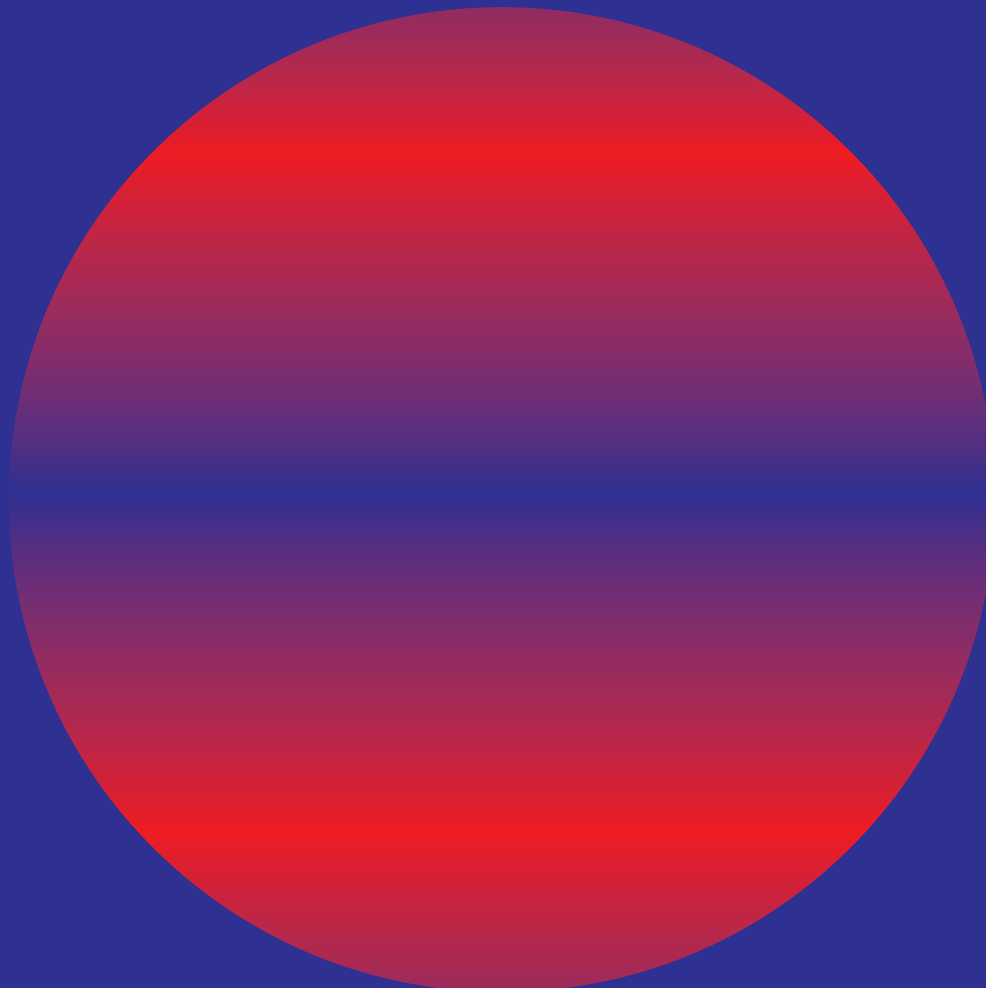


Hu & Okamoto (2003)

Reionization

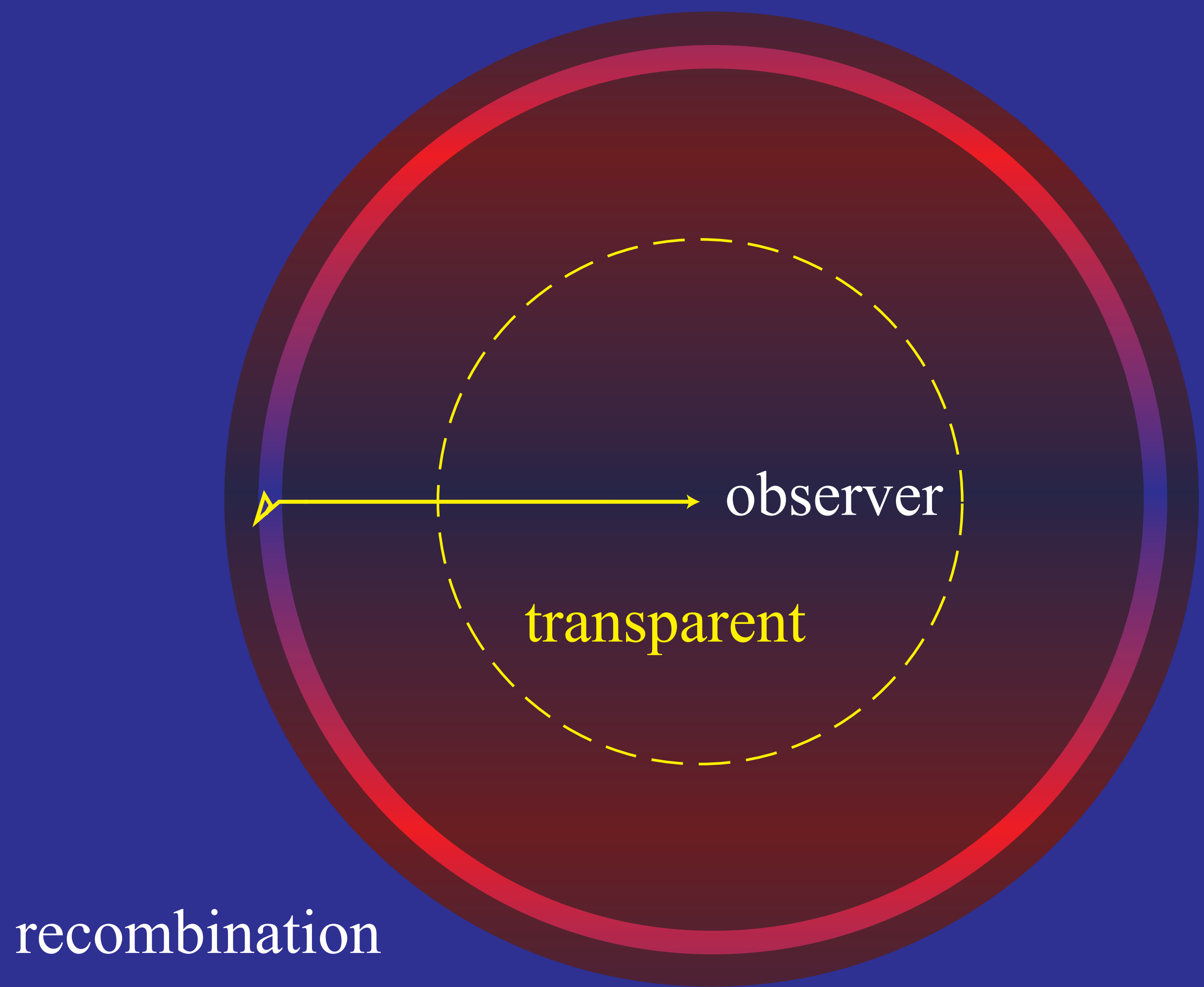
Temperature Inhomogeneity

- Temperature inhomogeneity reflects initial density perturbation on large scales
- Consider a single Fourier moment:



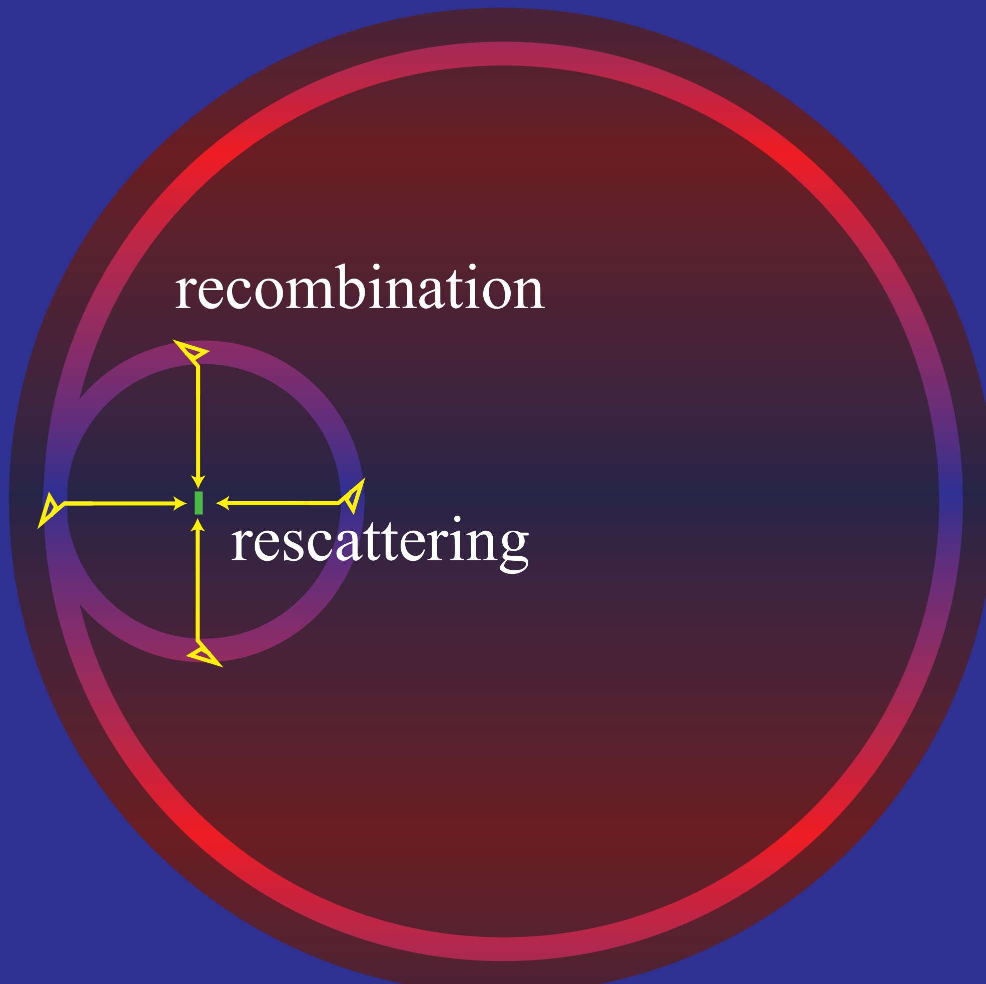
Locally Transparent

- Presently, the matter density is so low that a typical CMB photon will not scatter in a Hubble time (~age of universe)



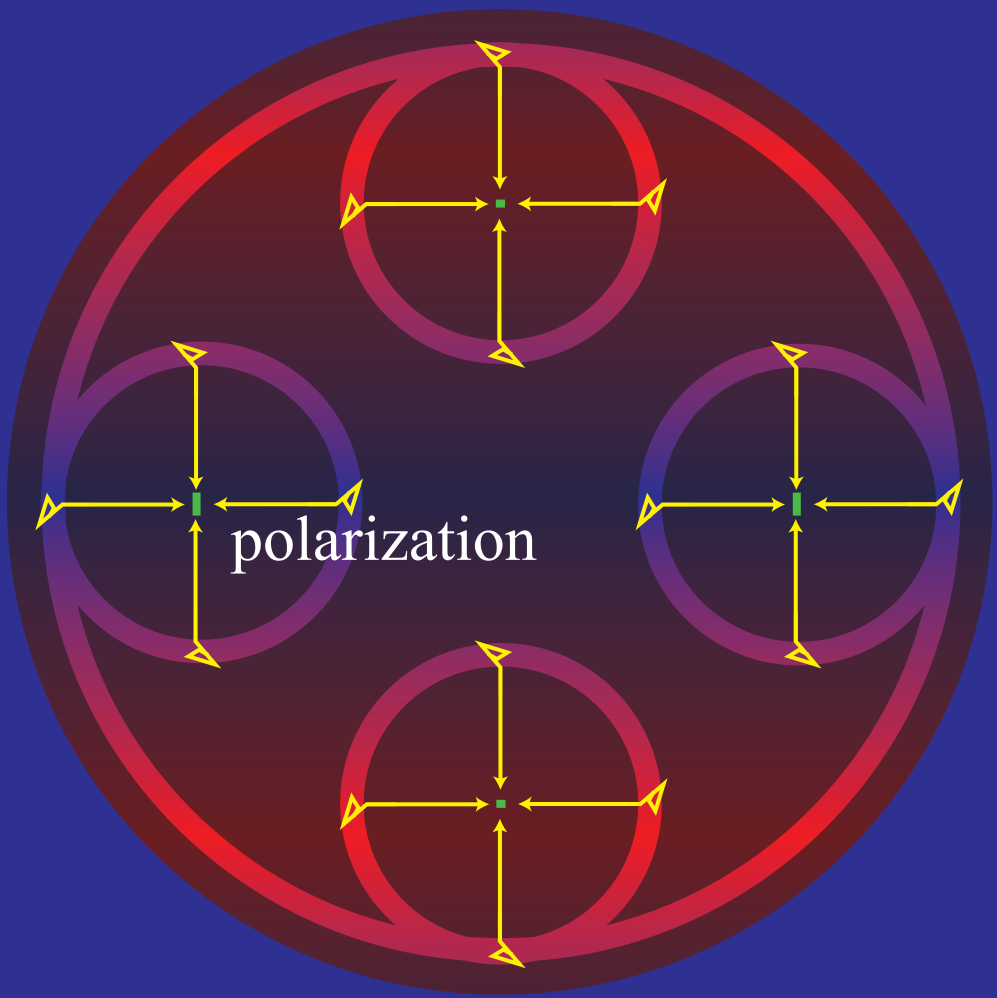
Reversed Expansion

- Free electron density in an ionized medium increases as scale factor a^{-3}; when the universe was a tenth of its current size CMB photons have a finite (~10%) chance to scatter



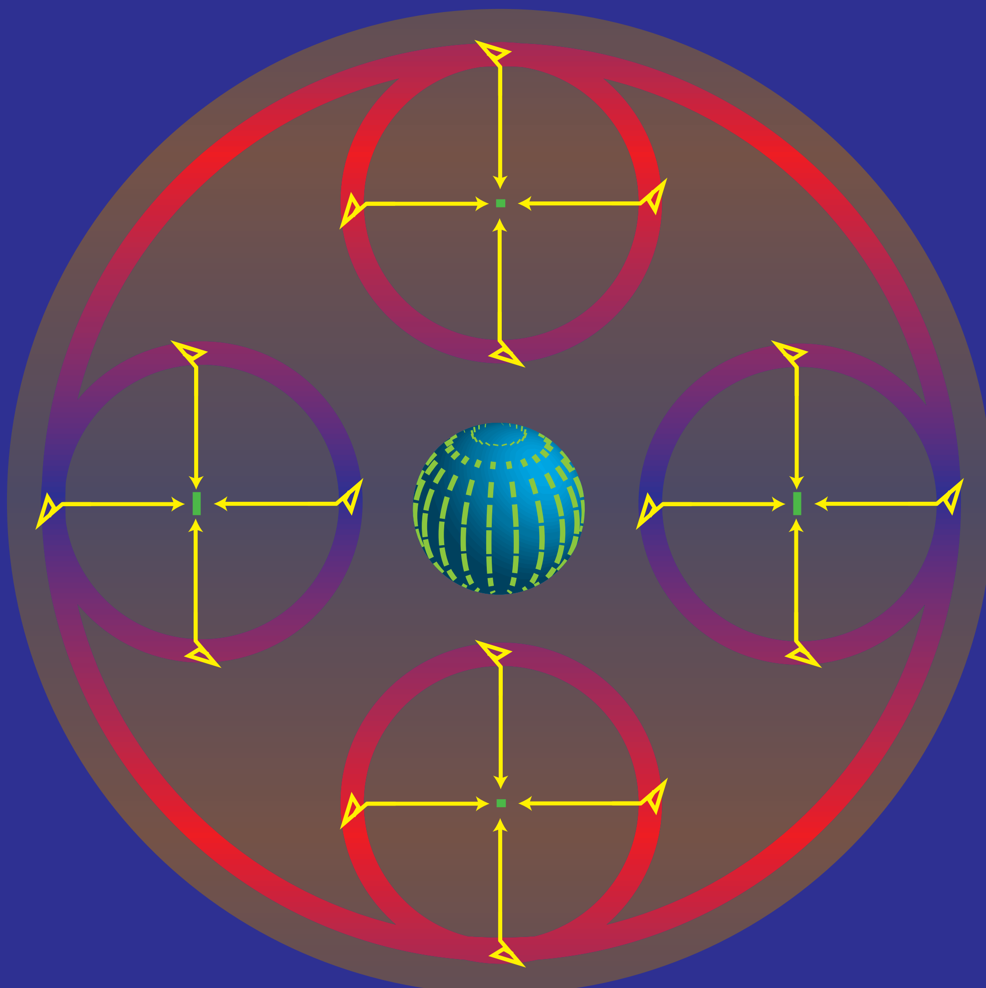
Polarization Anisotropy

- Electron sees the temperature anisotropy on its recombination surface and scatters it into a polarization



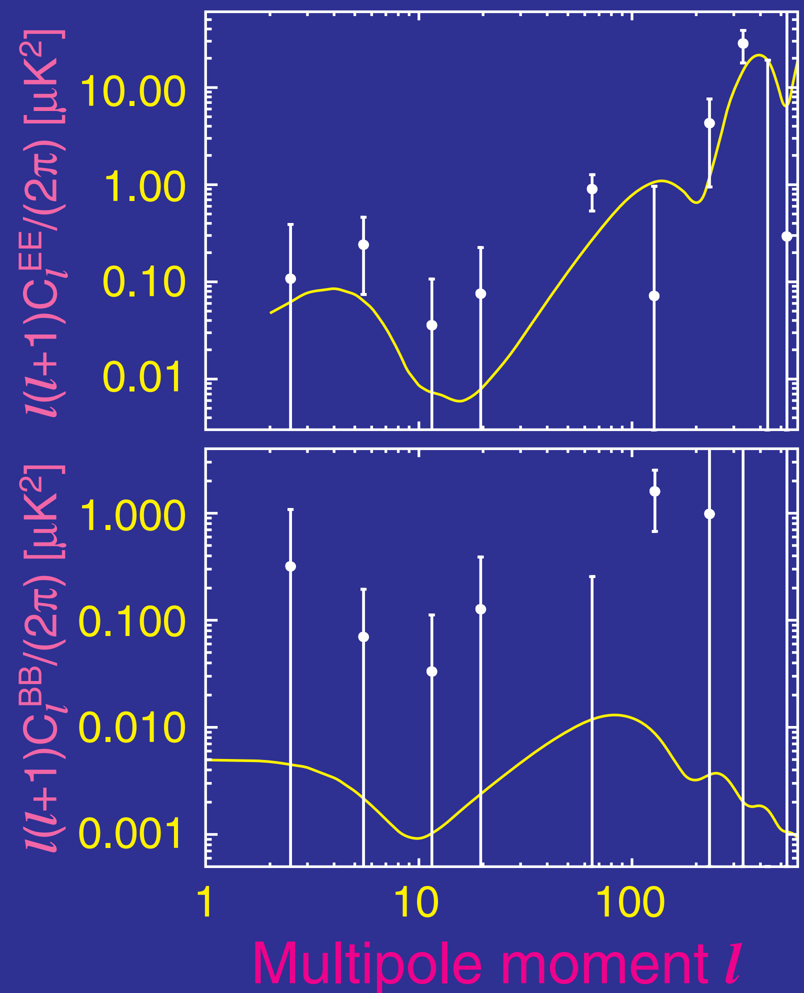
Temperature Correlation

- Pattern correlated with the temperature anisotropy that generates it; here an $m=0$ quadrupole



Instantaneous Reionization

- WMAP data constrains optical depth for instantaneous models of τ=0.087±0.017
- Upper limit on gravitational waves weaker than from temperature



Why Care?

- Early ionization is puzzling if due to ionizing radiation from normal stars; may indicate more exotic physics is involved

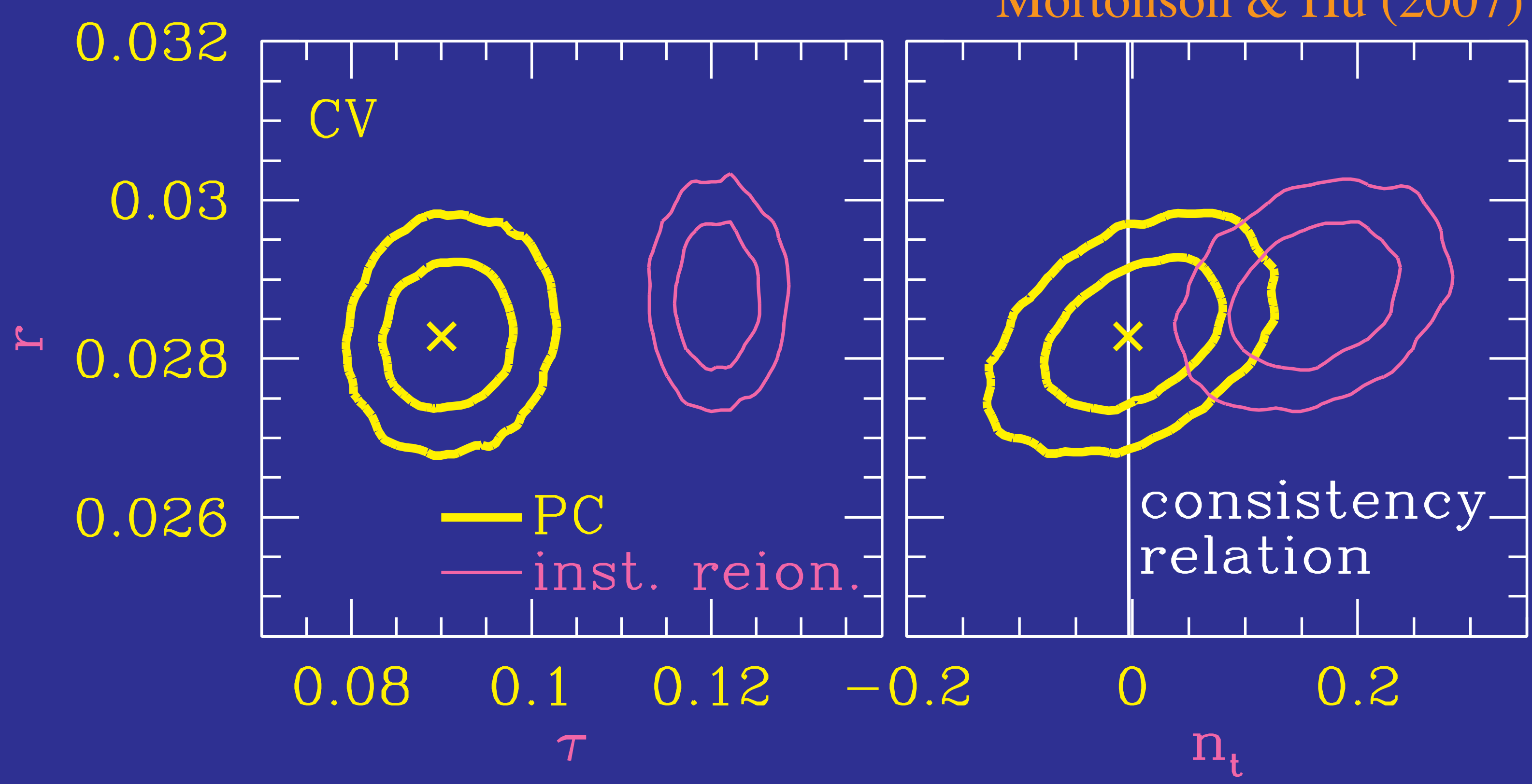
- Reionization screens temperature anisotropy on small scales making the true amplitude of initial fluctuations larger by e^{τ}
- Measuring the growth of fluctuations is one of the best ways of determining the neutrino masses and the dark energy

- Offers an opportunity to study the origin of the low multipole statistical anomalies
- Presents a second, and statistically cleaner, window on gravitational waves from the early universe

Consistency Relation & Reionization

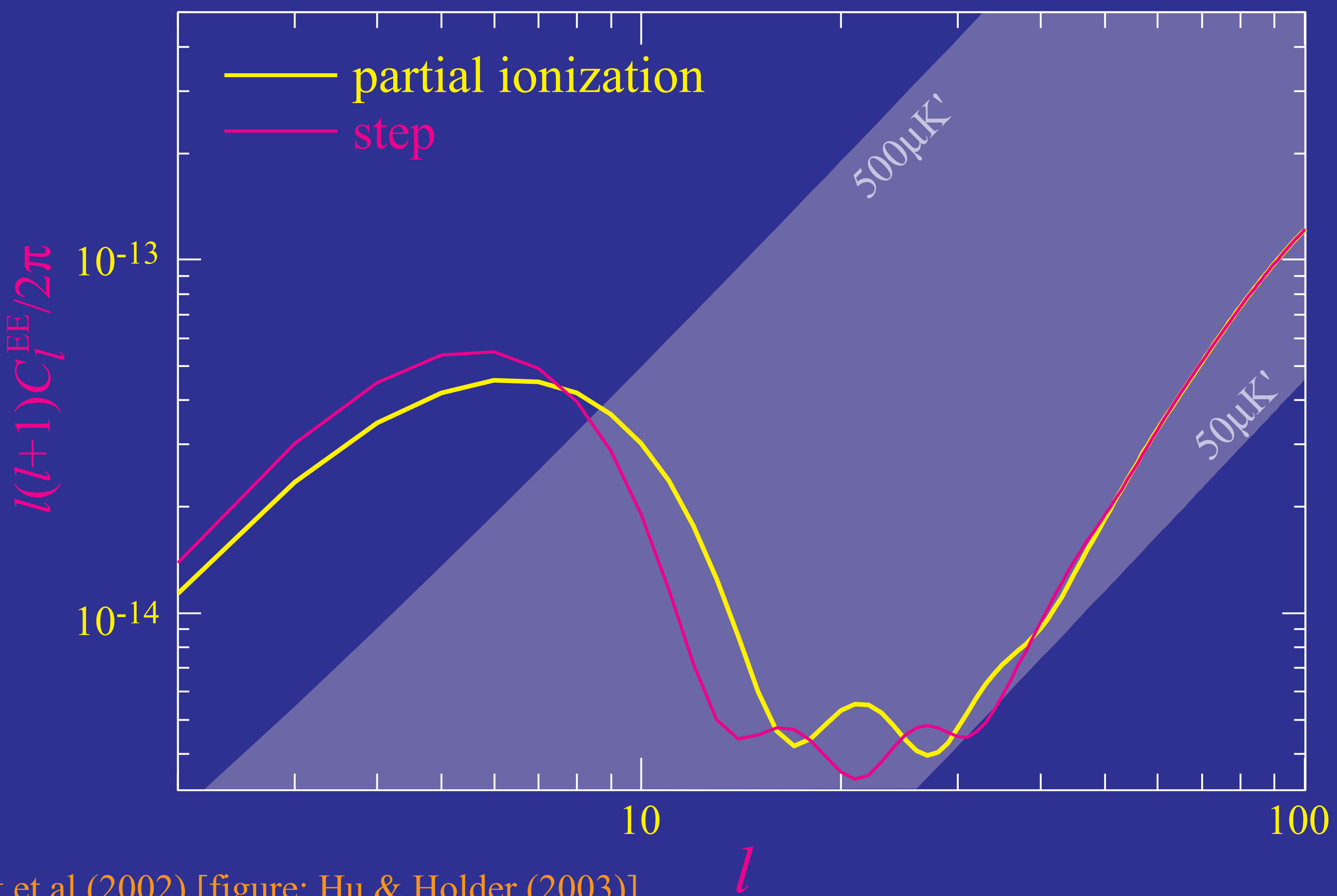
- By assuming the wrong ionization history can falsely rule out consistency relation
- Principal components eliminate possible biases

Mortonson & Hu (2007)



Polarization Power Spectrum

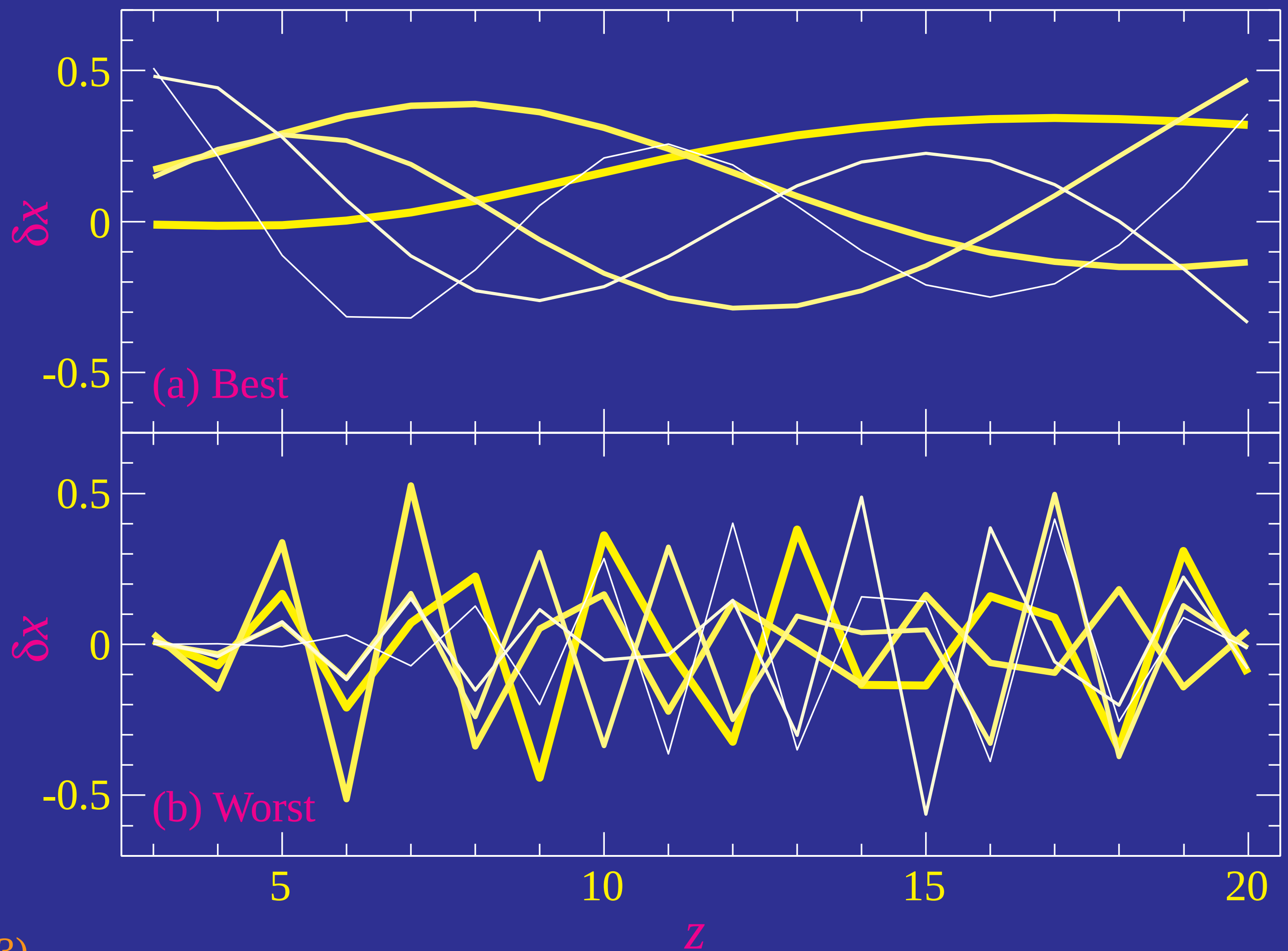
- Most of the information on ionization history is in the polarization (auto) power spectrum - two models with same optical depth but different ionization fraction



Kaplinghat et al (2002) [figure: Hu & Holder (2003)]

Principal Components

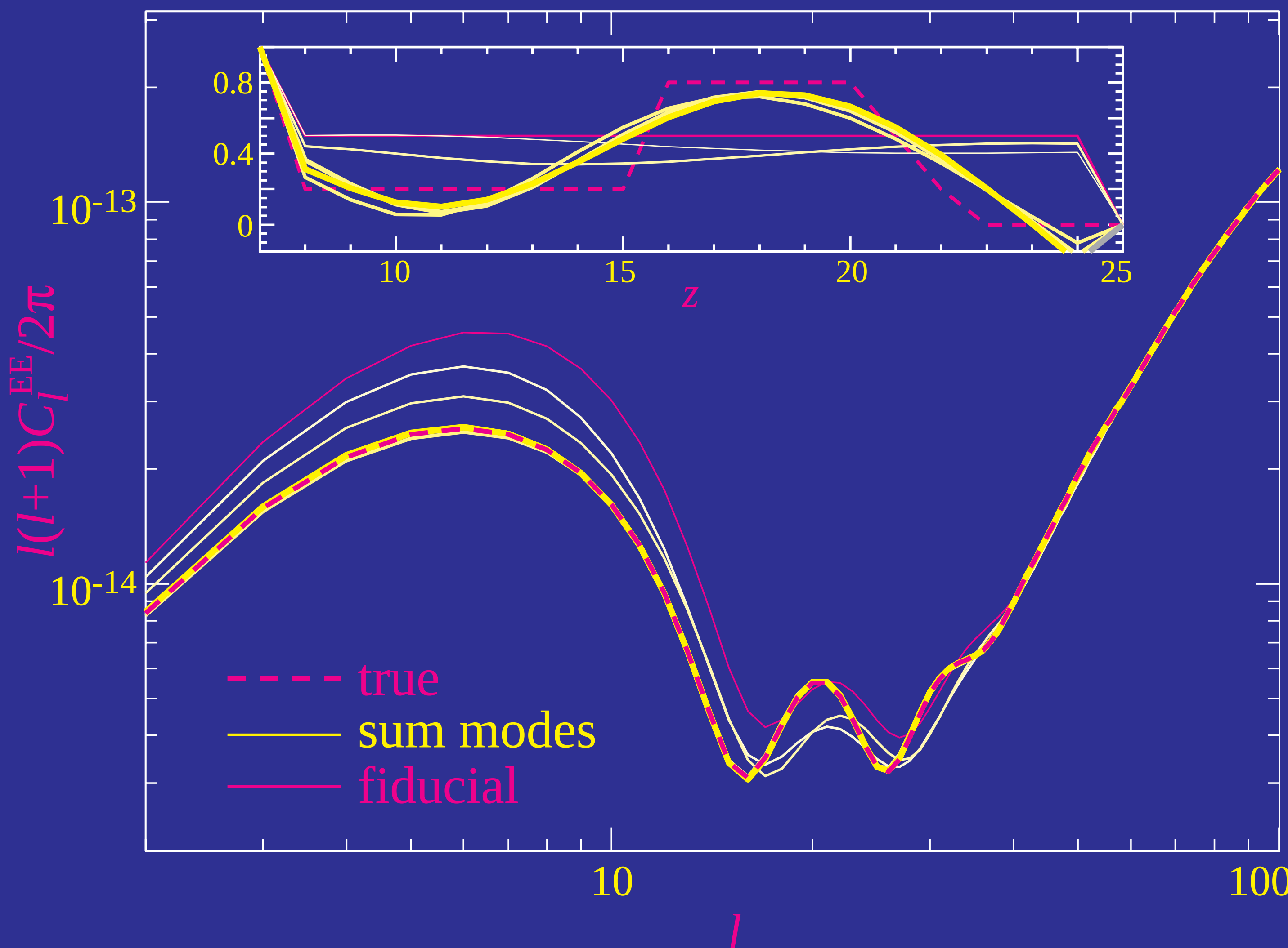
- Information on the ionization history is contained in ~5 numbers
  - essentially coefficients of first few Fourier modes



Hu & Holder (2003)

Representation in Modes

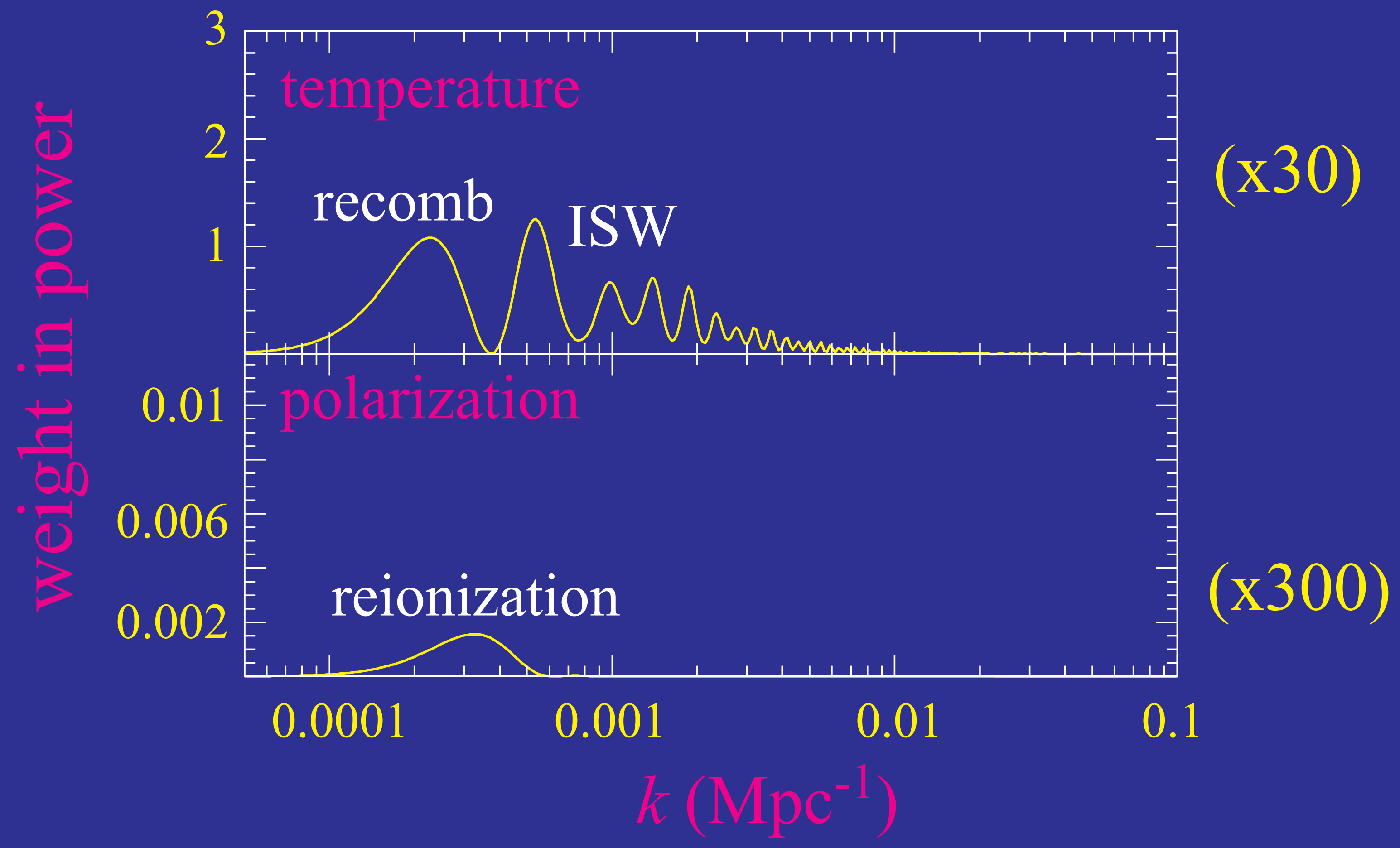
- Reproduces the power spectrum and net optical depth (actual τ=0.1375 vs 0.1377); indicates whether multiple physical mechanisms suggested



Hu & Holder (2003)

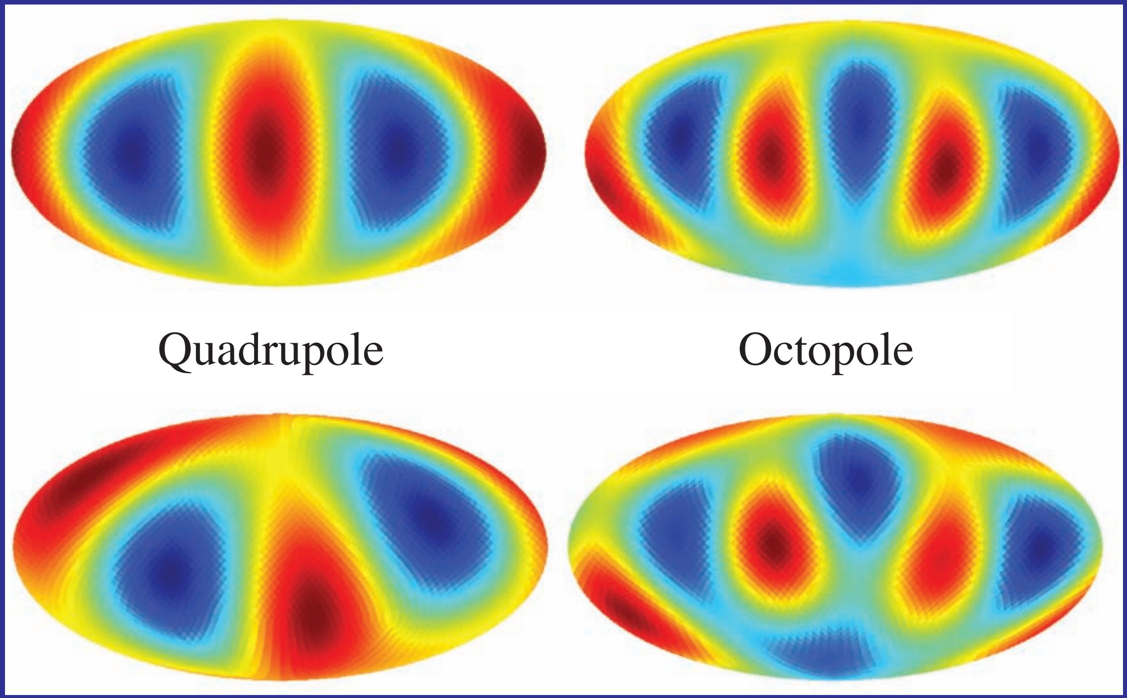
Temperature v. Polarization

- Quadrupole in polarization originates from a tight range of scales around the current horizon
- Quadrupole in temperature gets contributions from 2 decades in scale



Hu & Okamoto (2003)

Alignments



Dvorkin, Peiris, Hu (2007)

Gravitational Waves

Inflation Past

- Superhorizon correlations
  (acoustic coherence, polarization corr.)
- Spatially flat geometry
  (angular peak scale)
- Adiabatic fluctuations
  (peak morphology)
- Nearly scale invariant fluctuations
  (broadband power, small red tilt favored)
- Gaussian fluctuations
  (but f_{nl}>few would rule out single field slow roll)

Inflation Present

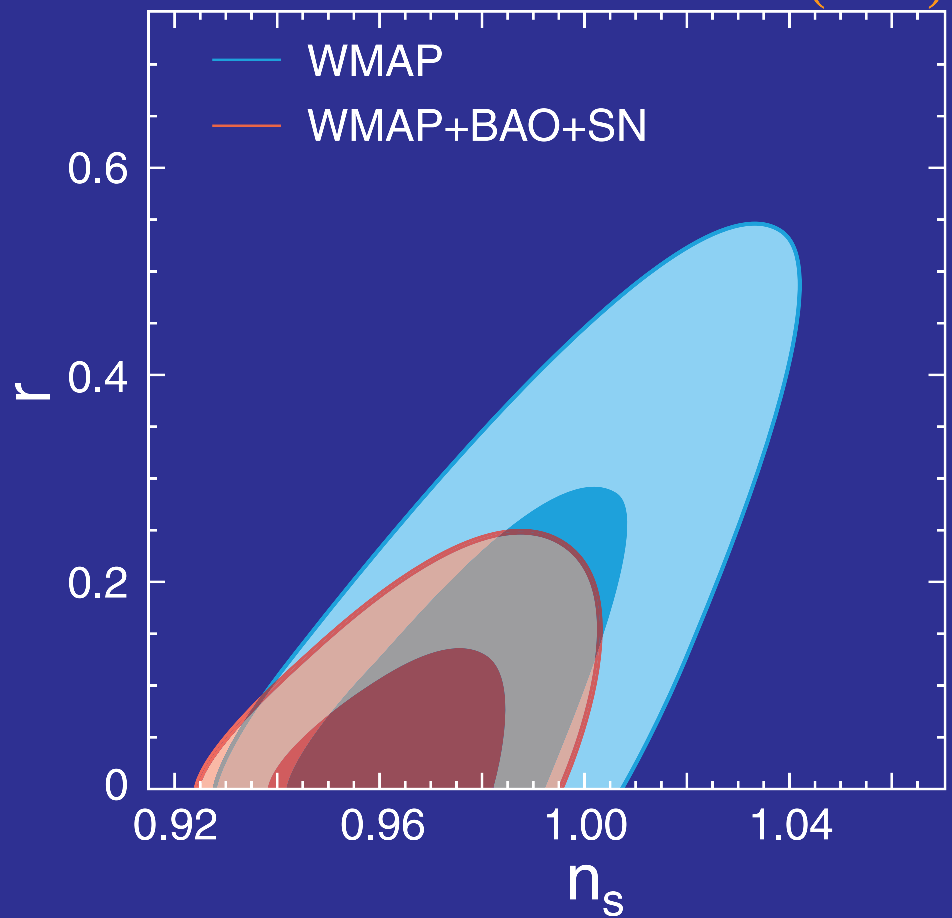
- Tilt (or gravitational waves) indicates that one of the slow roll parameters finite (ignoring exotic high-z reionization)
- Constraints in the r-n_s plane test classes of models
- Upper limit on gravity waves put an upper limit on V'/V and hence an upper limit on how far the inflaton rolls
- Given functional form of V, constraints on the flatness of potential when the horizon left the horizon predict too many (or few) efolds of further inflation

- Non-Gaussian fluctuations at f_{nl}~50-100?

Inflationary Constraints

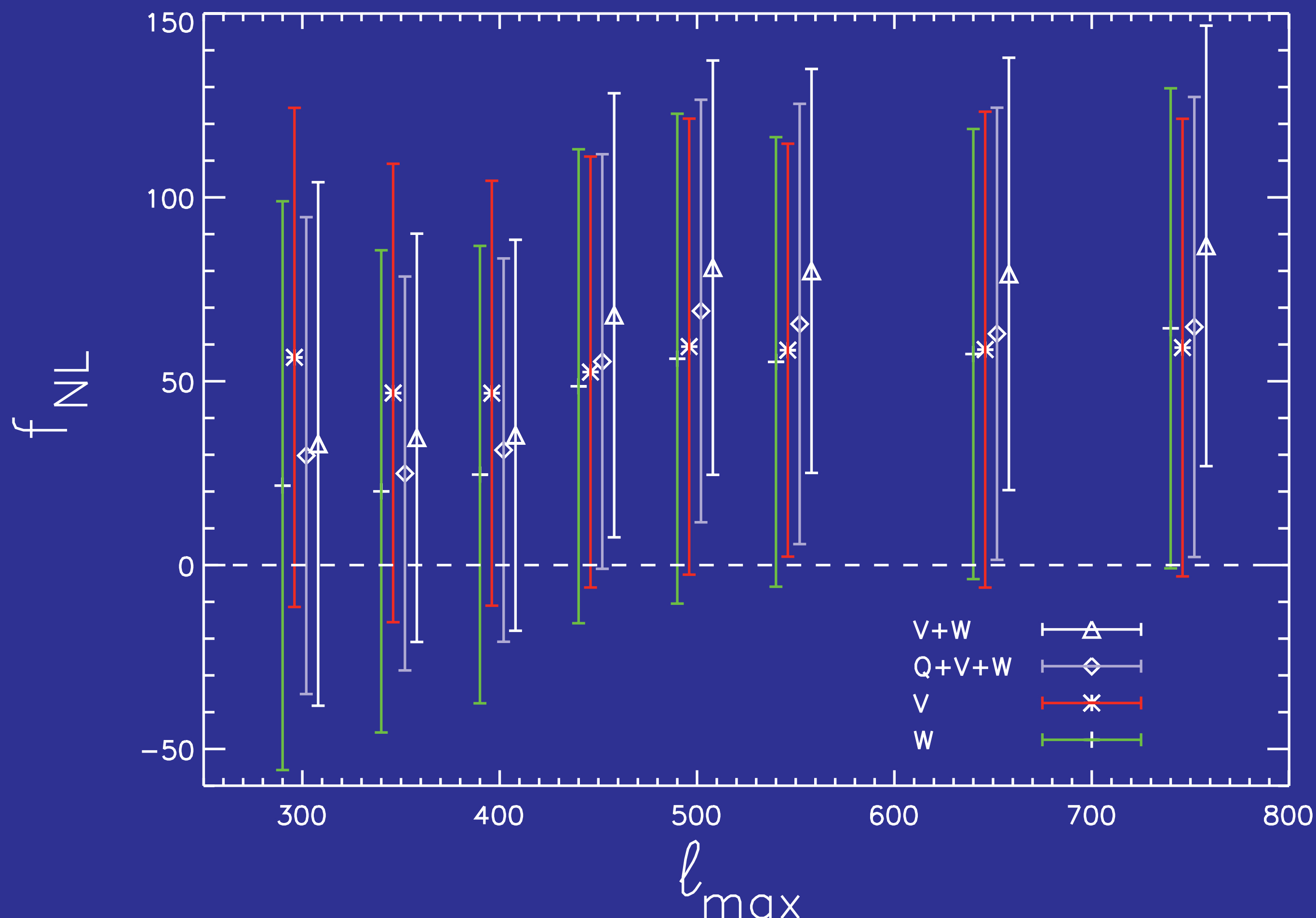
- Tilt mildly favored over tensors as explaining small scale suppression
- Specific models of inflation relate r-n_s through V', V''
- Small tensors and n_s~1 may make inflation continue for too many efolds

Komatsu et al (2008)



Primordial Non-Gaussianity f_{nl}

- Local second order non-Gaussianity: $\Phi_{nl}=\Phi+f_{nl}(\Phi^2-\langle\Phi^2\rangle)$
- WMAP3 Kp0+: $27<f_{nl}<147$ (95% CL) (Yadav & Wandelt 2007)
- WMAP5 KQ75: $-5<f_{nl}<111$ (95% CL) (Komatsu et al 2008)



Inflation Future

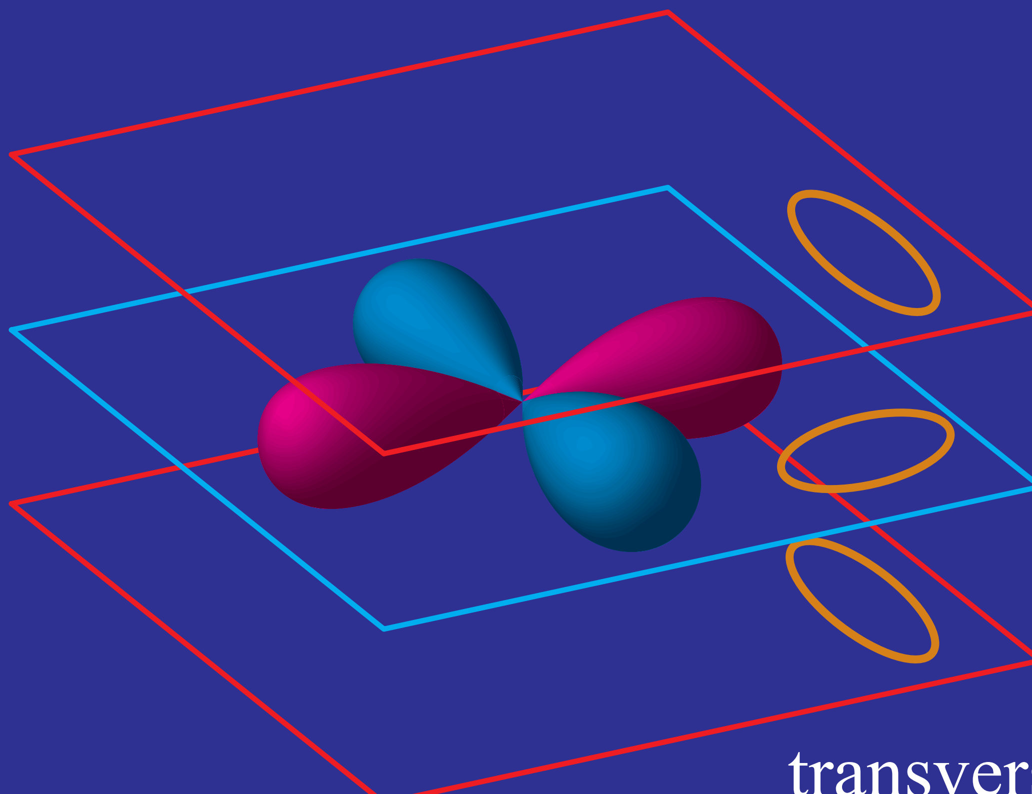
- Planck can test Gaussianity down to $f_{\rm nl}$~few

- Gravitational wave power proportional to energy scale to 4th power
- B-modes potentially observable for $V^{1/4}>3 \times 10^{15}$ GeV with removal of lensing B-modes and foregrounds

- Measuring both the reionization bump and recombination peak tests slow roll consistency relation by constraining tensor tilt
- Requires measurement and model-independent interpretation of reionization E-modes

Gravitational Waves

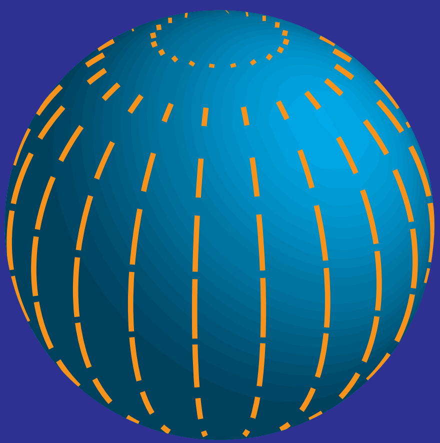
- Inflation predicts near scale invariant spectrum of gravitational waves
- Amplitude proportional to the square of the $E_\mathrm{i}=V^{1/4}$ energy scale
- If inflation is associated with the grand unification $E_\mathrm{i}\sim10^{16}$ GeV and potentially observable

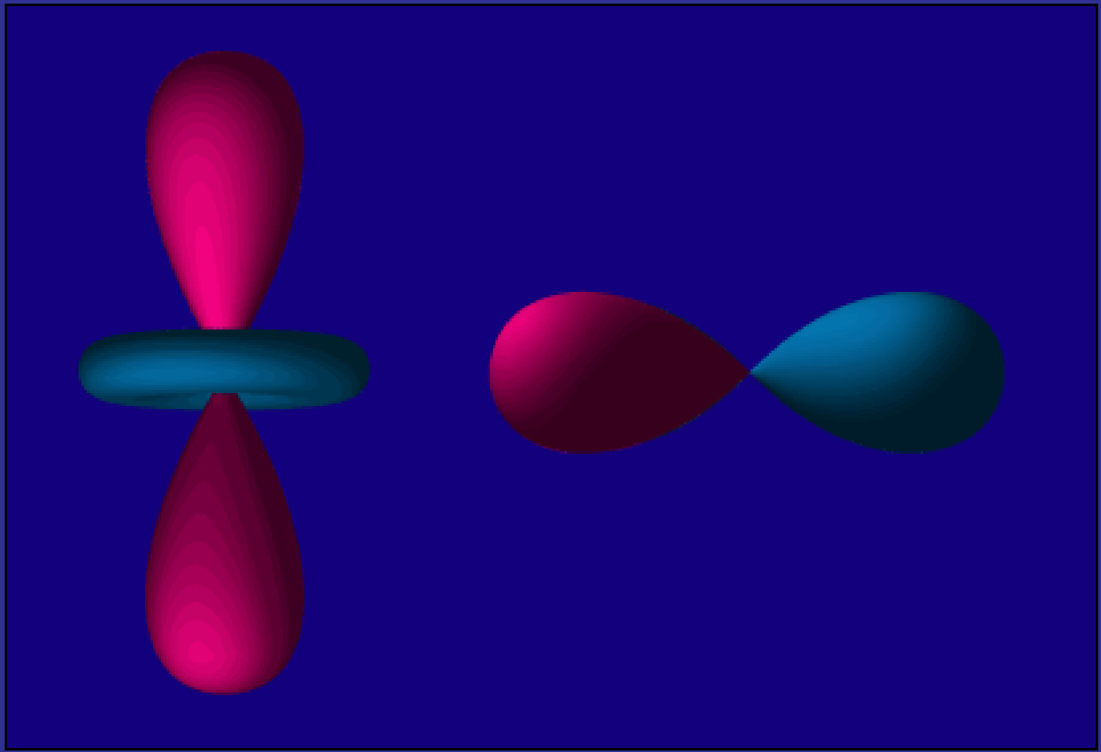


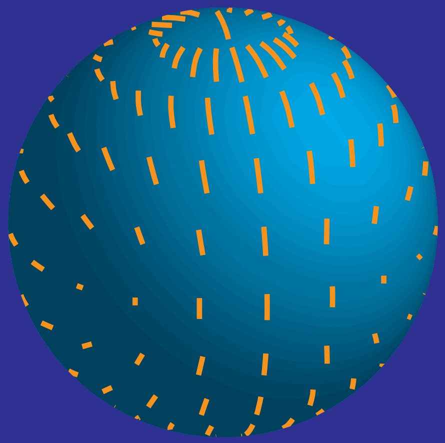
transverse-traceless distortion

Gravitational Wave Pattern

- Projection of the quadrupole anisotropy gives polarization pattern
- Transverse polarization of gravitational waves breaks azimuthal symmetry







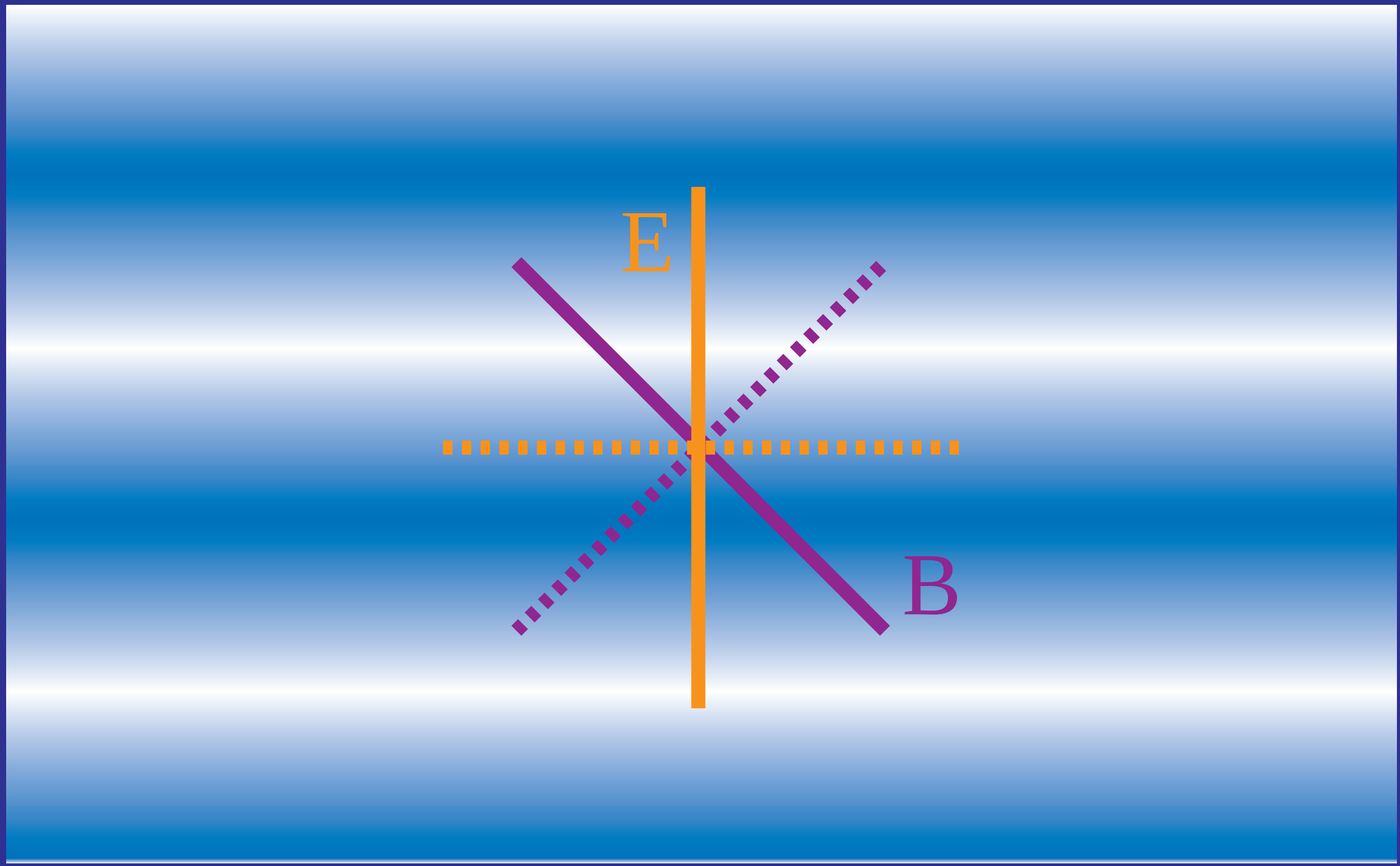
density perturbation

gravitational wave

Electric & Magnetic Polarization

(a.k.a. gradient & curl)

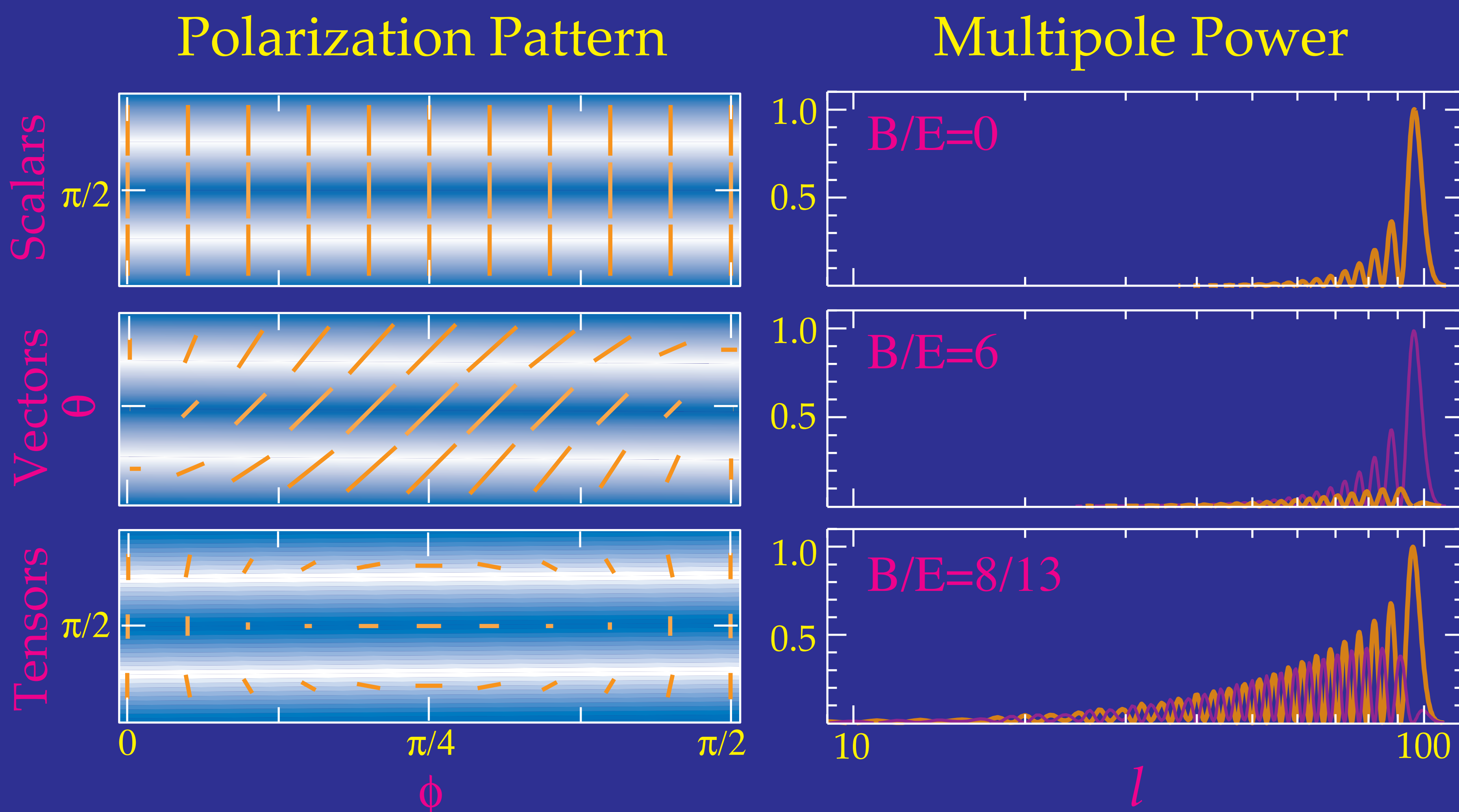
- Alignment of principal vs polarization axes
  (curvature matrix vs polarization direction)



Kamionkowski, Kosowsky, Stebbins (1997)
Zaldarriaga & Seljak (1997)

Patterns and Perturbation Types

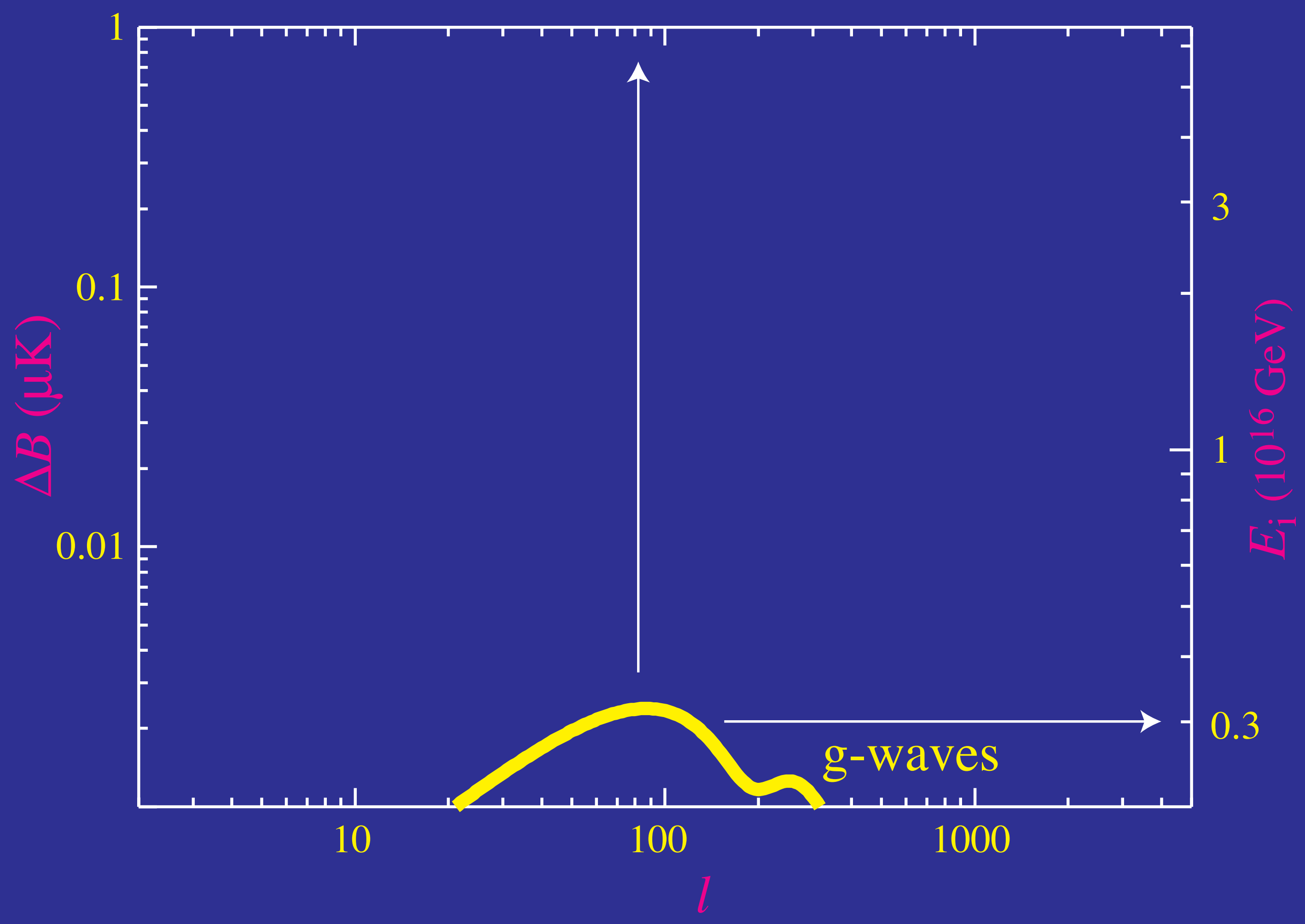
- Amplitude modulated by plane wave → Principal axis
- Direction detemined by perturbation type → Polarization axis



Kamionkowski, Kosowski, Stebbins (1997); Zaldarriaga & Seljak (1997); Hu & White (1997)

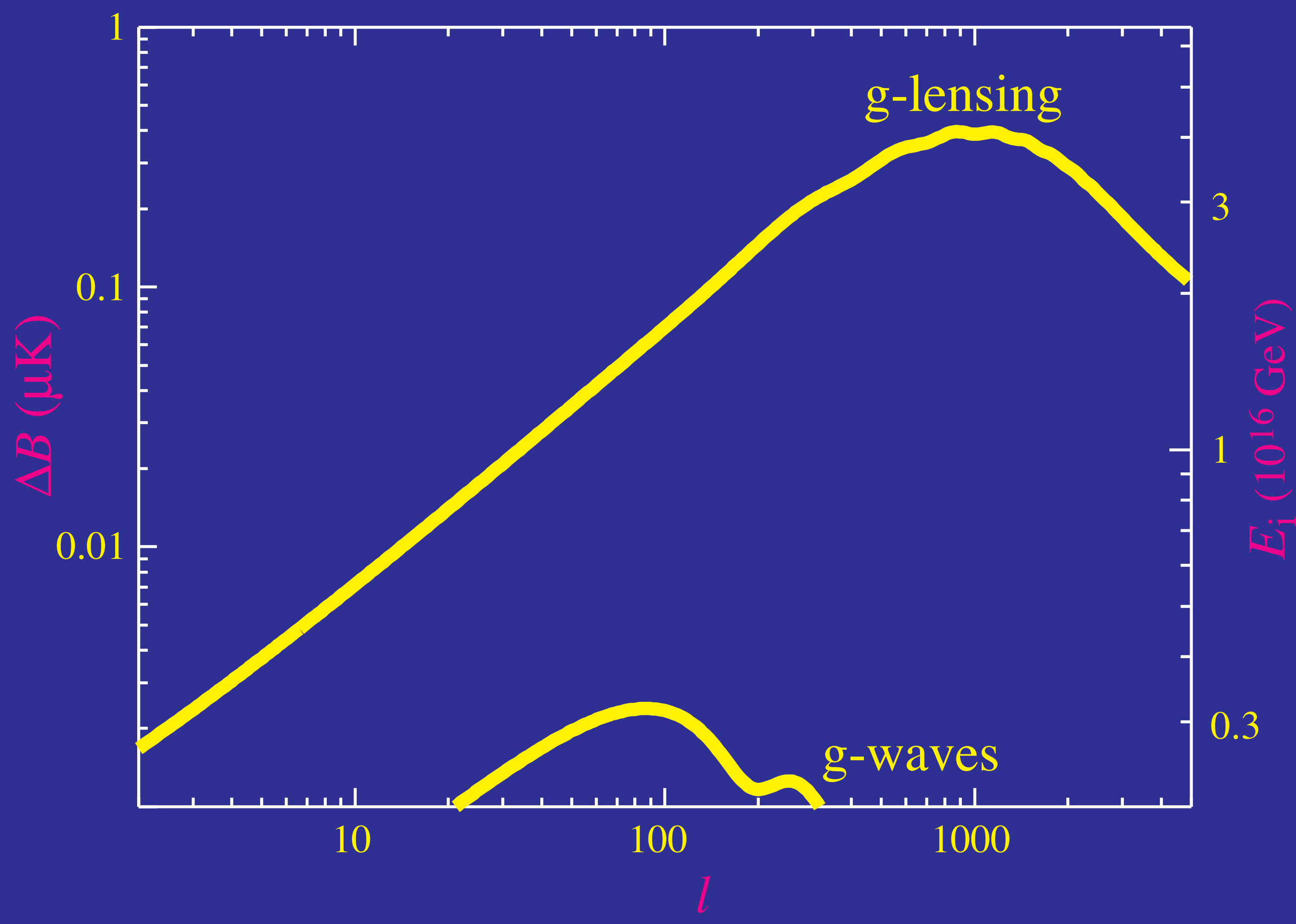
Scaling with Inflationary Energy Scale

- RMS B-mode signal scales with inflationary energy scale squared E_i^2



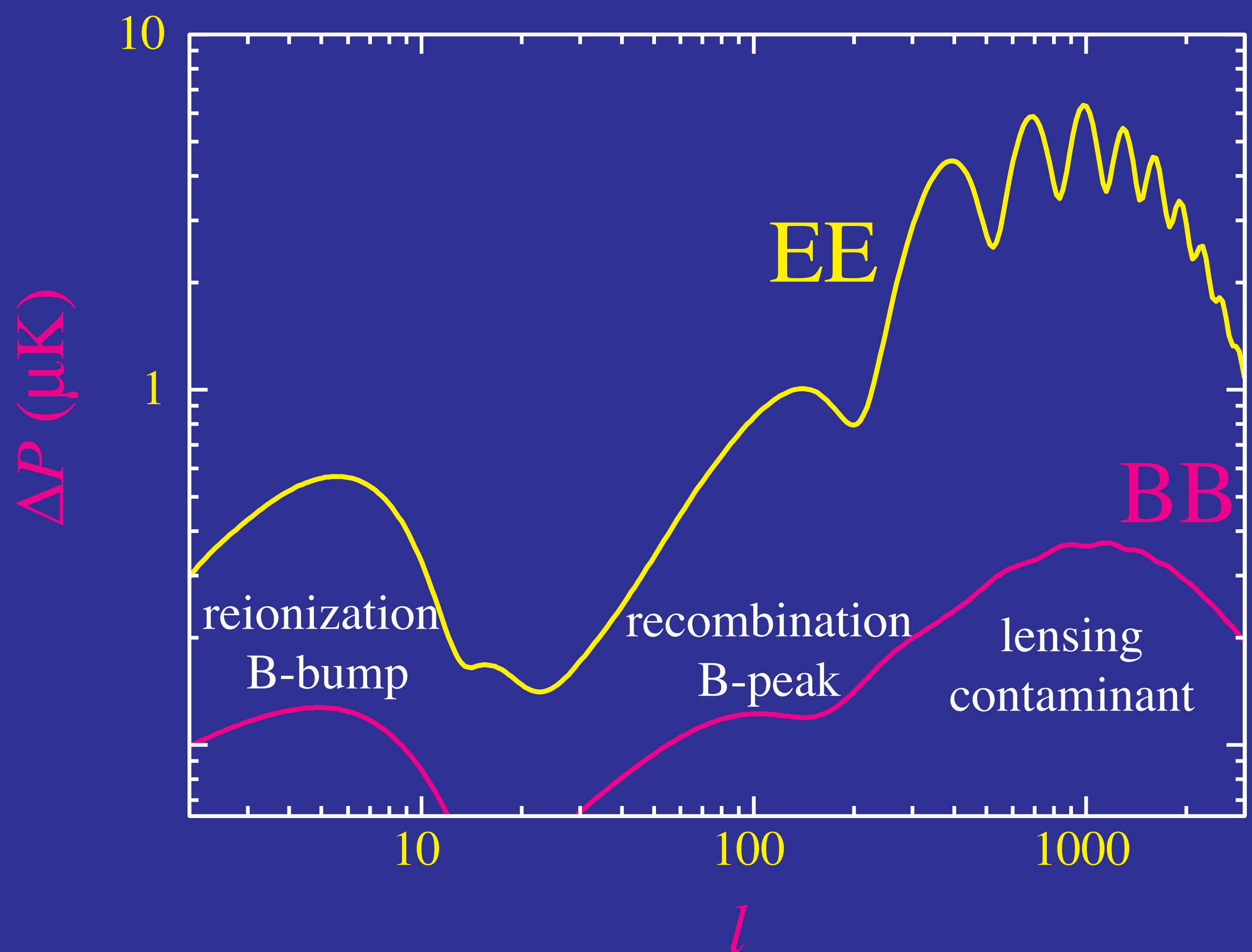
Contamination for Gravitational Waves

- Gravitational lensing contamination of B-modes from gravitational waves cleaned to E_i~0.3 x 10^{16} GeV
  Hu & Okamoto (2002) limits by Knox & Song (2002); Cooray, Kedsen, Kamionkowski (2002)



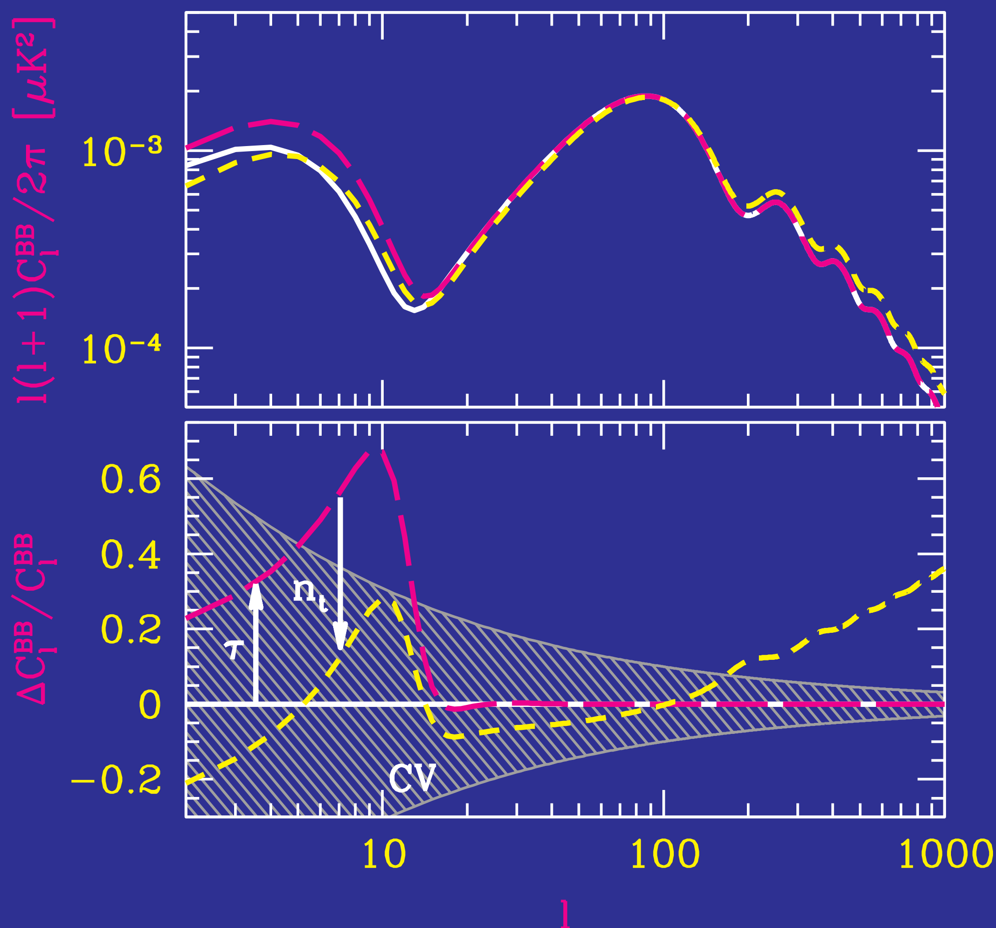
The B-Bump

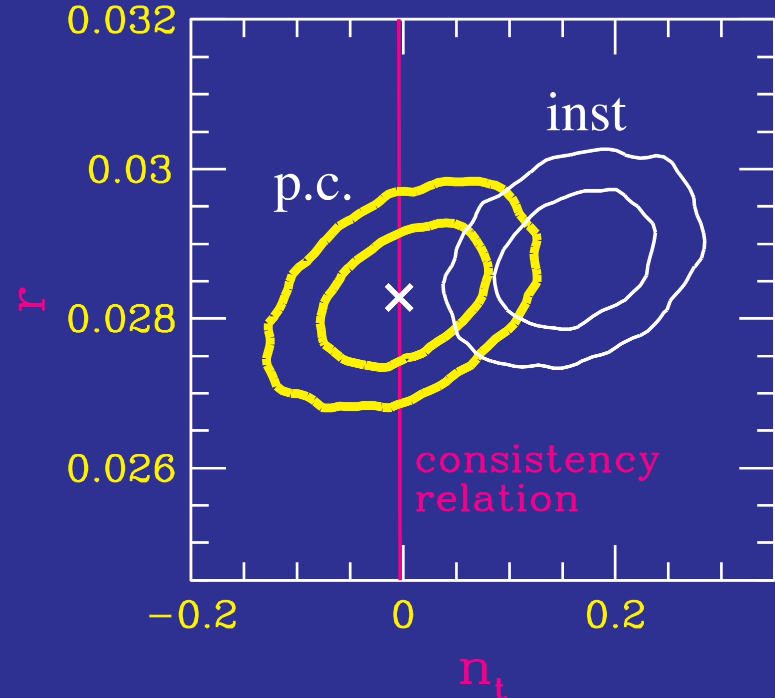
- Rescattering of gravitational wave anisotropy generates the B-bump
- Potentially the most sensitive probe of inflationary energy scale
- Potentially enables test of consistency relation (slow roll)



Slow Roll Consistency Relation

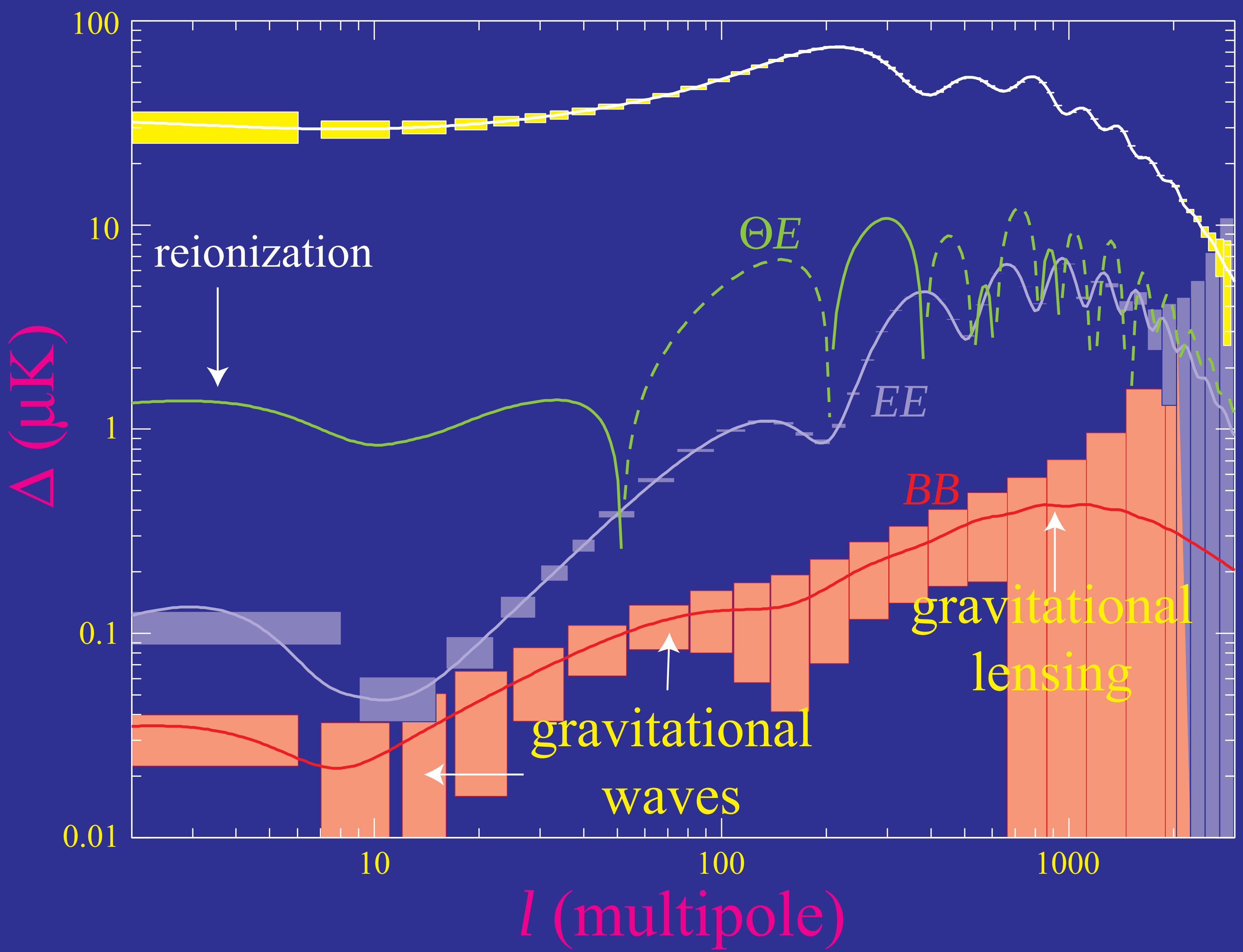
- Consistency relation between tensor-scalar ratio and tensor tilt $r = -8n_t$ tested by reionization
- Reionization uncertainties controlled by a complete p.c. analysis





Mortonson & Hu (2007)

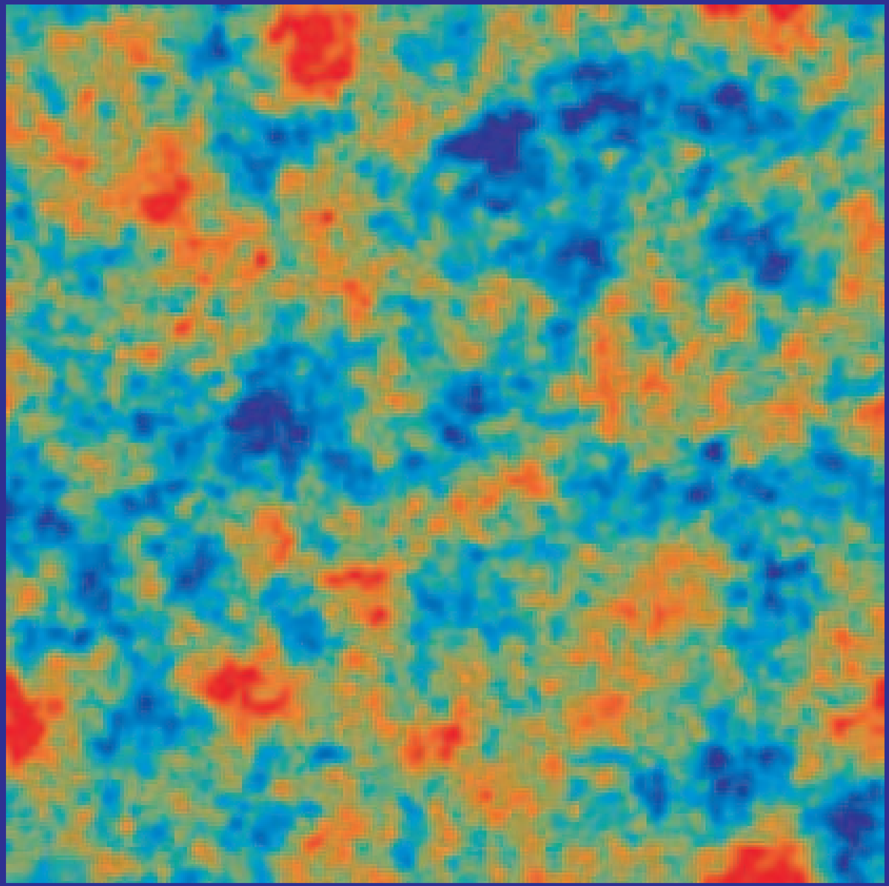
Temperature and Polarization Spectra



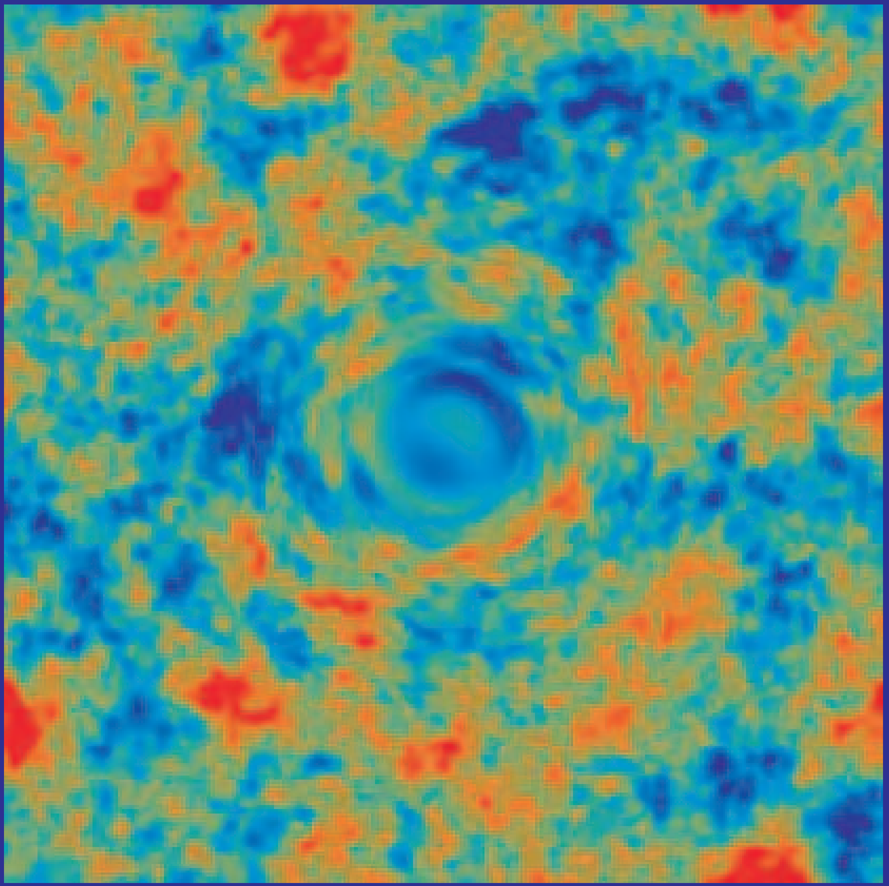
Gravitational Lensing

Gravitational Lensing

- Gravitational lensing by large scale structure distorts the observed temperature and polarization fields
- Exaggerated example for the temperature

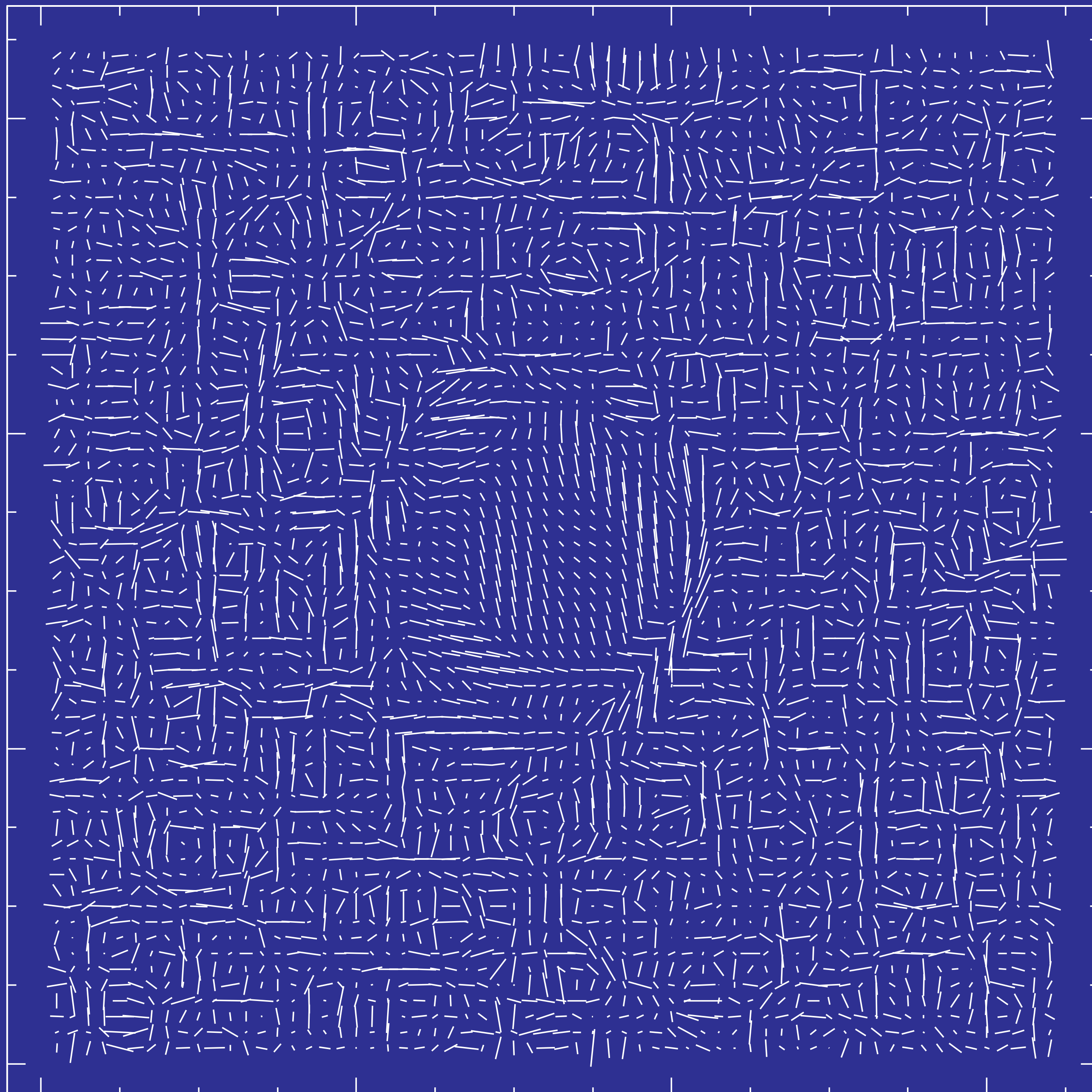


Original



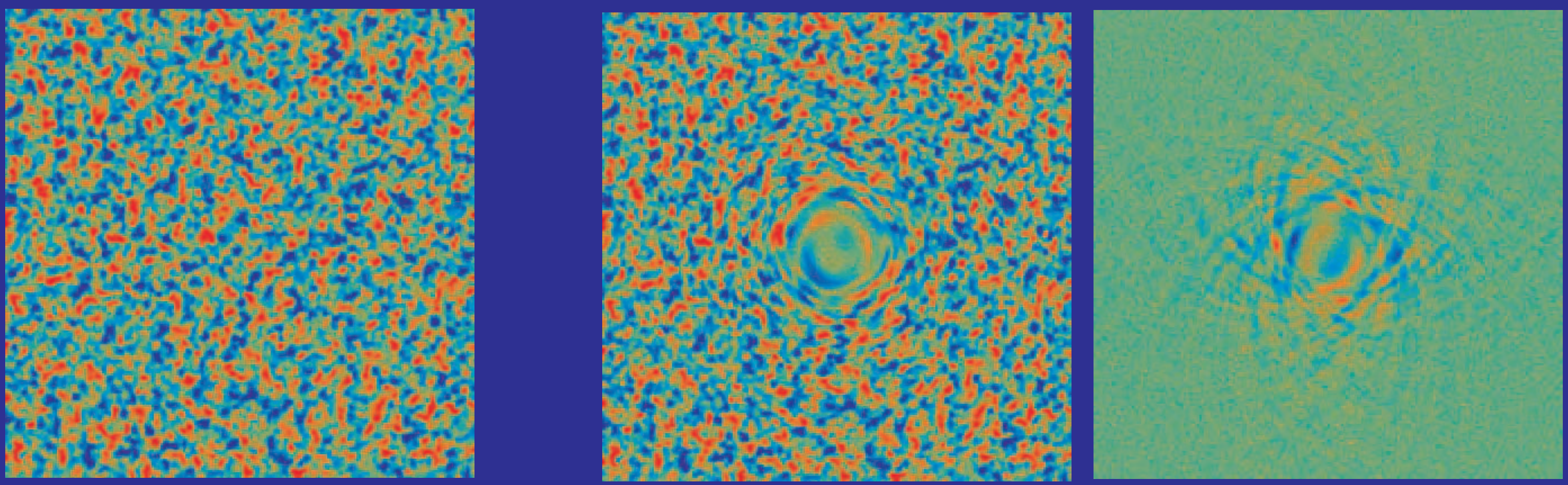
Lensed

Polarization Lensing



Polarization Lensing

- Since E and B denote the relationship between the polarization amplitude and direction, warping due to lensing creates B-modes

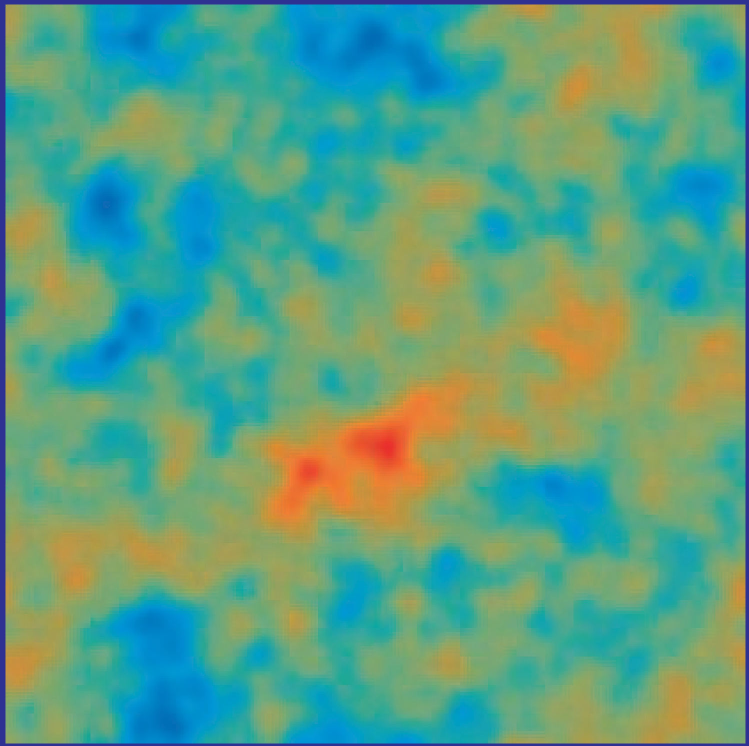


Original Lensed E Lensed B

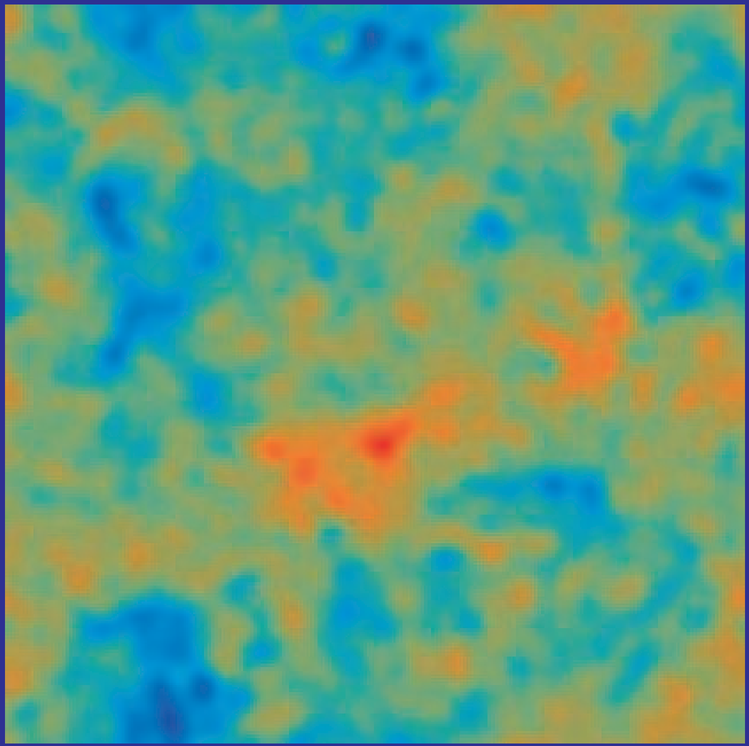
Zaldarriaga & Seljak (1998) [figure: Hu & Okamoto (2001)]

Reconstruction from Polarization

- Lensing B-modes correlated to the orignal E-modes in a specific way
- Correlation of E and B allows for a reconstruction of the lens
- Reference experiment of 4' beam, 1μK' noise and 100 deg^2



Original Mass Map



Reconstructed Mass Map

Hu & Okamoto (2001) [iterative improvement Hirata & Seljak (2003)]

Why Care

- Gravitational lensing sensitive to amount and hence growth of structure
- Examples: massive neutrinos - $d \ln C_\ell^{BB} / dm_\nu \approx -1/3$eV, dark energy - $d \ln C_\ell^{BB} / dw \approx -1/8$
- Mass reconstruction measures the large scale structure on large scales and the mass profile of objects on small scales
- Examples: large scale decontamination of the gravitational wave B modes; lensing by SZ clusters combined with optical weak lensing can make a distance ratio test of the acceleration

Lecture II: Summary

- Polarization by Thomson scattering of quadrupole anisotropy
- Quadrupole anisotropy only sustained in optically thin conditions of reionization and the end of recombination
- Reionization generates E-modes at low multipoles from and correlated to the Sachs-Wolfe anisotropy
- Reionization polarization enables study of ionization history, low multipole anomalies, gravitational waves
- Dissipation of acoustic waves during recombination generates quadrupoles and correlated polarization peaks
- Recombination polarization provides consistency checks, features in power spectrum, source of graviational lensing B modes
- Gravitational waves B-mode polarization sensitive to inflation energy scale and tests slow roll consistency relation