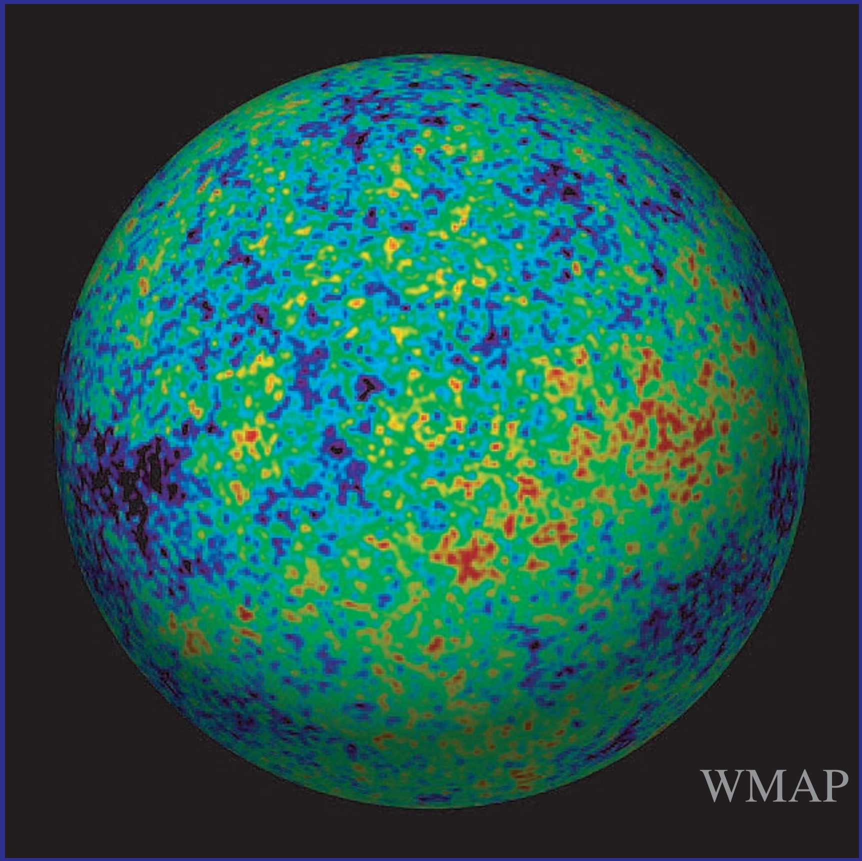
Lecture I

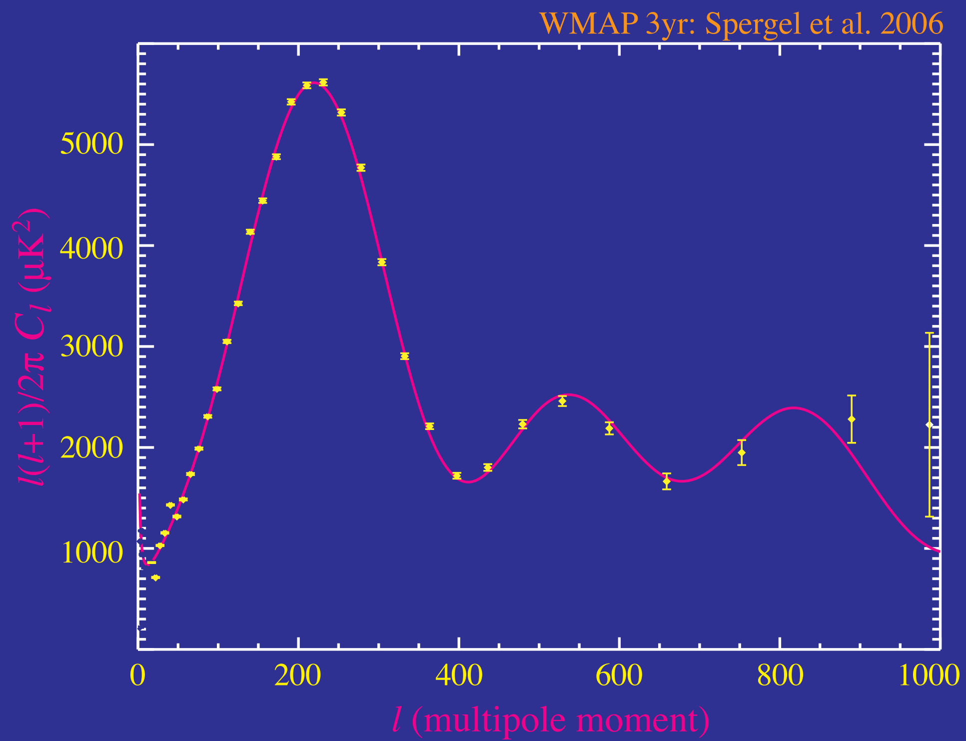


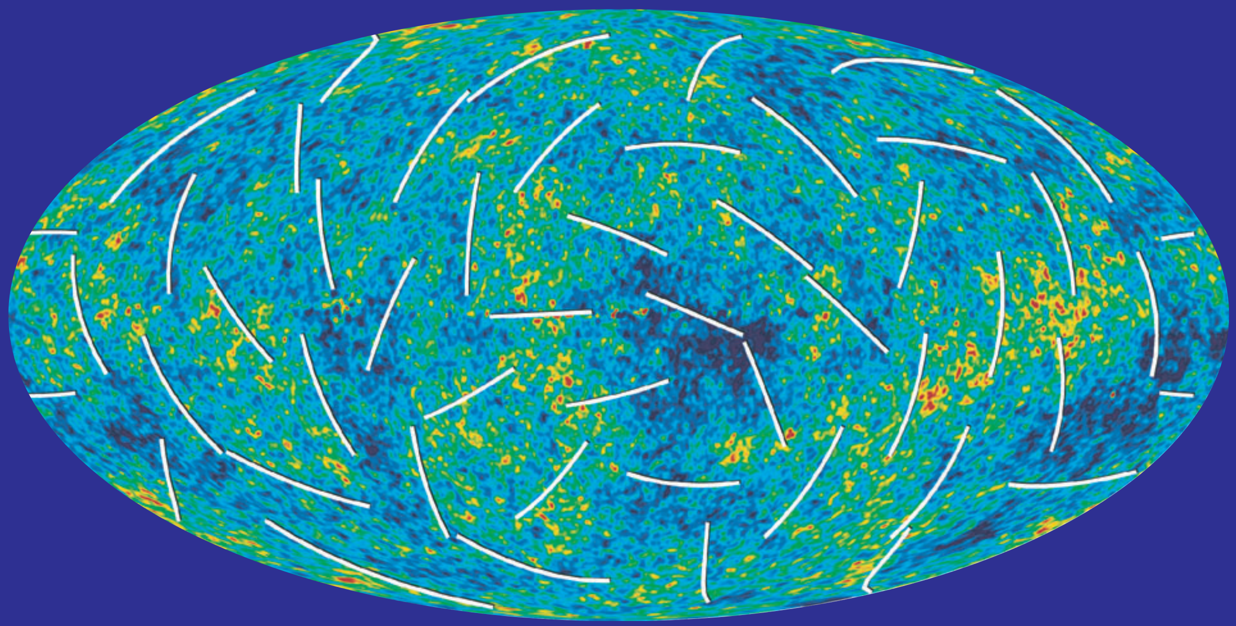
Temperature Anisotropy Spectrum

Wayne Hu

Crete, July 2008

WMAP 3yr Data





Standard Model: Vanilla ΛCDM

- 6 parameter ΛCDM model
- Fits WMAP and most other cosmological data

Parameter	3 Year Mean	5 Year Mean	5 Year Max Like
$100\Omega_b h^2$	2.229 ± 0.073	2.273 ± 0.062	2.27
$\Omega_c h^2$	0.1054 ± 0.0078	0.1099 ± 0.0062	0.108
Ω_Λ	0.759 ± 0.034	0.742 ± 0.030	0.751
n_s	0.958 ± 0.016	$0.963^{+0.014}_{-0.015}$	0.961
τ	0.089 ± 0.030	0.087 ± 0.017	0.089
$\Delta^2_{\mathcal{R}}$	$(2.35 \pm 0.13) \times 10^{-9}$	$(2.41 \pm 0.11) \times 10^{-9}$	2.41×10^{-9}
σ_8	0.761 ± 0.049	0.796 ± 0.036	0.787
Ω_m	0.241 ± 0.034	0.258 ± 0.030	0.249
$\Omega_m h^2$	0.128 ± 0.008	0.1326 ± 0.0063	0.131
H_0	$73.2^{+3.1}_{-3.2}$	$71.9^{+2.6}_{-2.7}$	72.4
z_{reion}	11.0 ± 2.6	11.0 ± 1.4	11.2
t_0	13.73 ± 0.16	13.69 ± 0.13	13.7

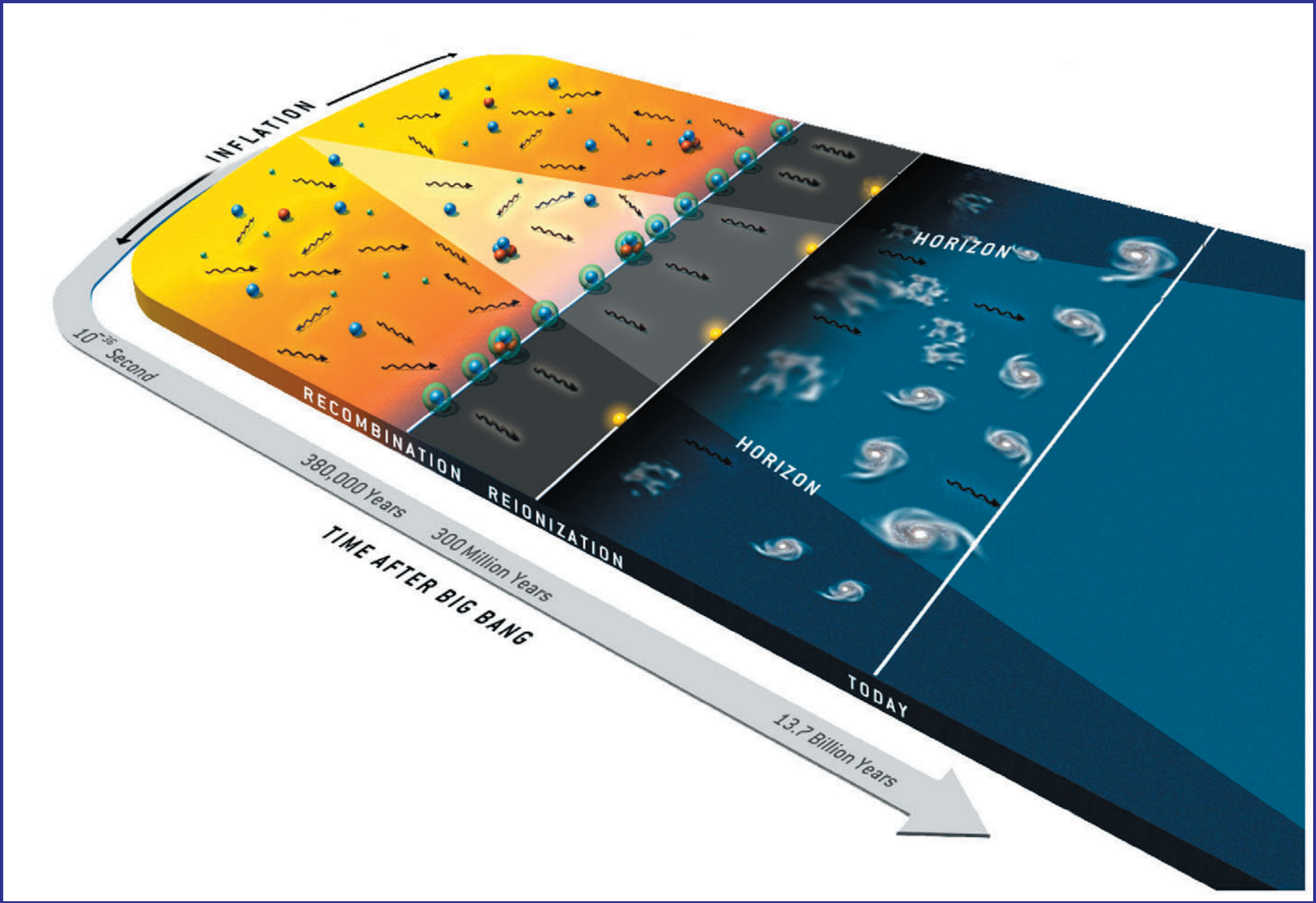
Milestones: Past & Present

• Large-scale anisotropy	COBE DMR	'92
• Degree-scale anisotropy	many	'93-'99
• First acoustic peak	Toco, Boom, Maxima	'99-'00
• Secondary acoustic peak(s)	DASI, Boom	'01
• Damping tail	CBI	'02
• Acoustic polarization	DASI	'02
• Secondary anisotropy?	CBI	'02
• Reionization	WMAP	'03
• ISW correlation	WMAP+LSS	'03
• Large scale anomalies?	WMAP (COBE)	'03
• Tilt (or finite slow roll param)	WMAP(+ext, LSS)	'06
• Lensing correlation	WMAP+LSS	'07
• Primordial non-Gaussianity?	WMAP	'07
• Lensing smoothing	ACBAR	'08

Milestones: Future

- Sunyaev-Zel’dovich cluster & secondaries surveys
- Polarization tests of large-scale temperature anomalies
- Lensing B-modes
- Lensing mass reconstruction
- Reionization history & inhomogeneity
- Gravitational wave B-modes

In the Beginning...



Hu & White (2004); artist:B. Christie/SciAm; available at http://background.uchicago.edu

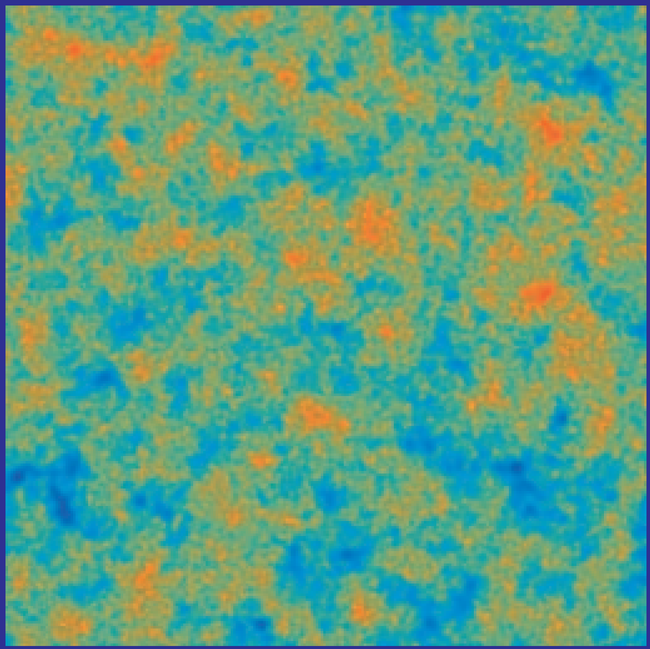
Anisotropy Formation

- Temperature inhomogeneities at recombination become anisotropy

Seeing Spots

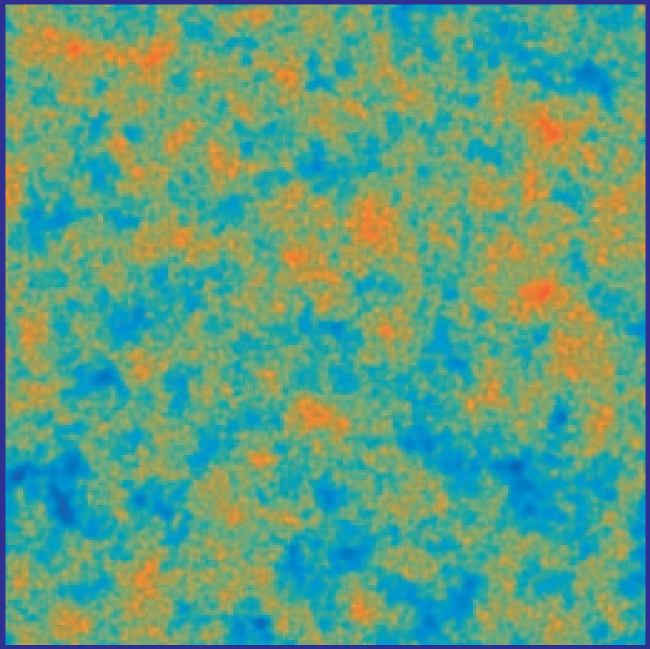
- 1 part in 100000 variations in temperature
- Spot sizes ranging from a fraction of a degree to 180 degrees

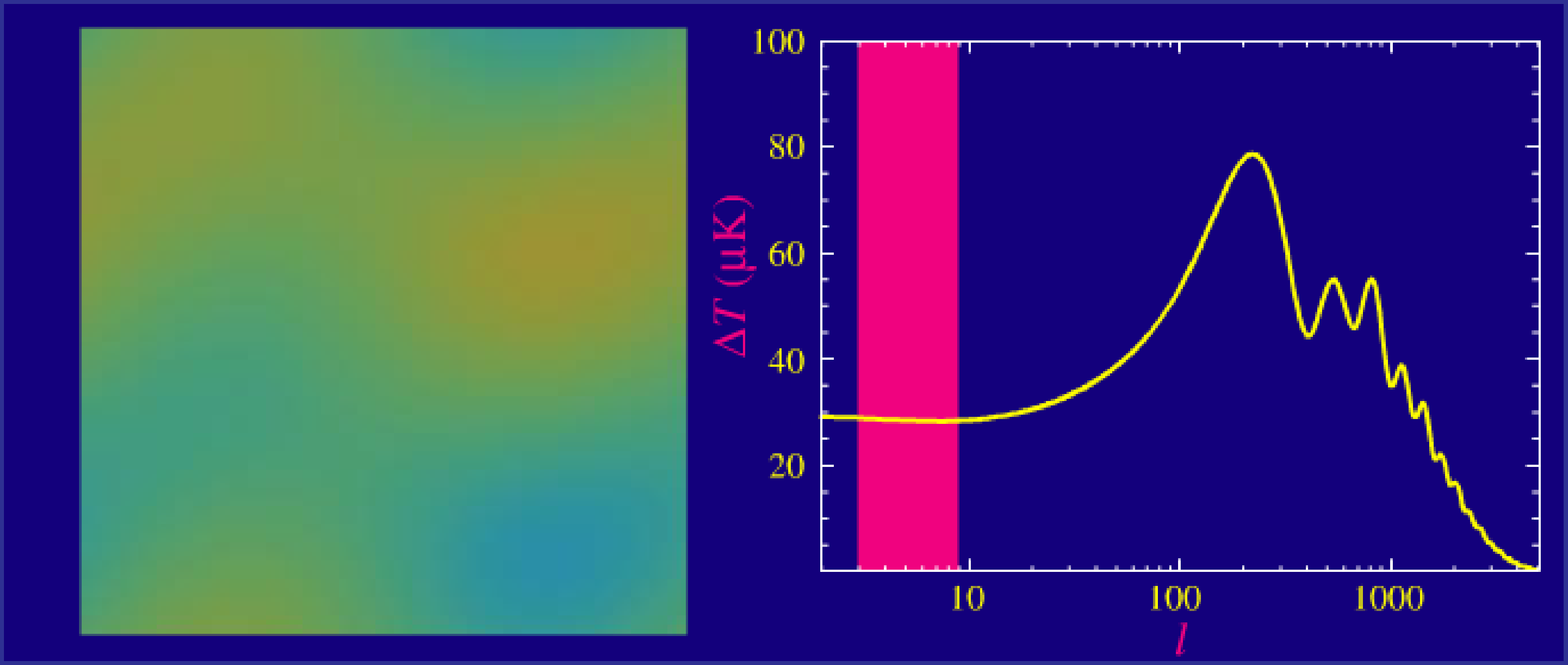
64°



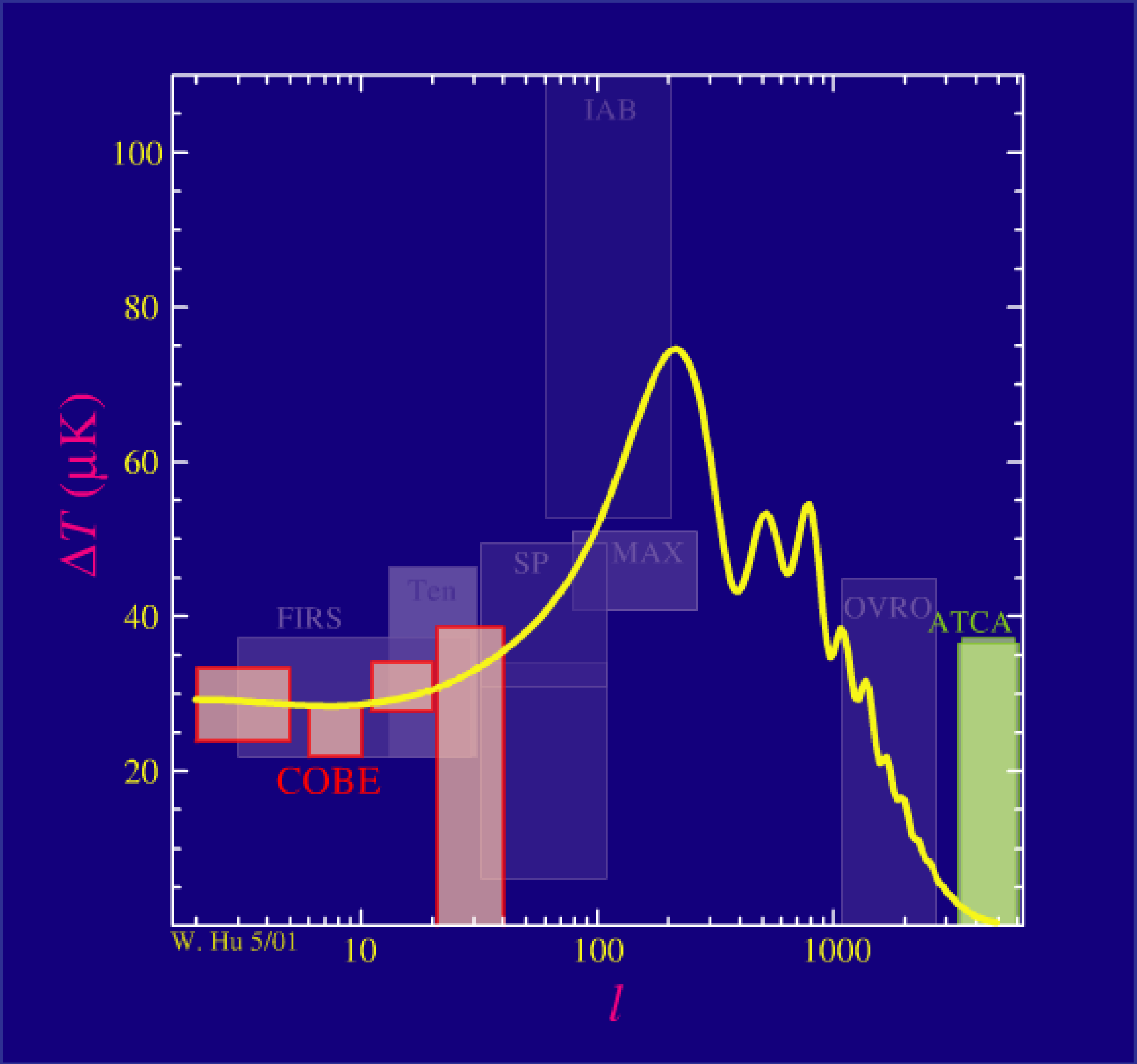
- Selecting only spots of a given range of sizes gives a power spectrum or frequency spectrum of the variations much like a graphic equalizer for sound.

Seeing Spots

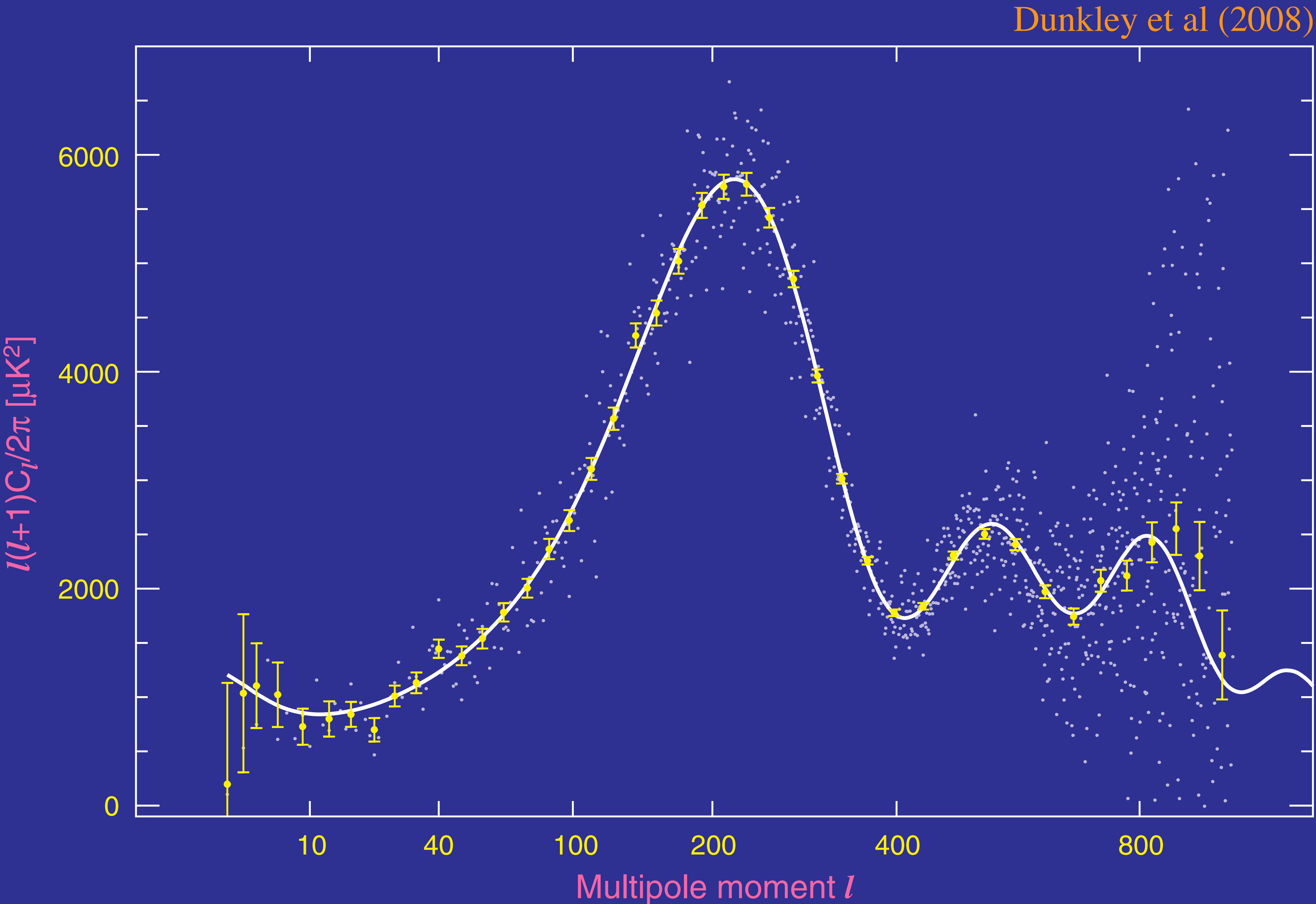




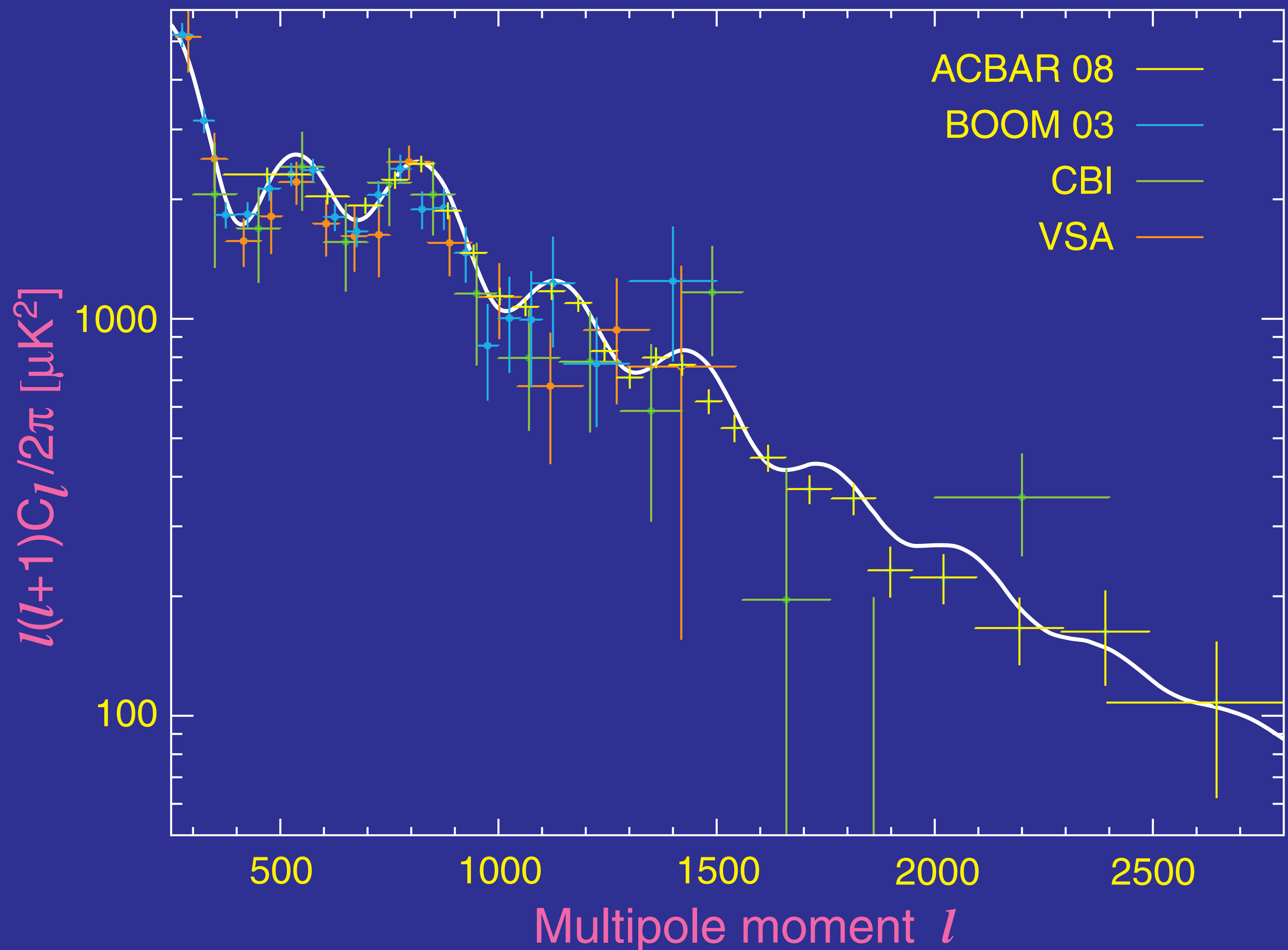
Theorist's Time-Ordered Data



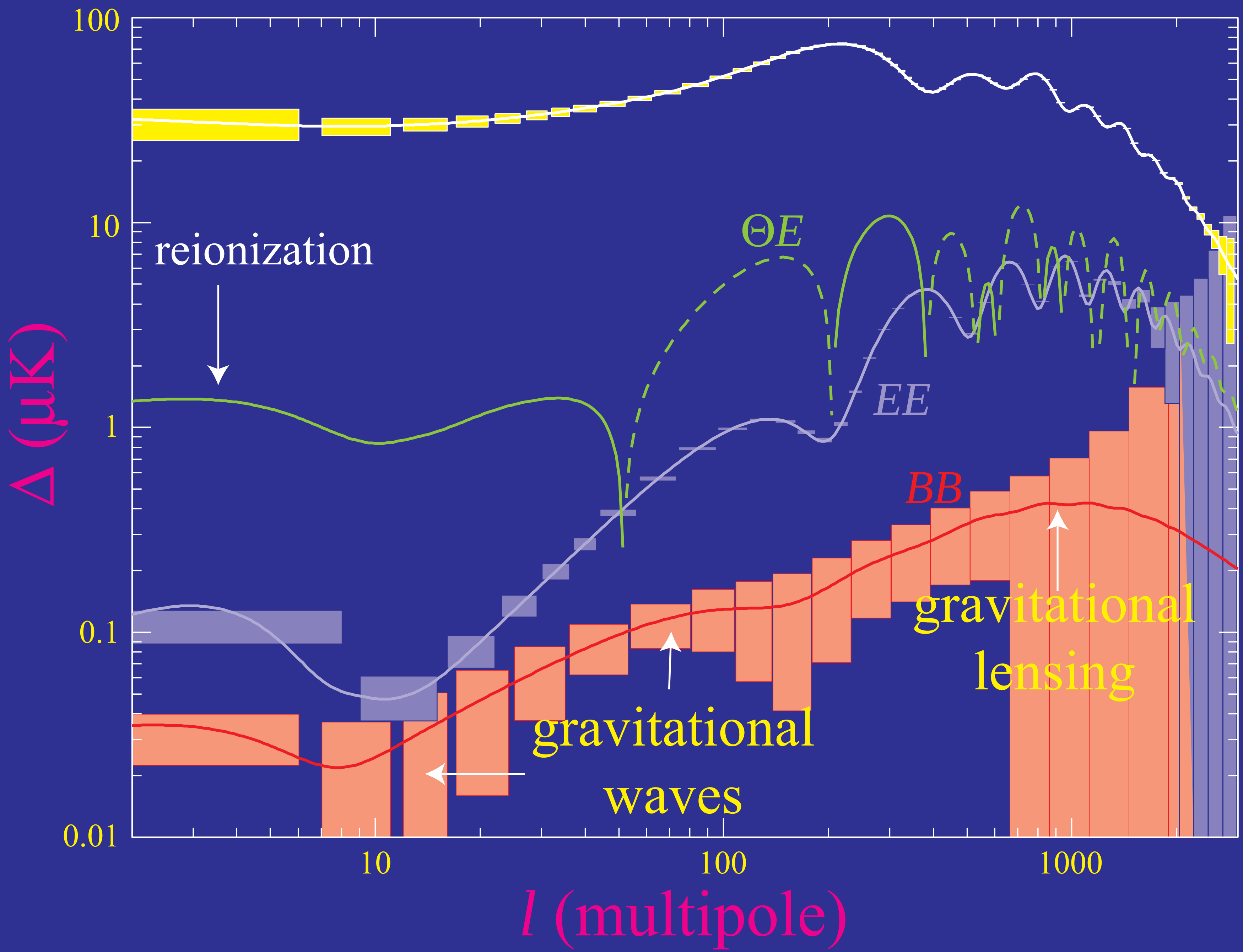
Power Spectrum Present



Power Spectrum Present

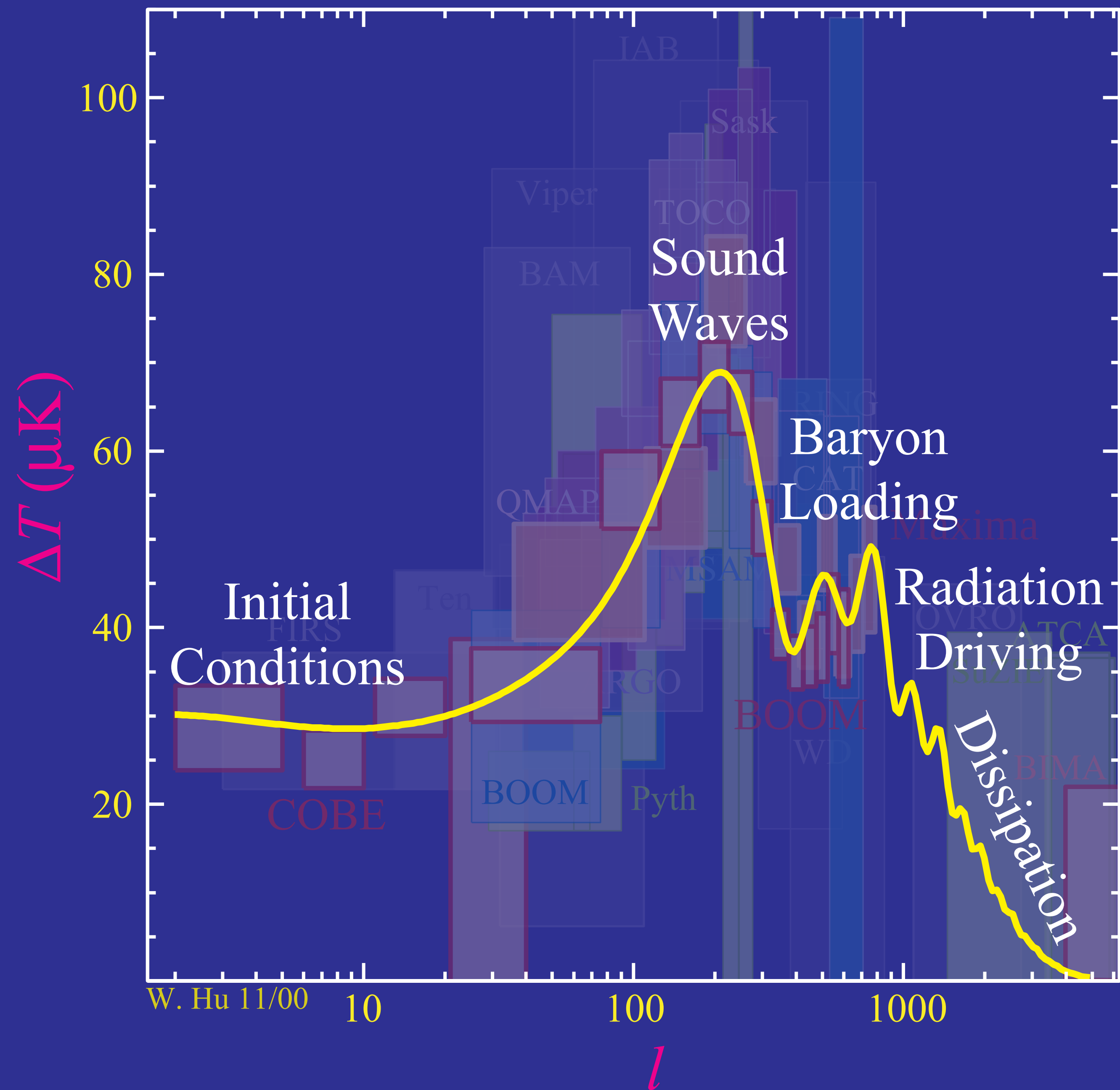


Power Spectrum Future



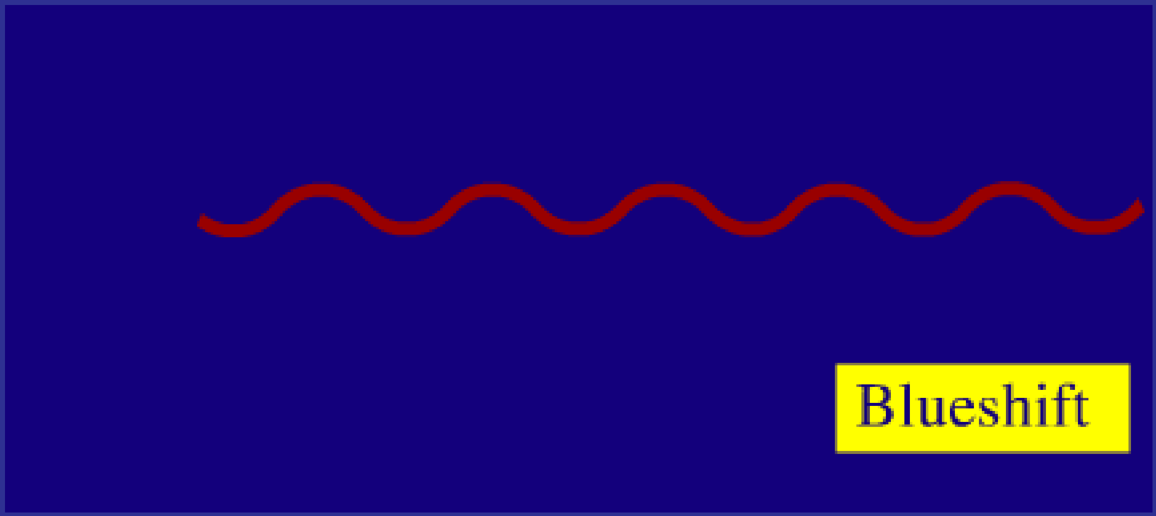
Angular Peaks

Physical Landscape



Seeing Sound

- Colliding electrons, protons and photons forms a plasma
- Acts as gas just like molecules in the air
- Compressional disturbance propagates in the gas through particle collisions



- Unlike sound in the air, we see the sound in the CMB
- Compression heats the gas resulting in a hot spot in the CMB

Thomson Scattering

- Thomson scattering of photons off of free electrons is the most important CMB process with a cross section (averaged over polarization states) of

$$\sigma_T = \frac{8\pi\alpha^2}{3m_e^2} = 6.65 \times 10^{-25}\text{cm}^2$$

- Density of free electrons in a fully ionized $x_e = 1$ universe

$$n_e = (1 - Y_p/2)x_e n_b \approx 10^{-5}\Omega_b h^2 (1+z)^3 \text{cm}^{-3}\,,$$

where $Y_p \approx 0.24$ is the Helium mass fraction, creates a high (comoving) Thomson opacity

$$\dot{\tau} \equiv n_e \sigma_T a$$

where dots are conformal time $\eta \equiv \int dt/a$ derivatives and τ is the optical depth.

Tight Coupling Approximation

- Near recombination $z \approx 10^3$ and $\Omega_b h^2 \approx 0.02$, the (comoving) mean free path of a photon

$$\lambda_C \equiv \frac{1}{\dot{\tau}} \sim 2.5\text{Mpc}$$

  small by cosmological standards!

- On scales $\lambda \gg \lambda_C$ photons are tightly coupled to the electrons by Thomson scattering which in turn are tightly coupled to the baryons by Coulomb interactions
- Specifically, their bulk velocities are defined by a single fluid velocity $v_\gamma = v_b$ and the photons carry no anisotropy in the rest frame of the baryons
- $\rightarrow$ No heat conduction or viscosity (anisotropic stress) in fluid

Zeroth Order Approximation

- Momentum density of a fluid is $(\rho + p)v$, where p is the pressure
- Neglect the momentum density of the baryons

$$R \equiv \frac{(\rho_b + p_b)v_b}{(\rho_\gamma + p_\gamma)v_\gamma} = \frac{\rho_b + p_b}{\rho_\gamma + p_\gamma} = \frac{3\rho_b}{4\rho_\gamma}$$

$$\approx 0.6 \left(\frac{\Omega_b h^2}{0.02}\right)\left(\frac{a}{10^{-3}}\right)$$

since $\rho_\gamma \propto T^4$ is fixed by the CMB temperature $T = 2.73(1+z)$K – OK substantially before recombination

- Neglect radiation in the expansion

$$\frac{\rho_m}{\rho_r} = 3.6 \left(\frac{\Omega_m h^2}{0.15}\right)\left(\frac{a}{10^{-3}}\right)$$

Number Continuity

- Photons are not created or destroyed. Without expansion

$$\dot{n}_\gamma + \nabla \cdot (n_\gamma \mathbf{v}_\gamma) = 0$$

but the expansion or Hubble flow causes $n_\gamma \propto a^{-3}$ or

$$\dot{n}_\gamma + 3n_\gamma \frac{\dot{a}}{a} + \nabla \cdot (n_\gamma \mathbf{v}_\gamma) = 0$$

- Linearize $\delta n_\gamma = n_\gamma - \bar{n}_\gamma$

$$(\delta n_\gamma)^{\cdot} = -3\delta n_\gamma \frac{\dot{a}}{a} - n_\gamma \nabla \cdot \mathbf{v}_\gamma$$

$$\left(\frac{\delta n_\gamma}{n_\gamma}\right)^{\cdot} = -\nabla \cdot \mathbf{v}_\gamma$$

Continuity Equation

- Number density $n_\gamma \propto T^3$ so define temperature fluctuation Θ

$$\frac{\delta n_\gamma}{n_\gamma} = 3\frac{\delta T}{T} \equiv 3\Theta$$

- Real space continuity equation

$$\dot{\Theta} = -\frac{1}{3}\nabla \cdot \mathbf{v}_\gamma$$

- Fourier space

$$\dot{\Theta} = -\frac{1}{3}i\mathbf{k} \cdot \mathbf{v}_\gamma$$

Momentum Conservation

- No expansion: $\dot{\mathbf{q}} = \mathbf{F}$
- De Broglie wavelength stretches with the expansion

$$\dot{\mathbf{q}} + \frac{\dot{a}}{a}\mathbf{q} = \mathbf{F}$$

  for photons this the redshift, for non-relativistic particles expansion drag on peculiar velocities

- Collection of particles: momentum $\rightarrow$ momentum density $(\rho_\gamma + p_\gamma)\mathbf{v}_\gamma$ and force $\rightarrow$ pressure gradient

$$[(\rho_\gamma + p_\gamma)\mathbf{v}_\gamma]^{\cdot} = -4\frac{\dot{a}}{a}(\rho_\gamma + p_\gamma)\mathbf{v}_\gamma - \nabla p_\gamma$$

$$\frac{4}{3}\rho_\gamma \dot{\mathbf{v}}_\gamma = \frac{1}{3}\nabla \rho_\gamma$$

$$\dot{\mathbf{v}}_\gamma = -\nabla\Theta$$

Euler Equation

- Fourier space

$$\dot{\mathbf{v}}_\gamma = -ik\Theta$$

- Pressure gradients (any gradient of a scalar field) generates a curl-free flow
- For convenience define velocity amplitude:

$$\mathbf{v}_\gamma \equiv -iv_\gamma \hat{\mathbf{k}}$$

- Euler Equation:

$$\dot{v}_\gamma = k\Theta$$

- Continuity Equation:

$$\dot{\Theta} = -\frac{1}{3}kv_\gamma$$

Oscillator: Take One

- Combine these to form the simple harmonic oscillator equation

$$\ddot{\Theta} + c_s^2 k^2 \Theta = 0$$

where the adiabatic sound speed is defined through

$$c_s^2 \equiv \frac{\dot{p}_\gamma}{\dot{\rho}_\gamma}$$

here $c_s^2 = 1/3$ since we are photon-dominated

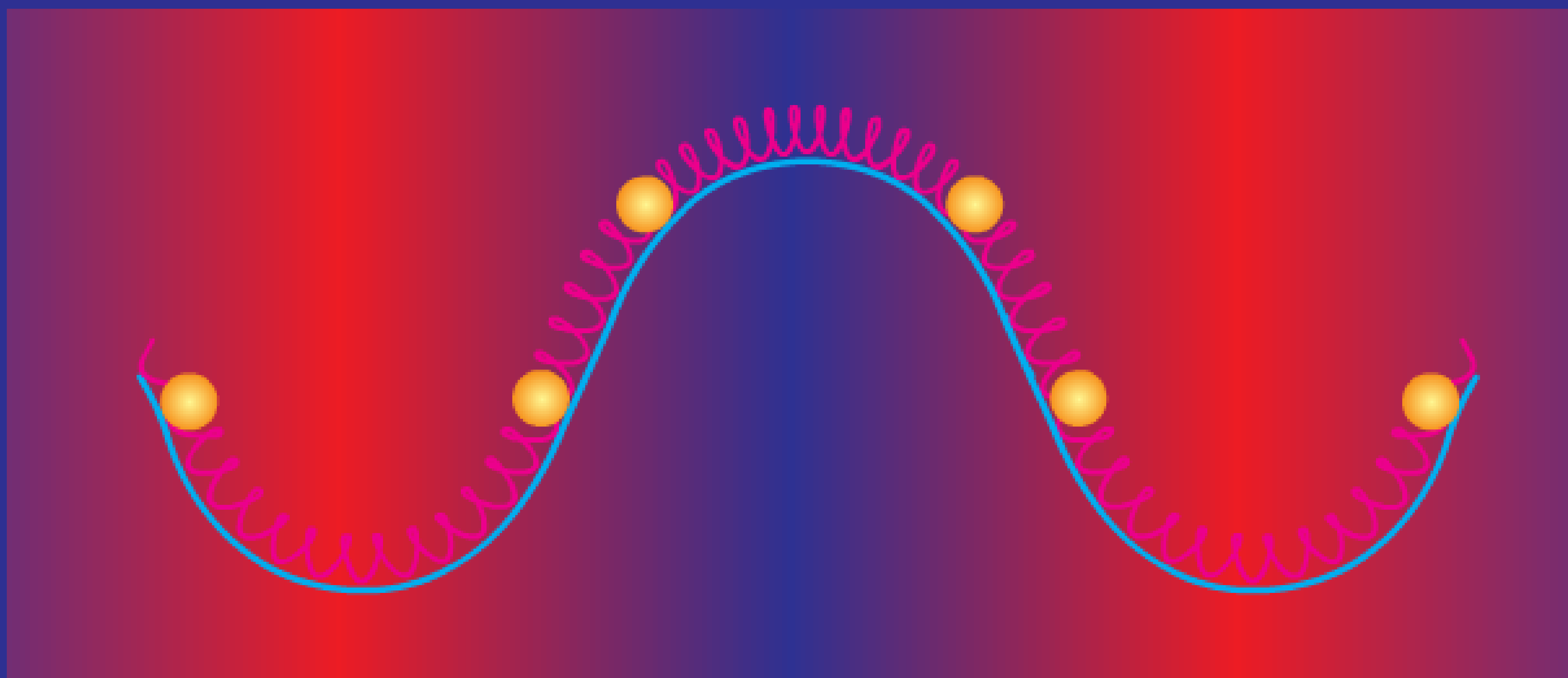
- General solution:

$$\Theta(\eta) = \Theta(0) \cos(ks) + \frac{\dot{\Theta}(0)}{kc_s} \sin(ks)$$

where the sound horizon is defined as $s \equiv \int c_s d\eta$

Seeing Sound

- Oscillations frozen at recombination
- Compression=hot spots, Rarefaction=cold spots



Harmonic Extrema

- All modes are frozen in at recombination (denoted with a subscript $*$) yielding temperature perturbations of different amplitude for different modes. For the adiabatic (curvature mode) $\dot{\Theta}(0) = 0$

$$\Theta(\eta_*) = \Theta(0) \cos(ks_*)$$

- Modes caught in the extrema of their oscillation will have enhanced fluctuations

$$k_n s_* = n\pi$$

yielding a fundamental scale or frequency, related to the inverse sound horizon

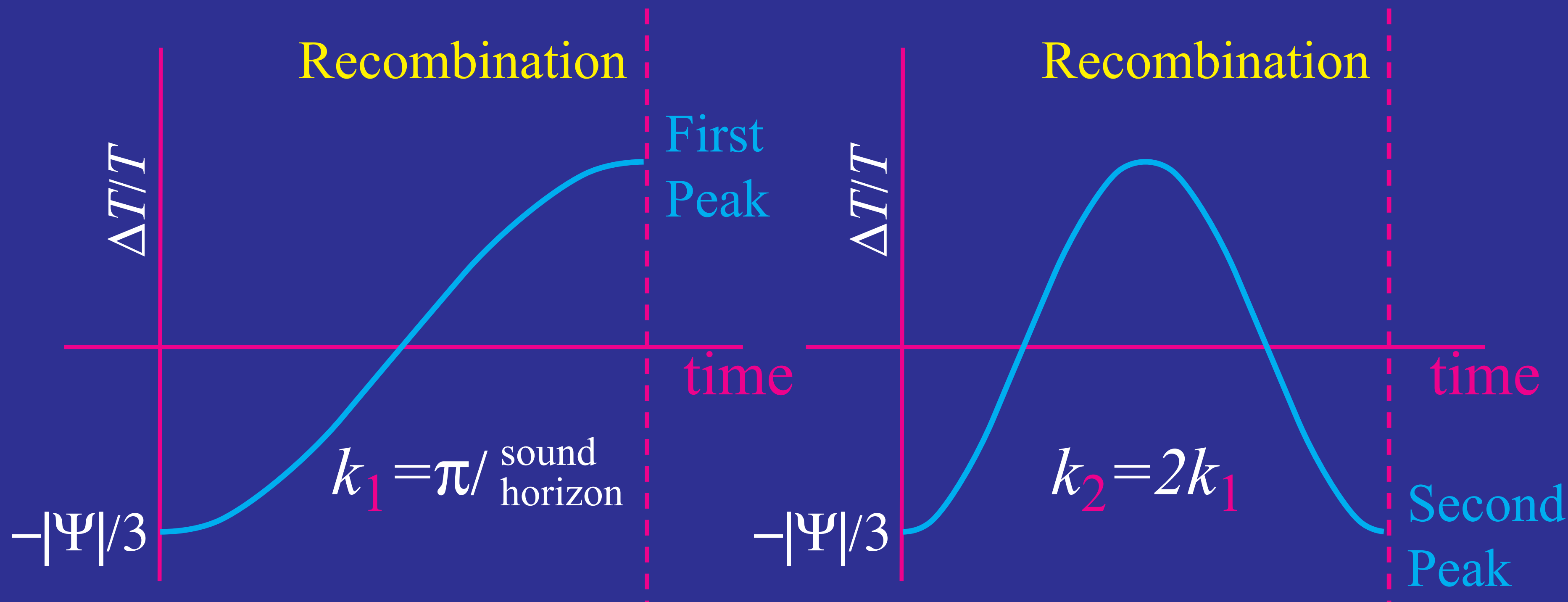
$$k_A = \pi / s_*$$

and a harmonic relationship to the other extrema as $1 : 2 : 3 \ldots$

The First Peak

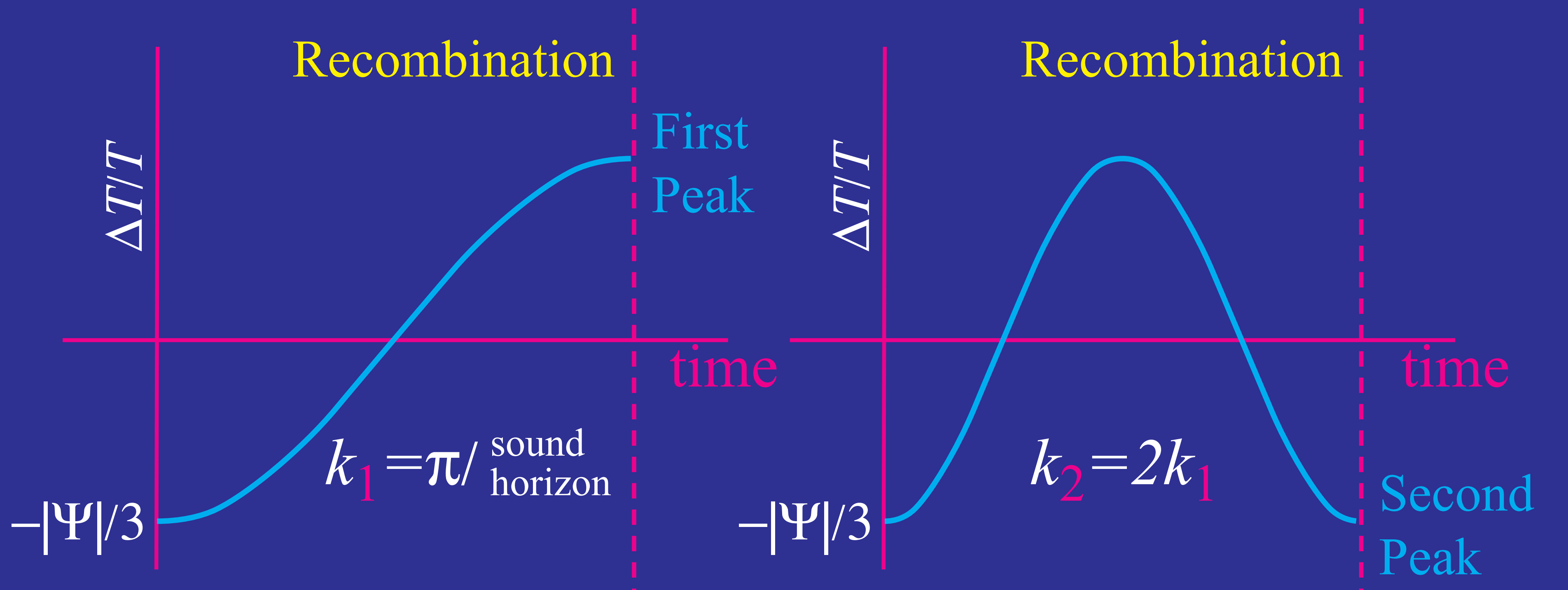
Extrema=Peaks

- First peak = mode that just compresses
- Second peak = mode that compresses then rarefies: twice the wavenumber



Extrema=Peaks

- First peak = mode that just compresses
- Second peak = mode that compresses then rarefies: twice the wavenumber
- Harmonic peaks: 1:2:3 in wavenumber



Peak Location

- The fundmental physical scale is translated into a fundamental angular scale by simple projection according to the angular diameter distance D_A

$$\begin{aligned} \theta_A &= \lambda_A / D_A \\ \ell_A &= k_A D_A \end{aligned}$$

- In a flat universe, the distance is simply $D_A = D \equiv \eta_0 - \eta_* \approx \eta_0$, the horizon distance, and $k_A = \pi / s_* = \sqrt{3}\pi / \eta_*$ so

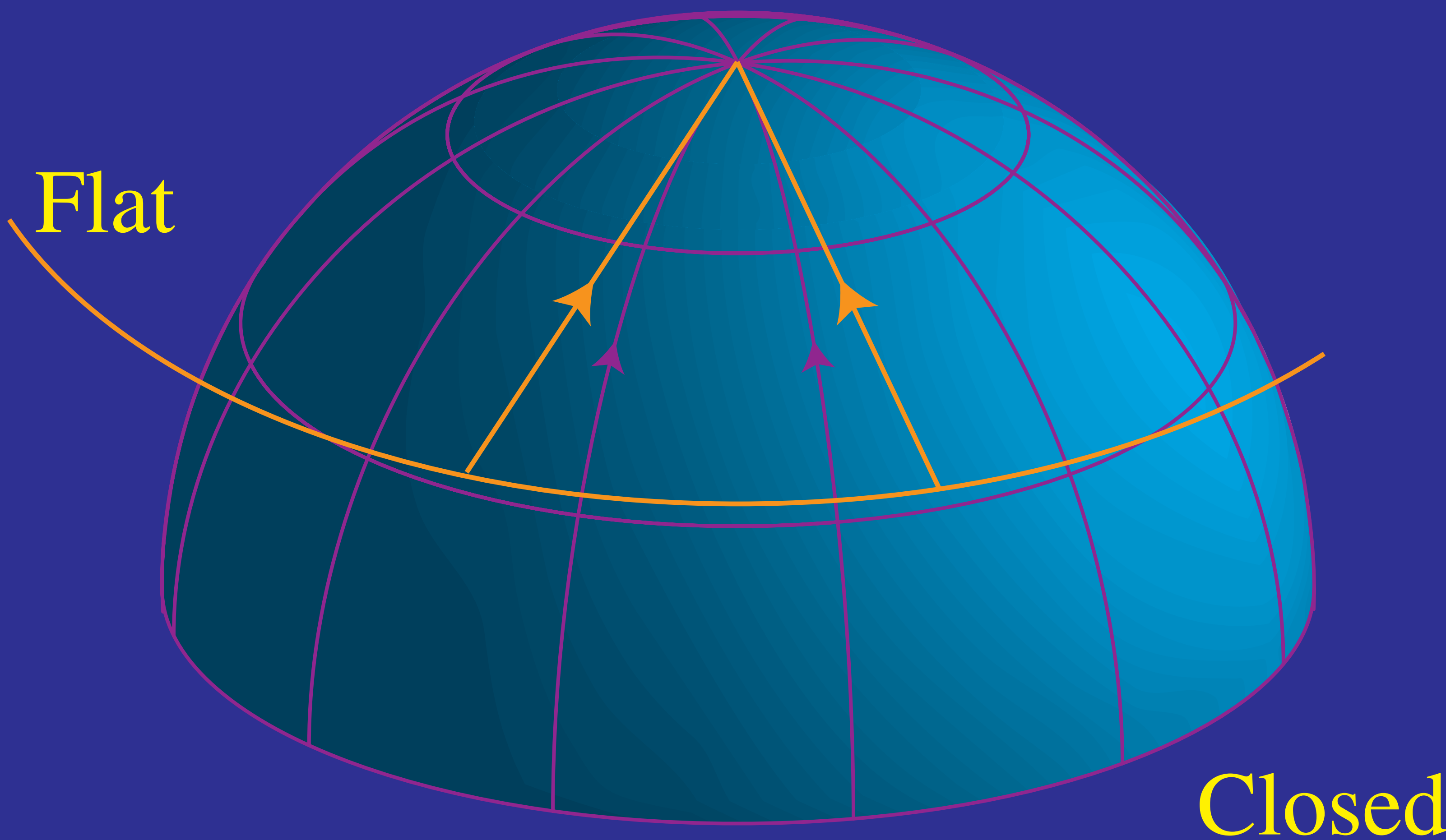
$$\theta_A \approx \frac{\eta_*}{\eta_0}$$

- In a matter-dominated universe $\eta \propto a^{1/2}$ so $\theta_A \approx 1/30 \approx 2^\circ$ or

$$\ell_A \approx 200$$

Spatial Curvature

- Physical scale of peak = distance sound travels
- Angular scale measured: comoving angular diameter distance test for curvature

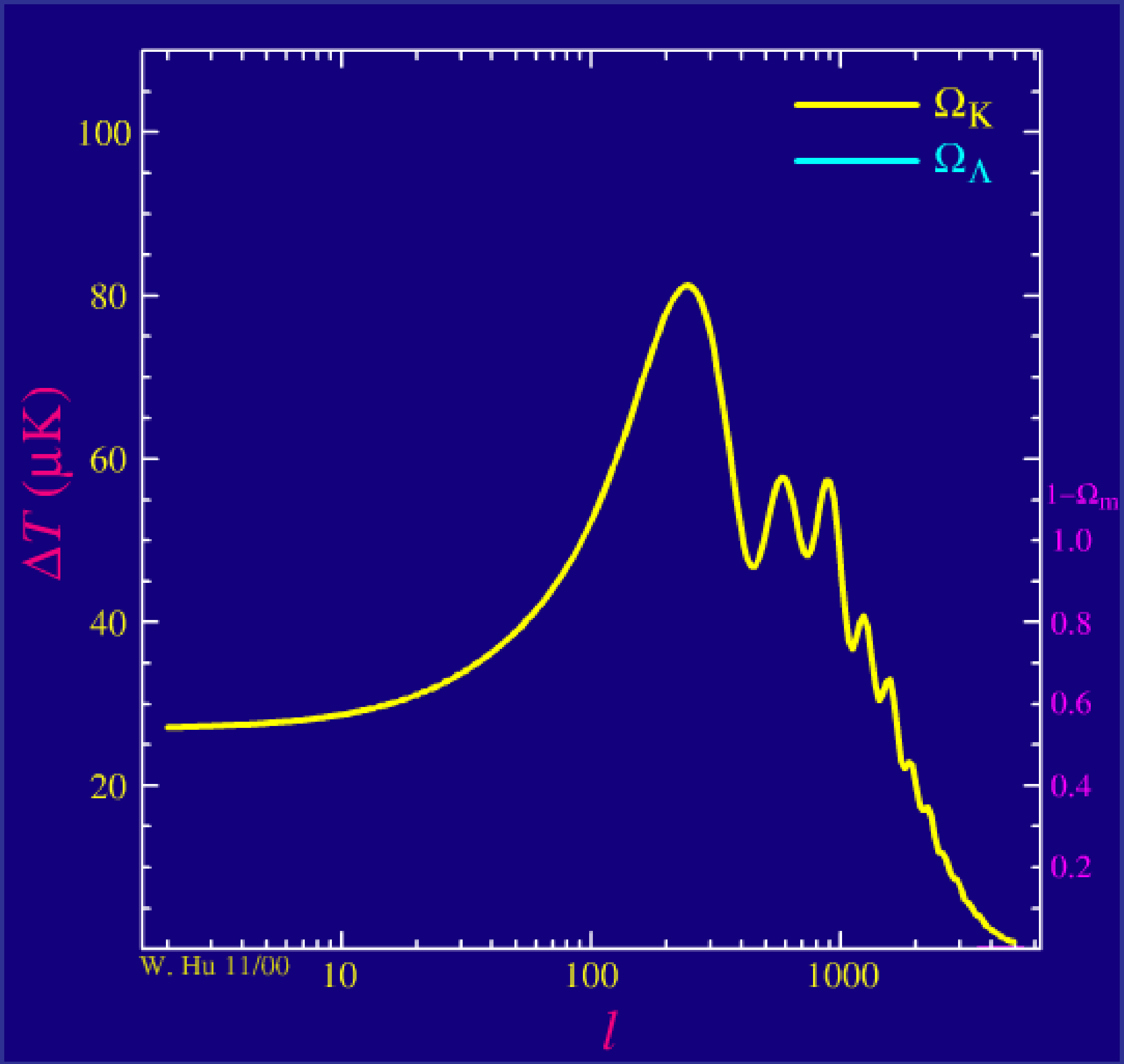


Curvature

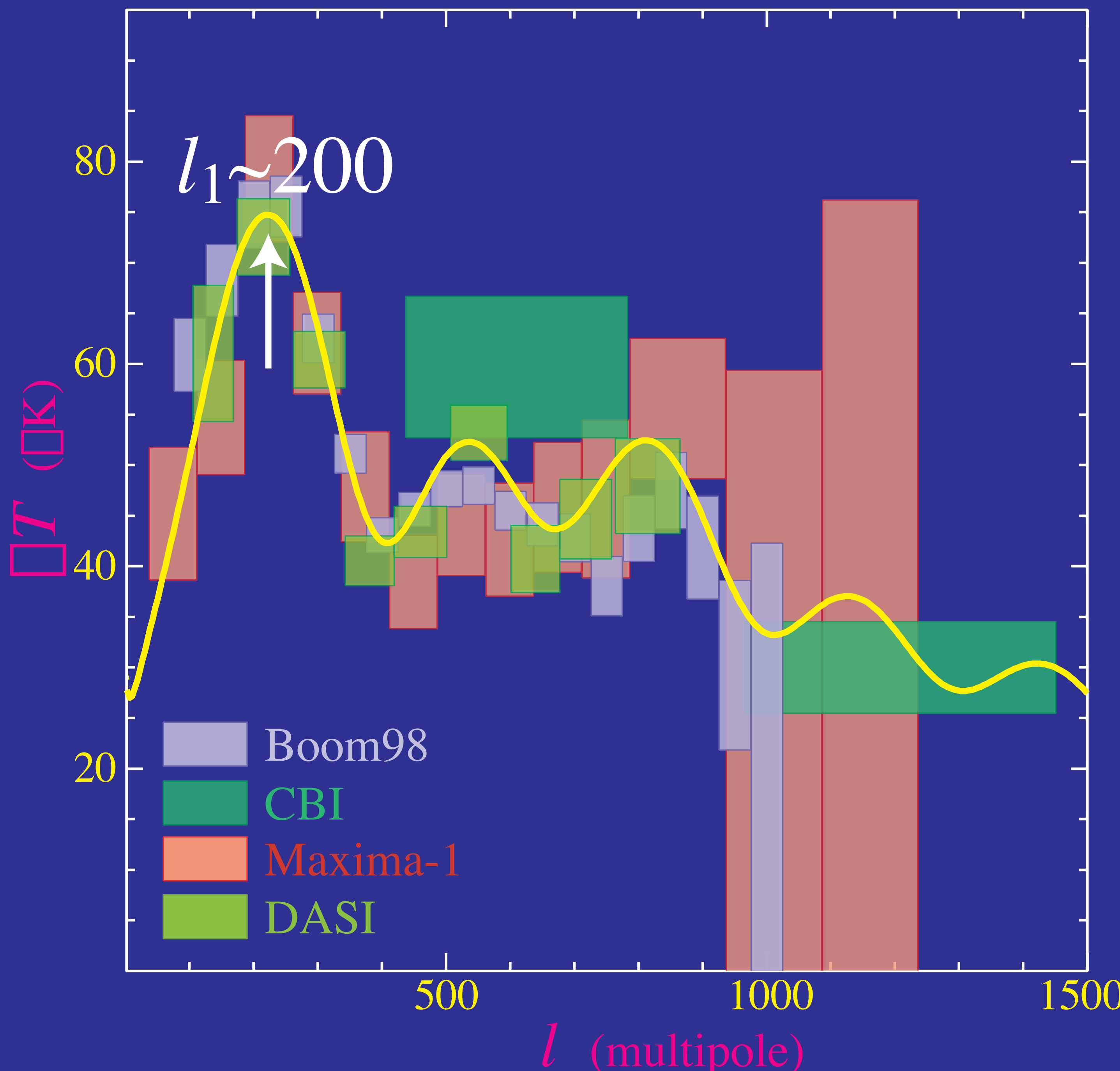
- In a curved universe, the apparent or angular diameter distance is no longer the conformal distance $D_A = R\sin(D/R) \neq D$
- Objects in a closed universe are further than they appear! gravitational lensing of the background...
- Curvature scale of the universe must be substantially larger than current horizon
- Flat universe indicates critical density and implies missing energy given local measures of the matter density "dark energy"
- D also depends on dark energy density $\Omega_{\rm DE}$ and equation of state $w = p_{\rm DE}/\rho_{\rm DE}$.
- Expansion rate at recombination or matter-radiation ratio enters into calculation of k_A.

Curvature in the Power Spectrum

- Features scale with angular diameter distance
- Angular location of the first peak

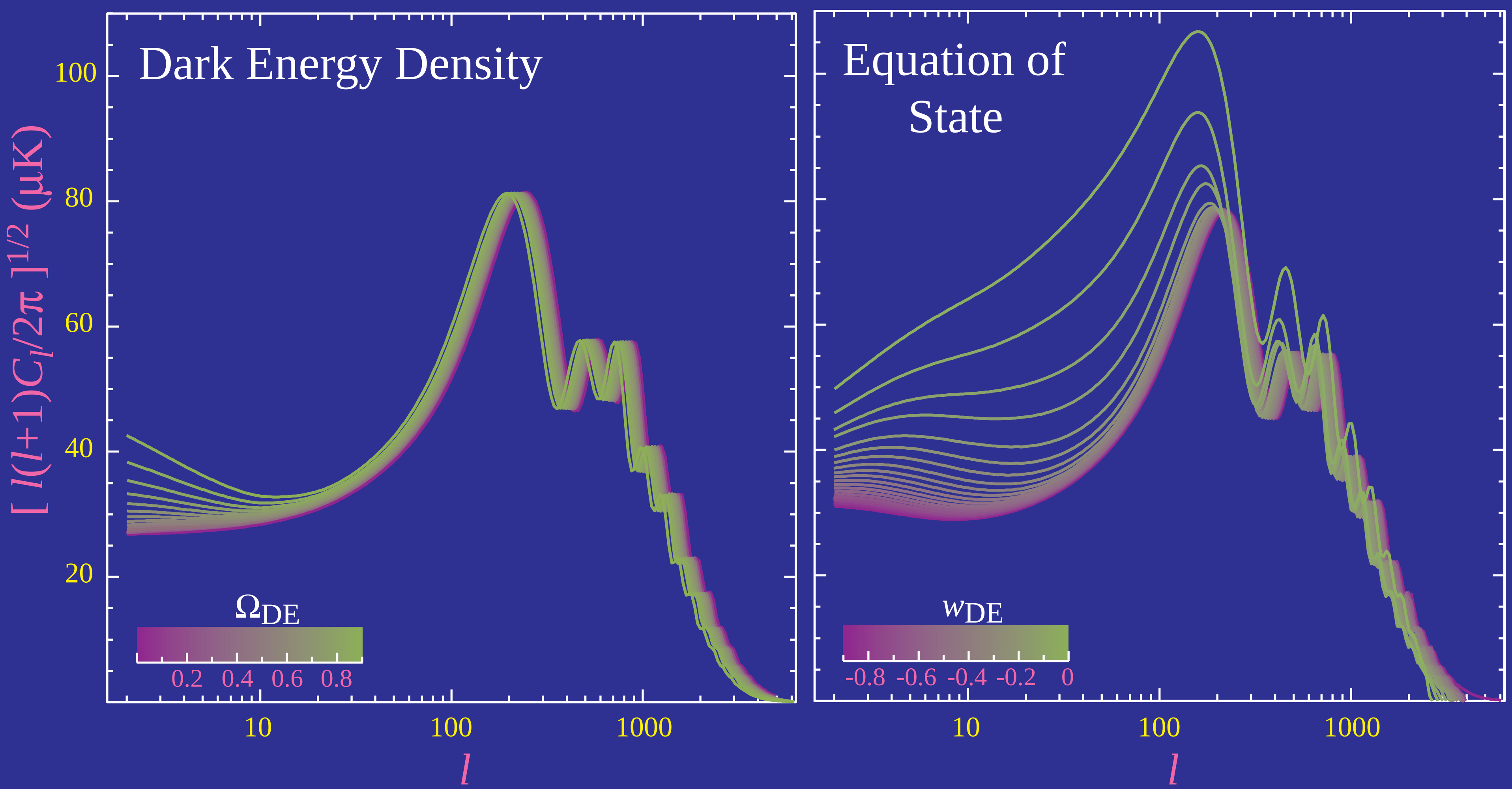


First Peak Precisely Measured



Dark Energy

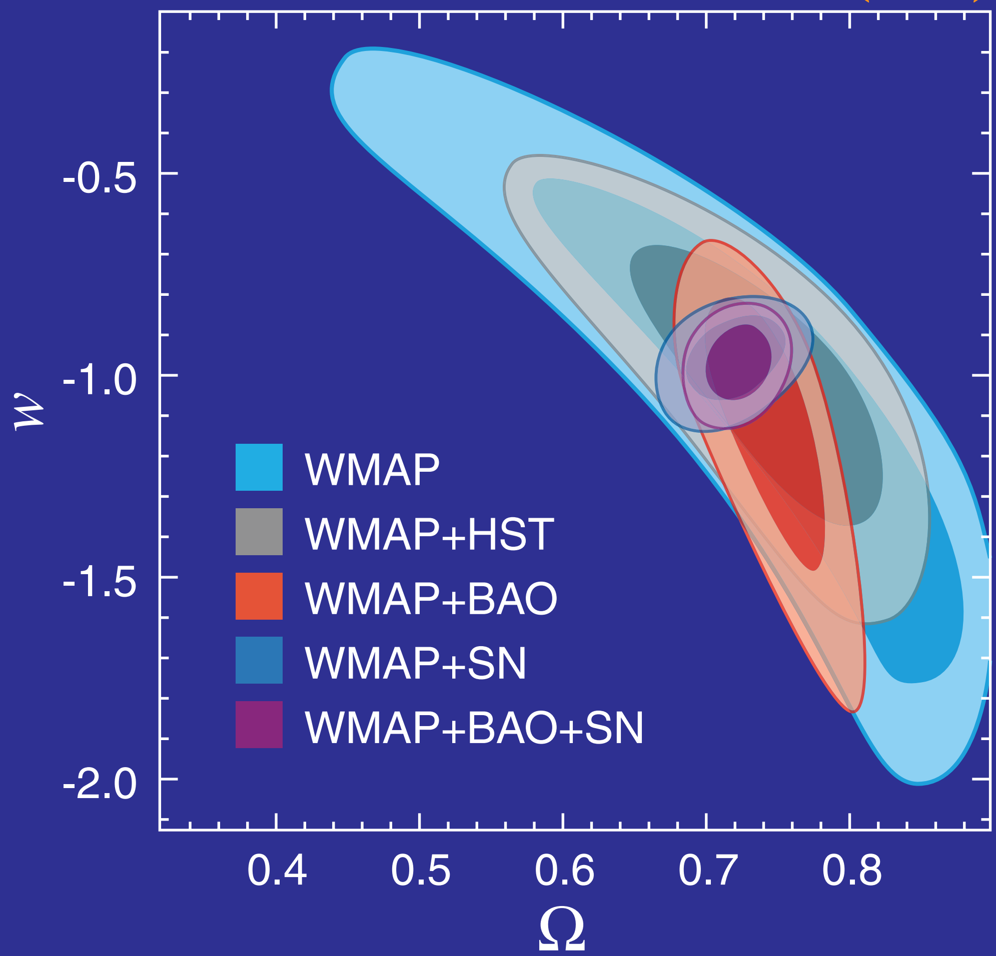
- Peaks measure distance to recombination
- ISW effect constrains dynamics of acceleration



Dark Energy

- Flat ΛCDM fully consistent with CMB and other distance measures
- Constant $w=p/\rho$ constrained as $-0.097<1+w<0.142$ (95% CL)

Komatsu et al (2008)



Doppler Peaks?

- Doppler effect for the photon dominated system is of equal amplitude and $\pi/2$ out of phase: extrema of temperature are turning points of velocity
- Effects add in quadrature:

$$\left(\frac{\Delta T}{T}\right)^2 = \Theta^2(0)[\cos^2(ks) + \sin^2(ks)] = \Theta^2(0)$$

- No peaks in k spectrum! However the Doppler effect carries an angular dependence that changes its projection on the sky $\hat{\mathbf{n}} \cdot \mathbf{v}_\gamma \propto \hat{\mathbf{n}} \cdot \hat{\mathbf{k}}$
- Coordinates where $\hat{\mathbf{z}} \parallel \hat{\mathbf{k}}$

$$Y_{10} Y_{\ell 0} \rightarrow Y_{\ell \pm 1\, 0}$$

recoupling $j_\ell' Y_{\ell 0}$: no peaks in Doppler effect

Doppler Effect

- Bulk motion of fluid changes the observed temperature via Doppler shifts

$$\left(\frac{\Delta T}{T}\right)_{\rm dop} = \hat{\mathbf{n}} \cdot \mathbf{v}_\gamma$$

- Averaged over directions

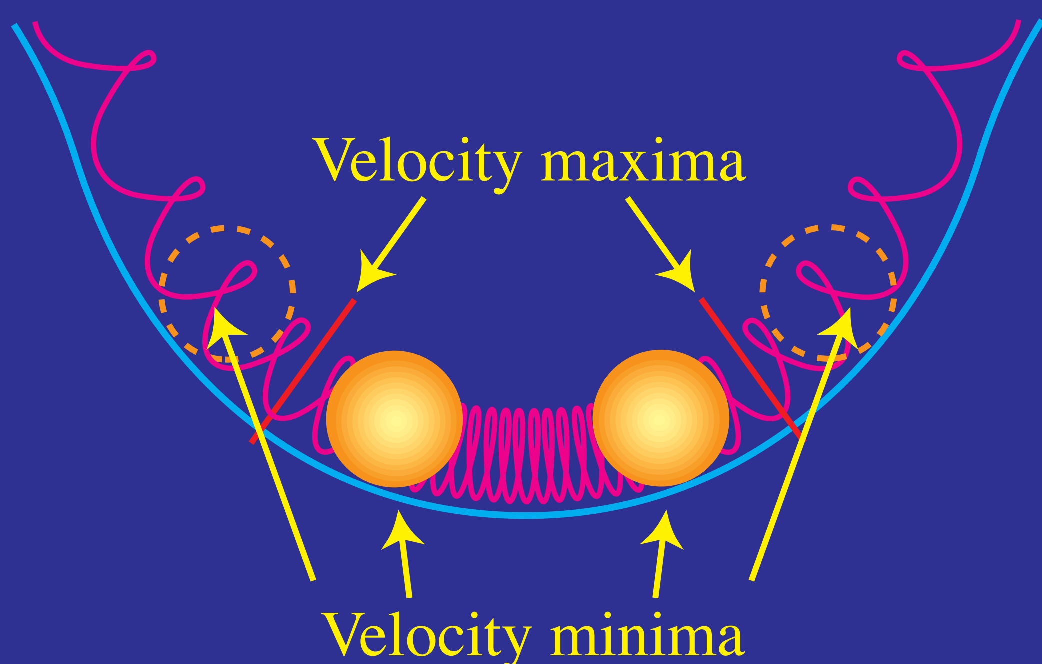
$$\left(\frac{\Delta T}{T}\right)_{\rm rms} = \frac{v_\gamma}{\sqrt{3}}$$

- Acoustic solution

$$\frac{v_\gamma}{\sqrt{3}} = -\frac{\sqrt{3}}{k}\dot{\Theta} = \frac{\sqrt{3}}{k} k c_s \, \Theta(0) \sin(ks)$$
$$= \Theta(0) \sin(ks)$$

Doppler Effect

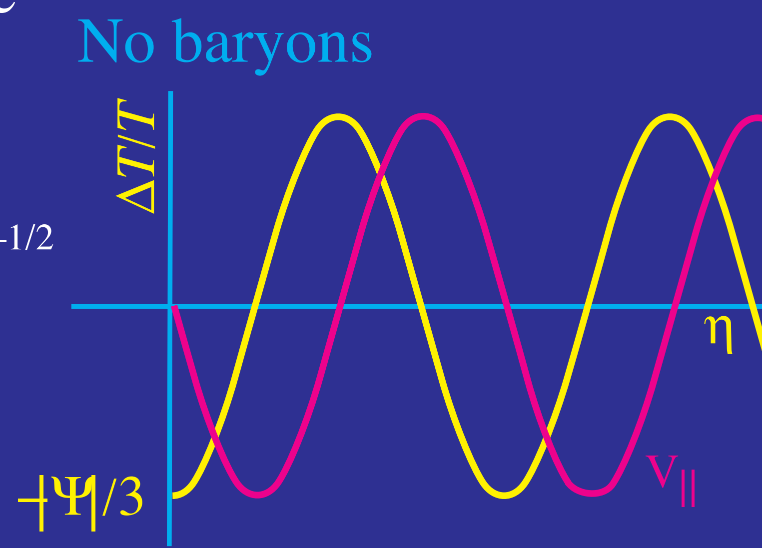
- Relative velocity of fluid and observer
- Extrema of oscillations are turning points or velocity zero points
- Velocity $\pi/2$ out of phase with temperature

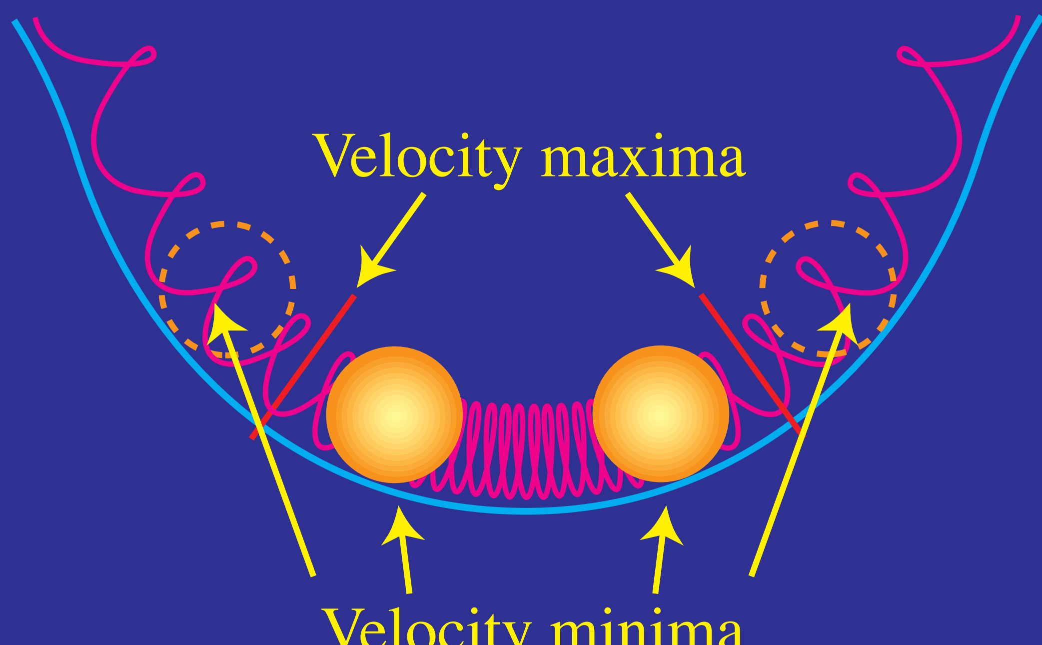


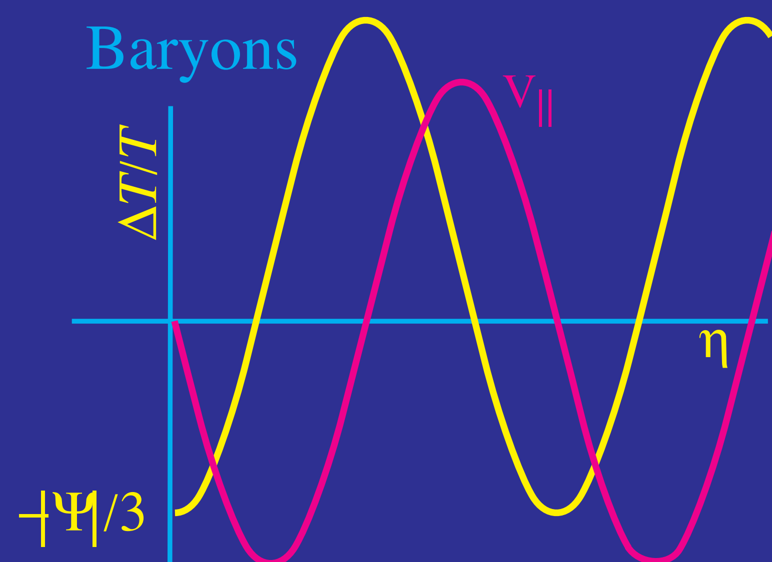
Doppler Effect

- Relative velocity of fluid and observer
- Extrema of oscillations are turning points or velocity zero points
- Velocity $\pi/2$ out of phase with temperature
- Zero point not shifted by baryon drag
- Increased baryon inertia decreases effect

$$m_{\mathrm{eff}} V^2 = \text{const.} \qquad V \propto m_{\mathrm{eff}}^{-1/2} = (1+R)^{-1/2}$$

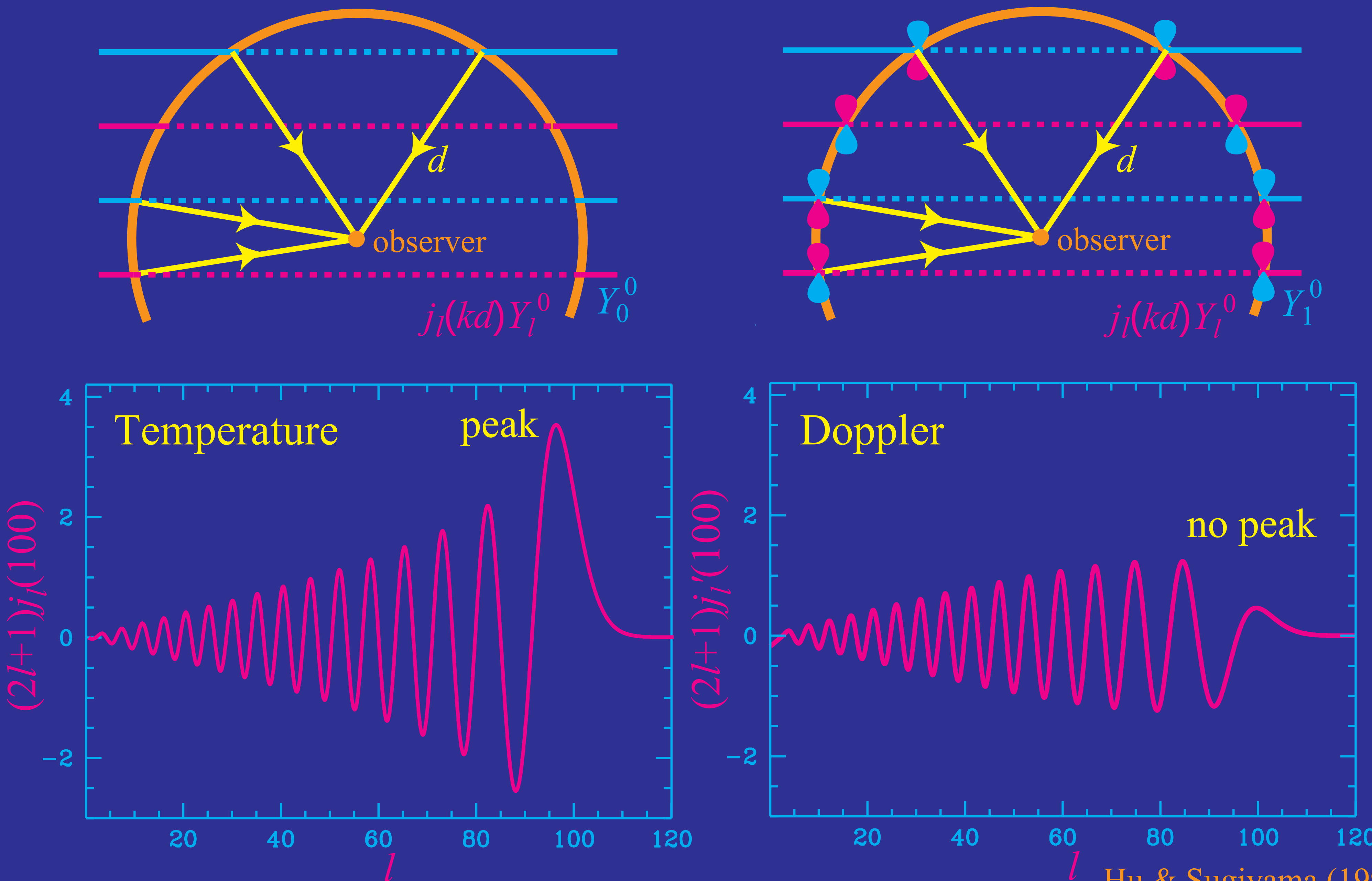






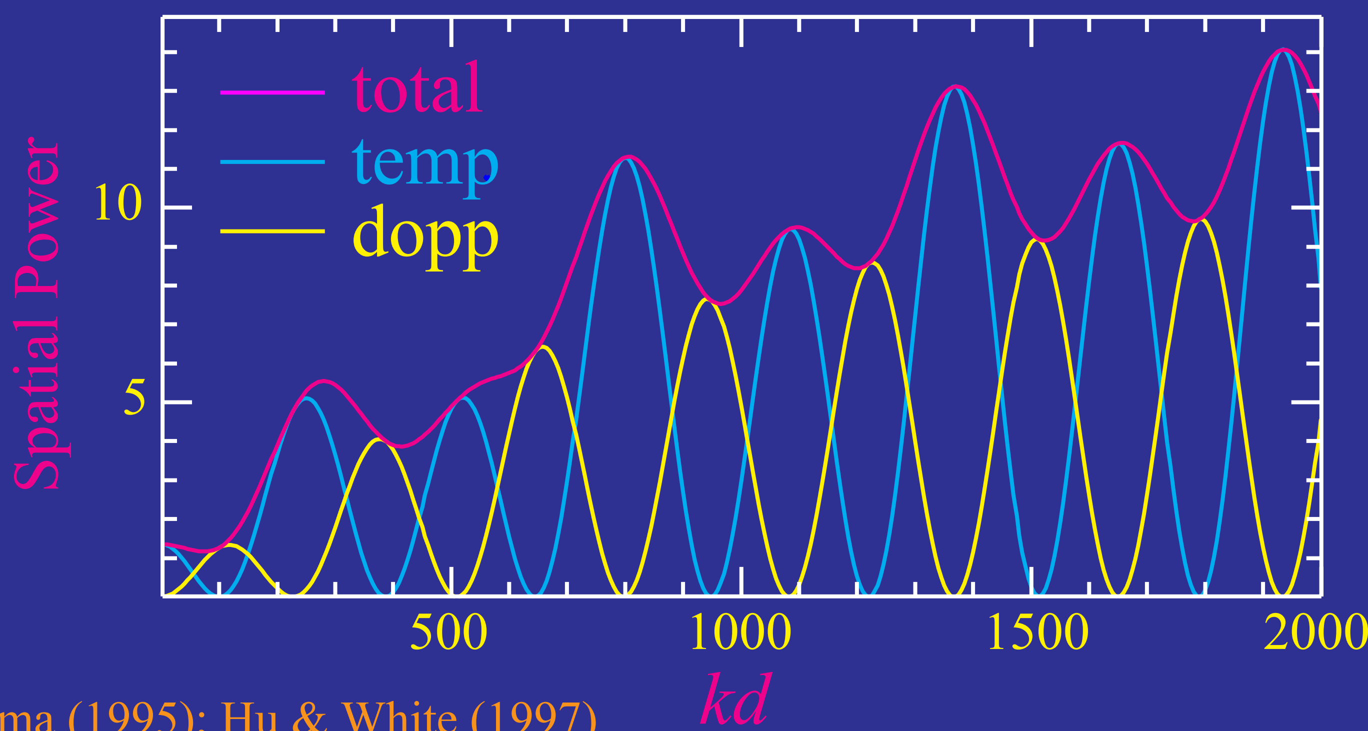
Doppler Peaks?

- Doppler effect has lower amplitude and weak features from projection



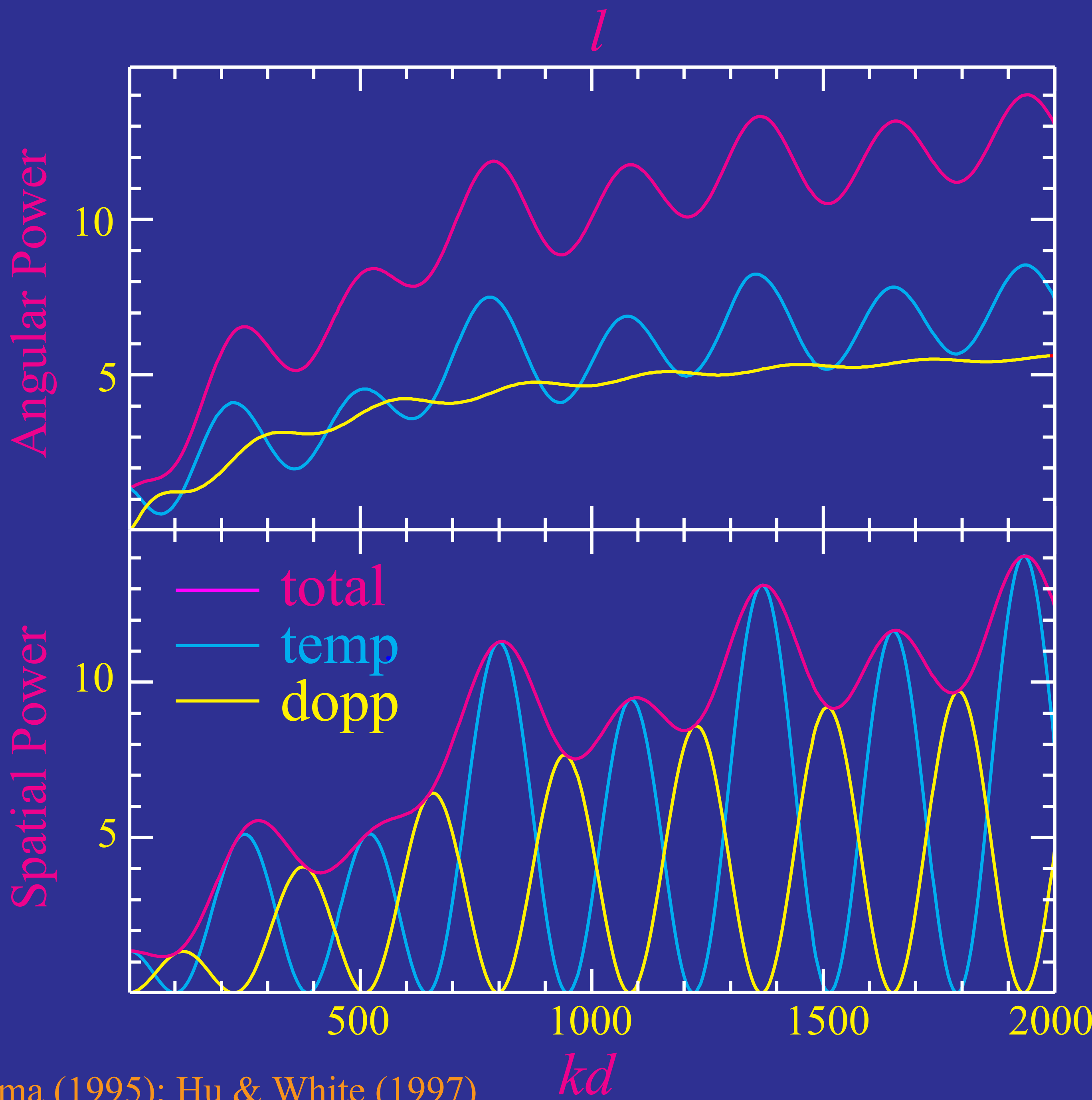
Hu & Sugiyama (1995)

Relative Contributions



Hu & Sugiyama (1995); Hu & White (1997)

Relative Contributions



Hu & Sugiyama (1995); Hu & White (1997)

Restoring Gravity: Continuity

- Take a simple photon dominated system with gravity
- Continuity altered since a gravitational potential represents a stretching of the spatial fabric that dilutes number densities – formally a spatial curvature perturbation
- Think of this as a perturbation to the scale factor $a \to a(1+\Phi)$ so that the cosmogical redshift is generalized to

$$\frac{\dot{a}}{a} \to \frac{\dot{a}}{a} + \dot{\Phi}$$

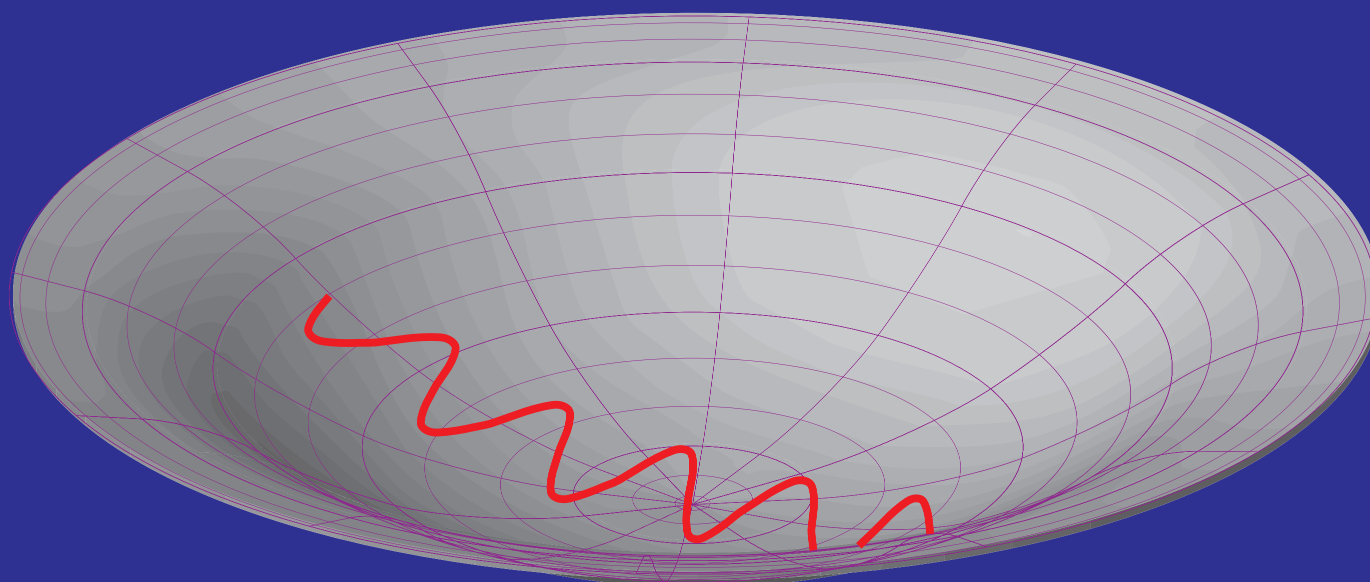
$$(\delta n_\gamma)^{\cdot} = -3\delta n_\gamma \frac{\dot{a}}{a} - 3n_\gamma \dot{\Phi} - n_\gamma \nabla \cdot \mathbf{v}_\gamma$$

so that the continuity equation becomes

$$\dot{\Theta} = -\frac{1}{3} k v_\gamma - \dot{\Phi}$$

Metric Stretch

- Potential wells curve or stretch space
- Like the expansion of the universe, changes in the potential change the wavelength of photons



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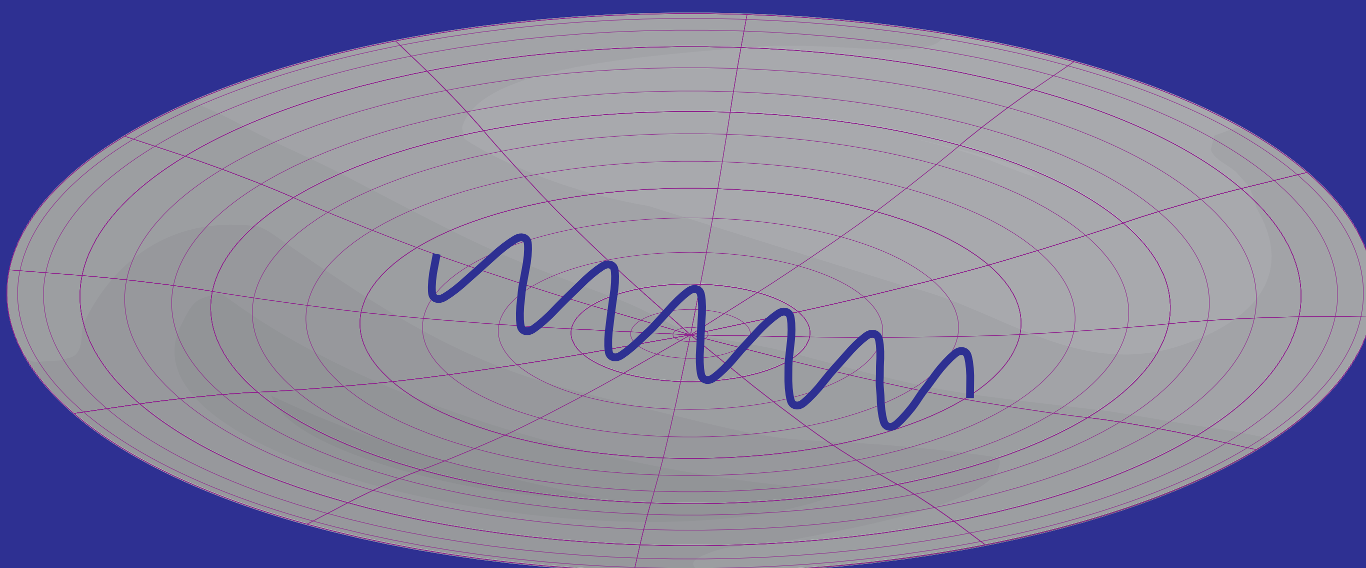


Restoring Gravity: Euler

- Gravitational force in momentum conservation $\mathbf{F} = -m\nabla\Psi$ generalized to momentum density modifies the Euler equation to

$$\dot{v}_\gamma = k(\Theta + \Psi)$$

- General relativity says that Φ and Ψ are the relativistic analogues of the Newtonian potential and that $\Phi \approx -\Psi$.
- In our matter-dominated approximation, Φ represents matter density fluctuations through the cosmological Poisson equation

$$k^2\Phi = 4\pi G a^2 \rho_m \Delta_m$$

where the difference comes from the use of comoving coordinates for k (a^2 factor), the removal of the background density into the background expansion $(\rho_m \Delta_m)$ and finally a coordinate subtlety that enters into the definition of Δ_m

Constant Potentials

- In the matter dominated epoch potentials are constant because infall generates velocities as $v_m \sim k\eta\Psi$
- Velocity divergence generates density perturbations as $\Delta_m \sim -k\eta v_m \sim -(k\eta)^2\Psi$
- And density perturbations generate potential fluctuations as $\Phi \sim \Delta_m/(k\eta)^2 \sim -\Psi$, keeping them constant. Note that because of the expansion, density perturbations must grow to keep potentials constant.
- Here we have used the Friedman equation $H^2 = 8\pi G\rho_m/3$ and $\eta = \int d\ln a/(aH) \sim 1/(aH)$
- More generally, if stress perturbations are negligible compared with density perturbations ($\delta p \ll \delta\rho$) then potential will remain roughly constant – more specifically a variant called the Bardeen or comoving curvature ζ is constant

Oscillator: Take Two

- Combine these to form the simple harmonic oscillator equation

$$\ddot{\Theta} + c_s^2 k^2 \Theta = -\frac{k^2}{3}\Psi - \ddot{\Phi}$$

- In a CDM dominated expansion $\dot{\Phi} = \dot{\Psi} = 0$. Also for photon domination $c_s^2 = 1/3$ so the oscillator equation becomes

$$\ddot{\Theta} + \ddot{\Psi} + c_s^2 k^2 (\Theta + \Psi) = 0$$

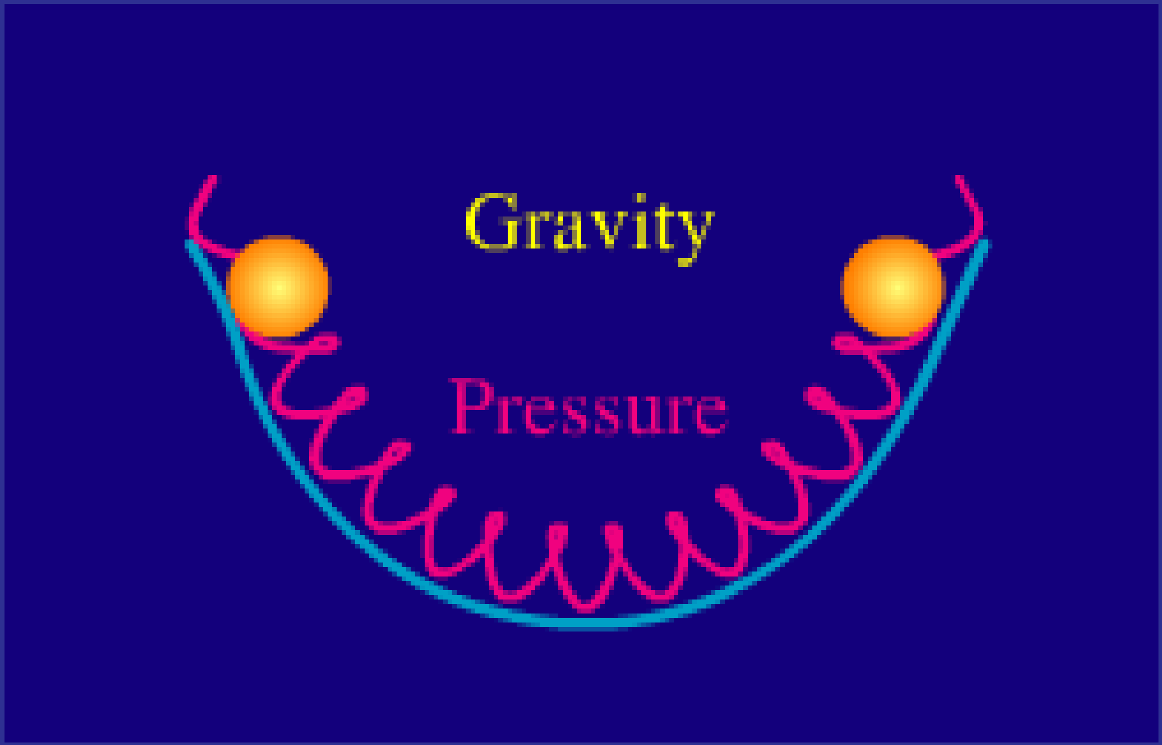
- Solution is just an offset version of the original

$$[\Theta + \Psi](\eta) = [\Theta + \Psi](0)\, \cos(ks)$$

- $\Theta + \Psi$ is also the observed temperature fluctuation since photons lose energy climbing out of gravitational potentials at recombination

Gravitational Ringing

- Potential wells = inflationary seeds of structure
- Fluid falls into wells, pressure resists: acoustic oscillations



Effective Temperature

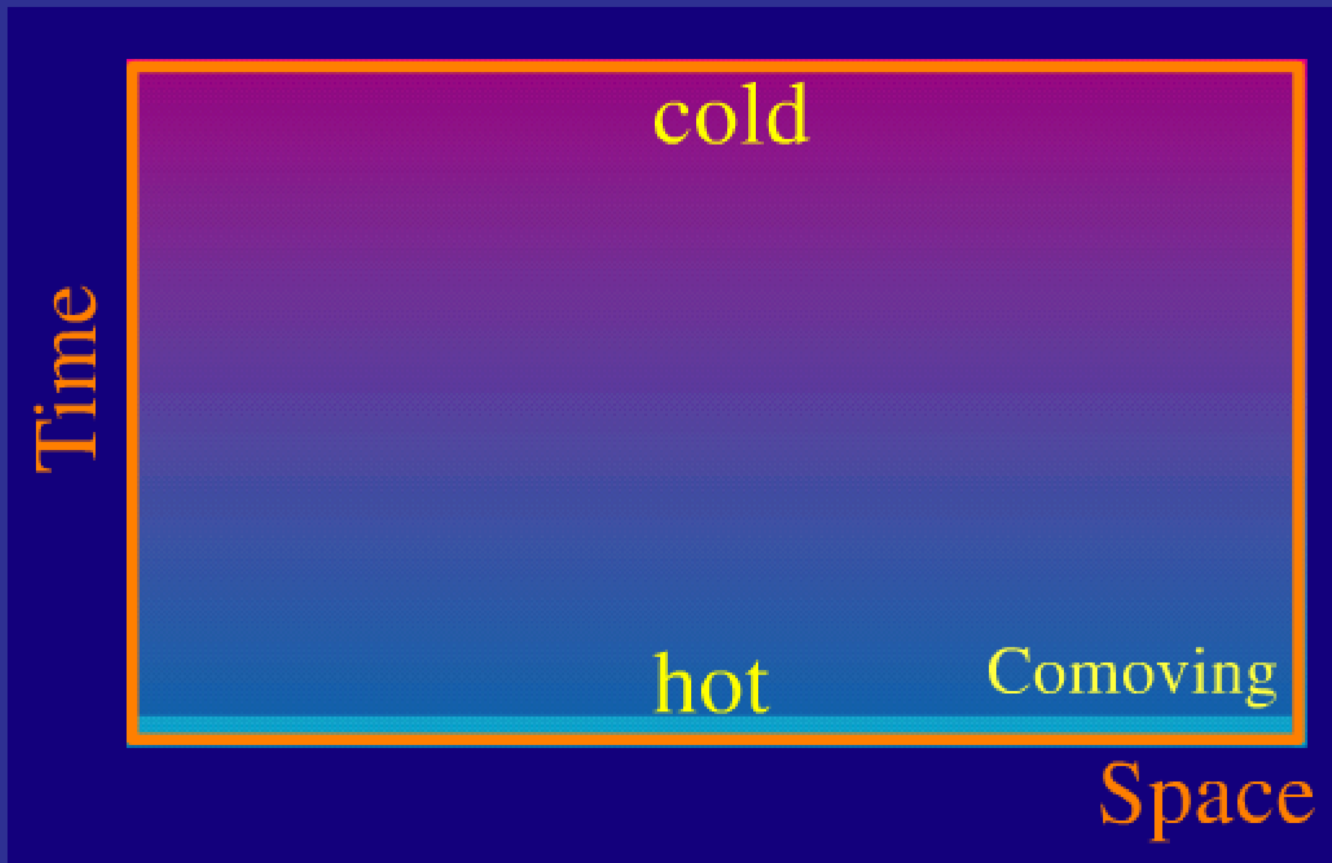
- Photons climb out of potential wells at last scattering
- Lose energy to gravitational redshifts
- Observed or effective temperature

$$\Theta + \Psi$$

- Effective temperature oscillates around zero with amplitude given by the initial conditions
- Note: initial conditions are set when the perturbation is outside of horizon, need inflation or other modification to matter-radiation FRW universe.
- GR says that initial temperature is given by initial potential

Inflation and the Initial Conditions

- Inflation: (nearly) scale-invariant curvature (potential) perturbations
- Superluminal expansion → superhorizon scales → "initial conditions"
- Accompanying temperture perturbations due to cosmological redshift



- Potential perturbation Ψ = time-time metric perturbation

  $\delta t/t = \Psi \quad \rightarrow \quad \delta T/T = -\delta a/a = -2/3\,\delta t/t = -2/3\,\Psi$

Sachs & Wolfe (1967); White & Hu (1997)

Sachs-Wolfe Effect and the Magic 1/3

- A gravitational potential is a perturbation to the temporal coordinate [formally a gauge transformation]

$$\frac{\delta t}{t} = \Psi$$

- Convert this to a perturbation in the scale factor,

$$t = \int \frac{da}{aH} \propto \int \frac{da}{a\rho^{1/2}} \propto a^{3(1+w)/2}$$

where $w \equiv p/\rho$ so that during matter domination

$$\frac{\delta a}{a} = \frac{2}{3}\frac{\delta t}{t}$$

- CMB temperature is cooling as $T \propto a^{-1}$ so

$$\Theta + \Psi \equiv \frac{\delta T}{T} + \Psi = -\frac{\delta a}{a} + \Psi = \frac{1}{3}\Psi$$

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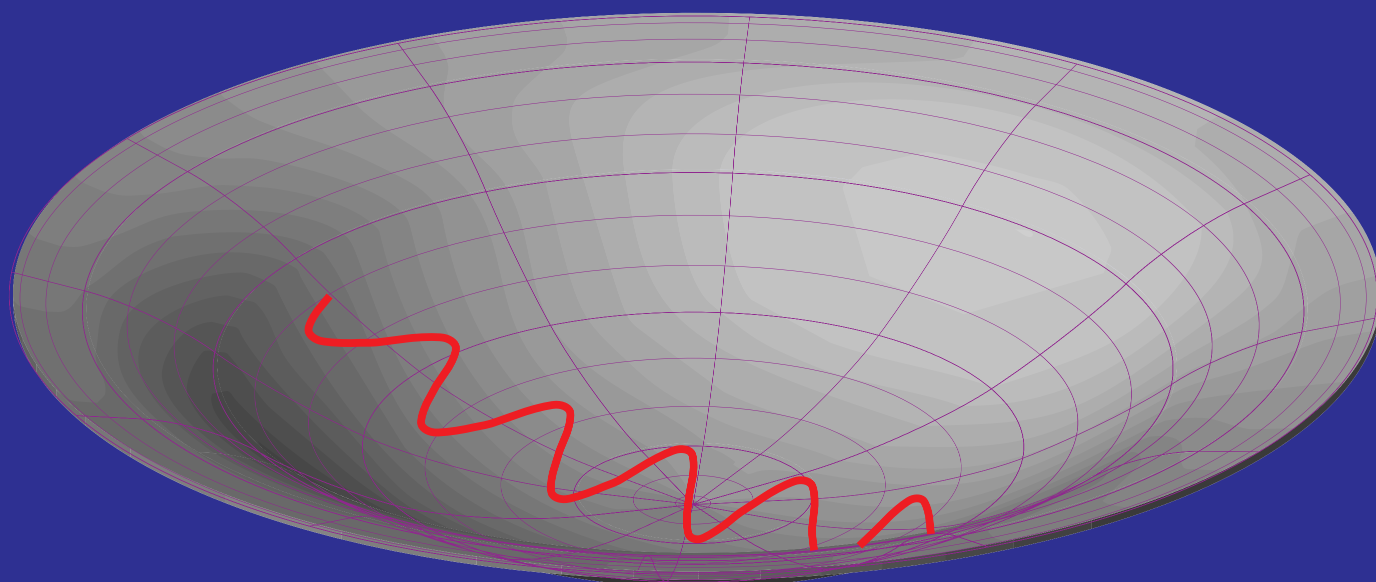
Smooth Energy Density & Potential Decay

- A smooth component contributes
  density ρ to the expansion

  but not
  density fluctuation $\delta\rho$ to the Poisson equation
- Imbalance causes potential to decay once smooth component dominates the expansion

ISW Effect

- Gravitational blueshift on infall does not cancel redshift on climbing out
- Contraction of spatial metric doubles the effect: $\Delta T/T = 2\Delta\Phi$
- Effect from potential hills and wells cancel on small scales



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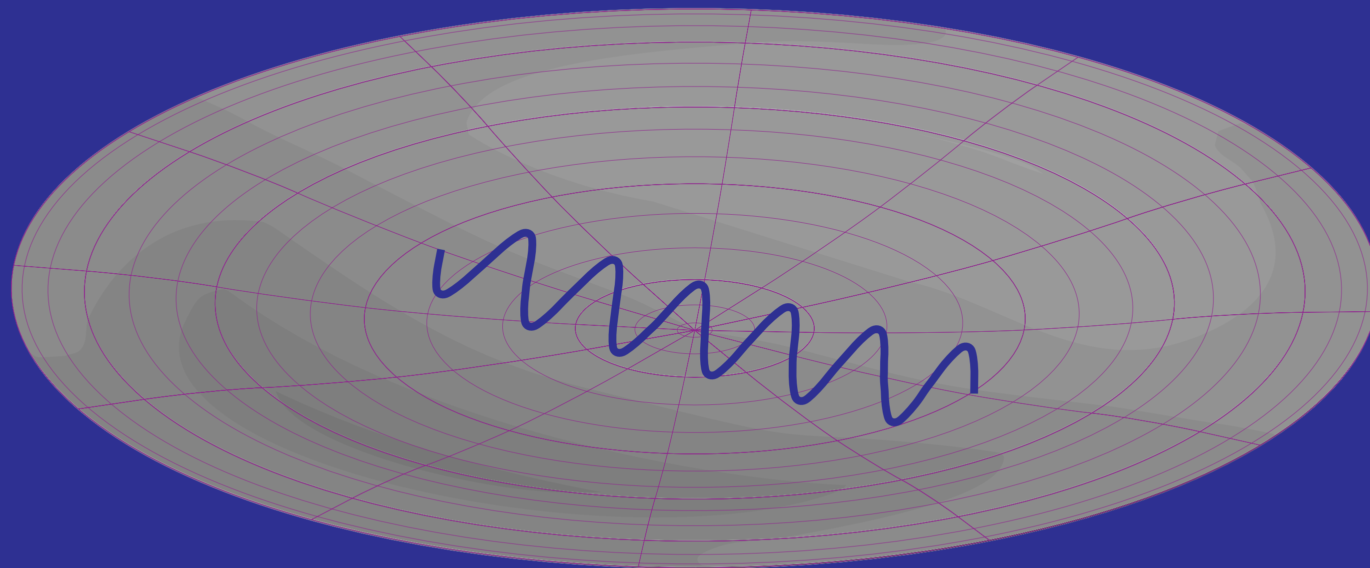
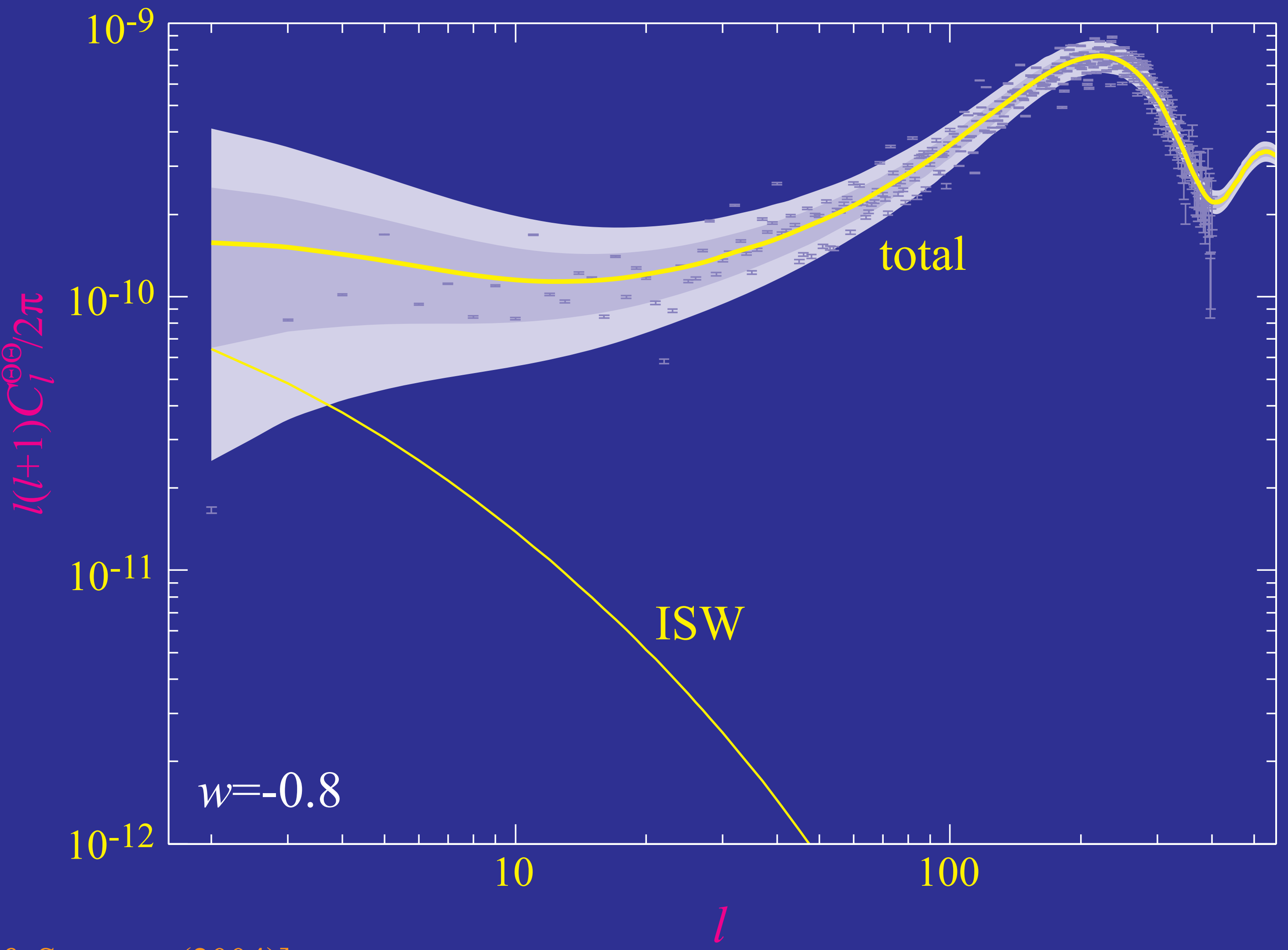


ISW Effect

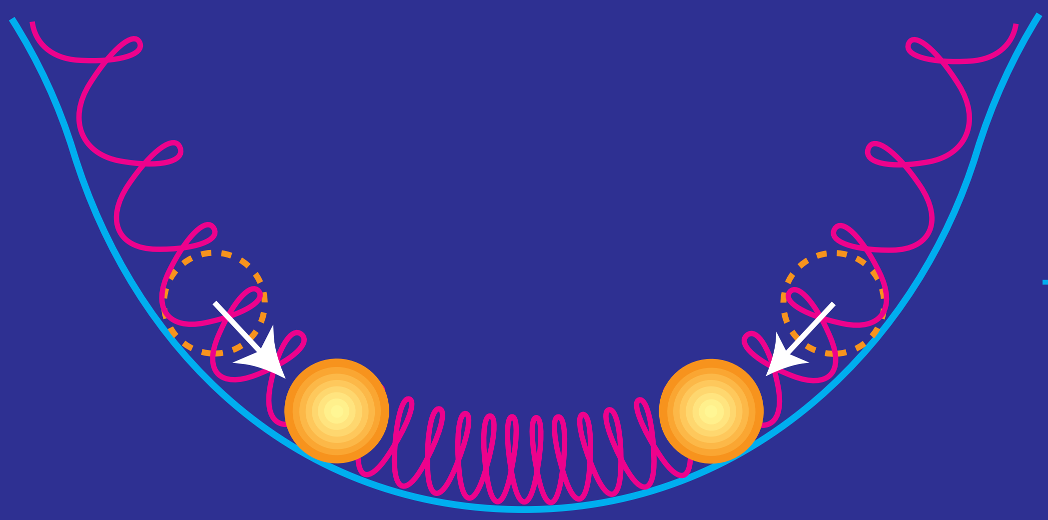
- ISW effect hidden in the temperature power spectrum by primary anisotropy and cosmic variance

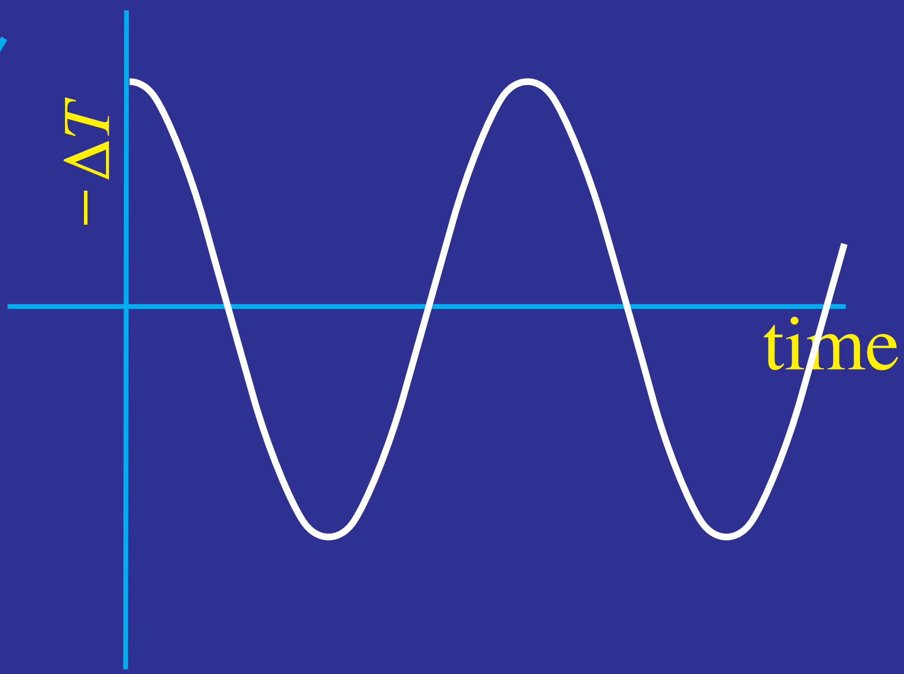


[plot: Hu & Scranton (2004)]

Effective Temperature

- Effective temperature initially $\Theta+\Psi=\Psi/3$ and is negative in an overdensity
- Effective temperature oscillates around zero
- Effective temperature becomes observed temperature after gravitational redshift





The Second Peak

Baryon Loading

- Baryons add extra mass to the photon-baryon fluid
- Controlling parameter is the momentum density ratio:

$$R \equiv \frac{p_b + \rho_b}{p_\gamma + \rho_\gamma} \approx 30\Omega_b h^2 \left(\frac{a}{10^{-3}} \right)$$

of order unity at recombination

- Momentum density of the joint system is conserved

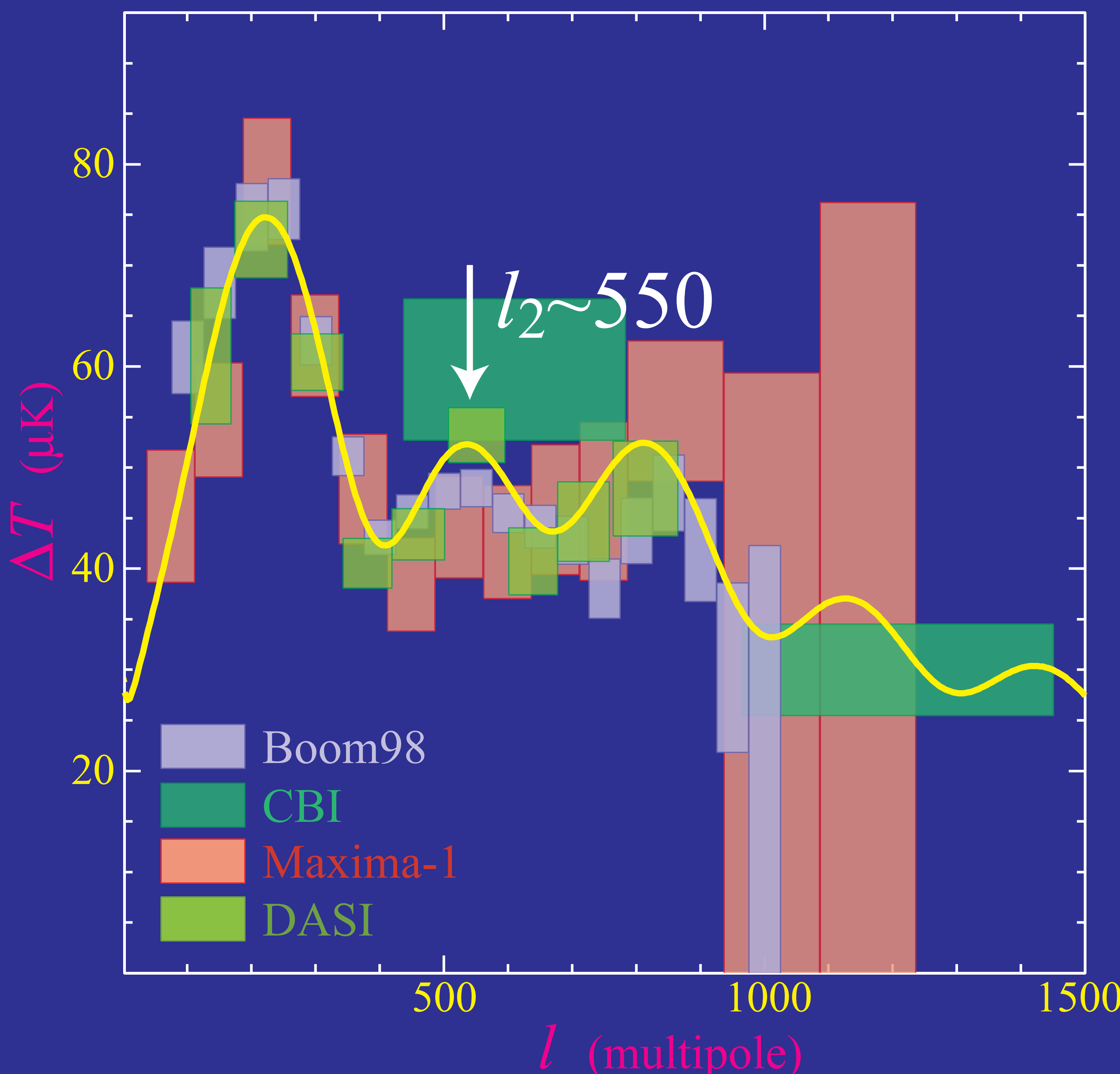
$$\begin{aligned}(\rho_\gamma + p_\gamma)v_\gamma + (\rho_b + p_b)v_b &\approx (p_\gamma + p_\gamma + \rho_b + \rho_\gamma)v_\gamma \\ &= (1 + R)(\rho_\gamma + p_\gamma)v_{\gamma b}\end{aligned}$$

where the controlling parameter is the momentum density ratio:

$$R \equiv \frac{p_b + \rho_b}{p_\gamma + \rho_\gamma} \approx 30\Omega_b h^2 \left(\frac{a}{10^{-3}} \right)$$

of order unity at recombination

Second Peak First Measured



New Euler Equation

- Momentum density ratio enters as

$$[(1+R)(\rho_\gamma + p_\gamma)\mathbf{v}_{\gamma b}]^{\cdot} = -4\frac{\dot{a}}{a}(1+R)(\rho_\gamma + p_\gamma)\mathbf{v}_{\gamma b} - \nabla p_\gamma - (1+R)(\rho_\gamma + p_\gamma)\nabla\Psi$$

same as before except for $(1+R)$ terms so

$$[(1+R)v_{\gamma b}]^{\cdot} = k\Theta + (1+R)k\Psi$$

- Photon continuity remains the same

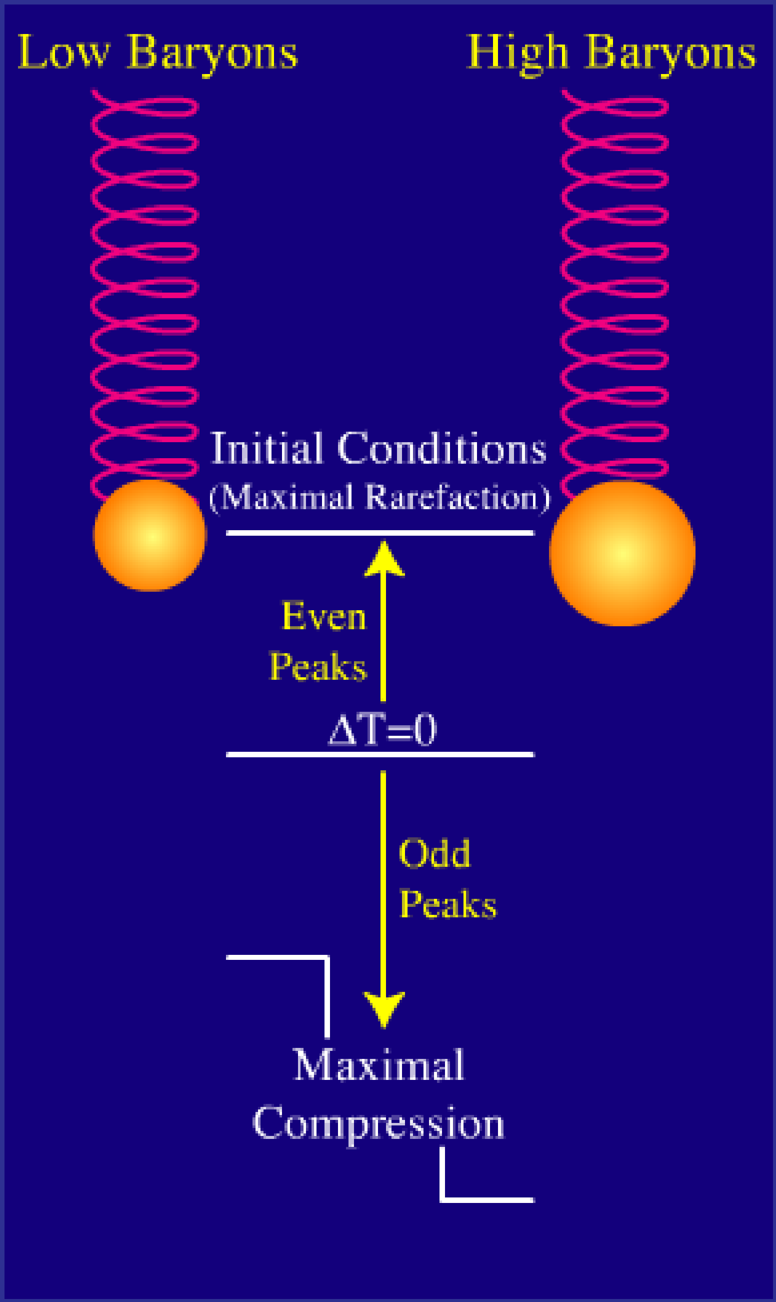
$$\dot{\Theta} = -\frac{k}{3}v_{\gamma b} - \dot{\Phi}$$

- Modification of oscillator equation

$$[(1+R)\dot{\Theta}]^{\cdot} + \frac{1}{3}k^2\Theta = -\frac{1}{3}k^2(1+R)\Psi - [(1+R)\dot{\Phi}]^{\cdot}$$

Baryon & Inertia

- Baryons add inertia to the fluid
- Equivalent to adding mass on a spring
- Same initial conditions
- Same null in fluctuations
- Unequal amplitudes of extrema



Oscillator: Take Three

- Combine these to form the not-quite-so simple harmonic oscillator equation

$$c_s^2 \frac{d}{d\eta}(c_s^{-2}\dot{\Theta}) + c_s^2 k^2 \Theta = -\frac{k^2}{3}\Psi - c_s^2 \frac{d}{d\eta}(c_s^{-2}\dot{\Phi})$$

where $c_s^2 \equiv \dot{p}_{\gamma b}/\dot{\rho}_{\gamma b}$

$$c_s^2 = \frac{1}{3}\frac{1}{1+R}$$

- In a CDM dominated expansion $\dot{\Phi} = \dot{\Psi} = 0$ and the adiabatic approximation $\dot{R}/R \ll \omega = kc_s$

$$[\Theta + (1+R)\Psi](\eta) = [\Theta + (1+R)\Psi](0)\cos(ks)$$

Baryon Peak Phenomenology

- Photon-baryon ratio enters in three ways
- Overall larger amplitude:

$$[\Theta + (1+R)\Psi](0) = \frac{1}{3}(1+3R)\Psi(0)$$

- Even-odd peak modulation of effective temperature

$$[\Theta + \Psi]_{\rm peaks} = [\pm(1+3R) - 3R]\,\frac{1}{3}\Psi(0)$$

$$[\Theta + \Psi]_1 - [\Theta + \Psi]_2 = [-6R]\frac{1}{3}\Psi(0)$$

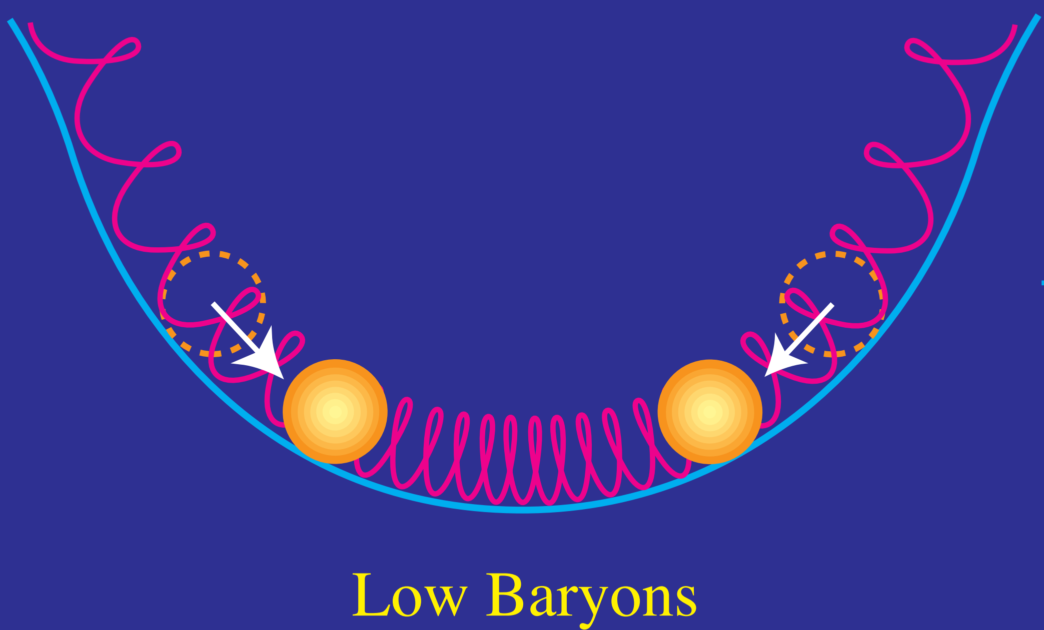
- Shifting of the sound horizon down or ℓ_A up

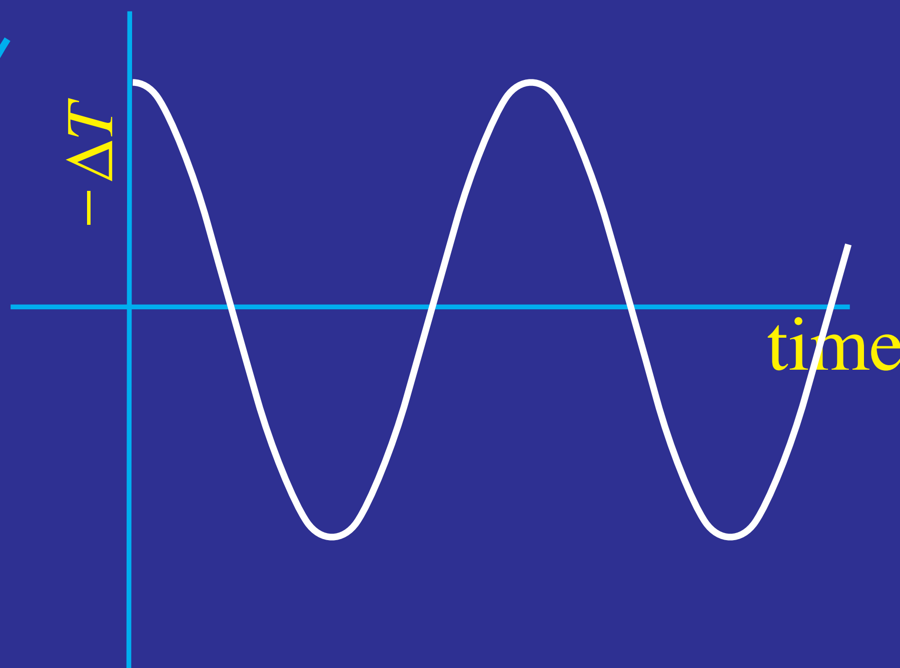
$$\ell_A \propto \sqrt{1+R}$$

- Actual effects smaller since R evolves

A Baryon-meter

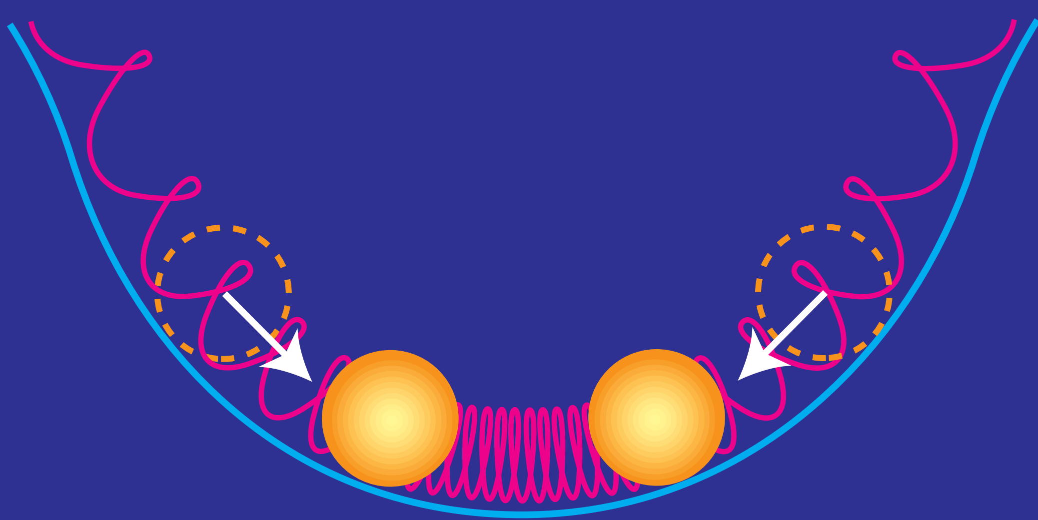
- Low baryons: symmetric compressions and rarefactions



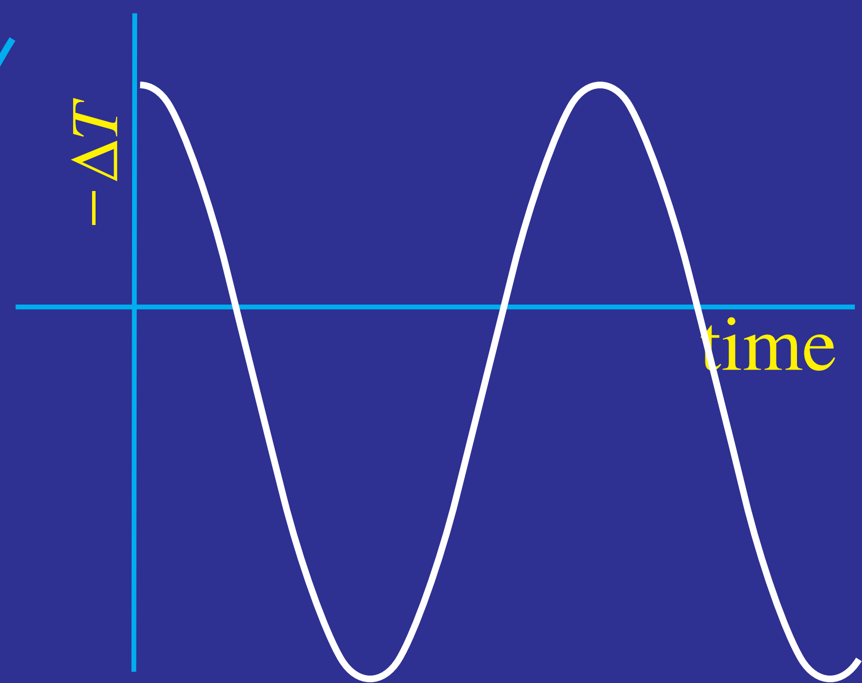


A Baryon-meter

- Load the fluid adding to gravitational force
- Enhance compressional peaks (odd) over rarefaction peaks (even)



Baryon Loading



A Baryon-meter

- Enhance compressional peaks (odd) over rarefaction peaks (even)

e.g. relative suppression of second peak

Photon Baryon Ratio Evolution

- Oscillator equation has time evolving mass

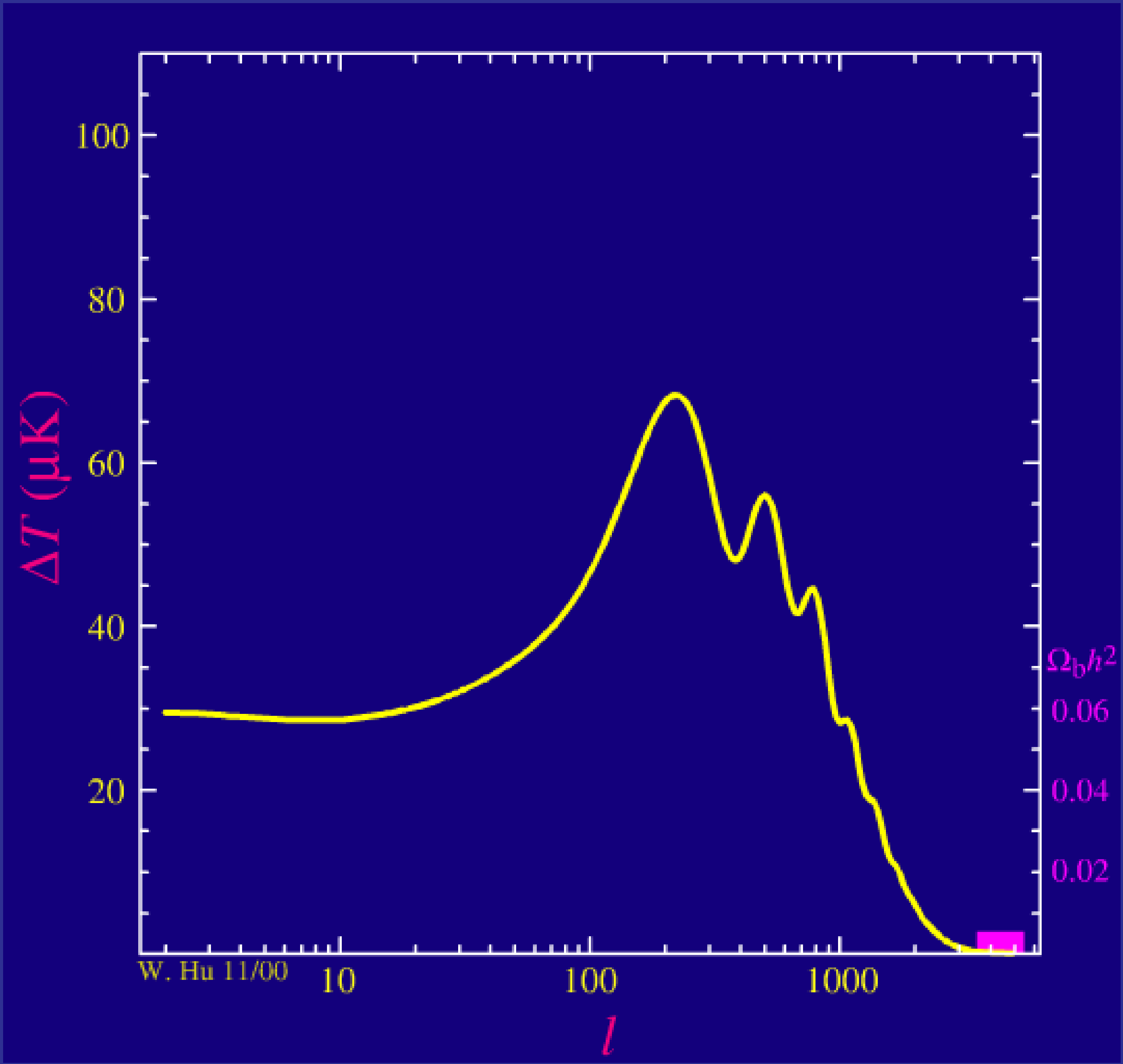
$$c_s^2 \frac{d}{d\eta}(c_s^{-2}\dot{\Theta}) + c_s^2 k^2 \Theta = 0$$

- Effective mass is is $m_{\rm eff} = 3c_s^{-2} = (1 + R)$
- Adiabatic invariant

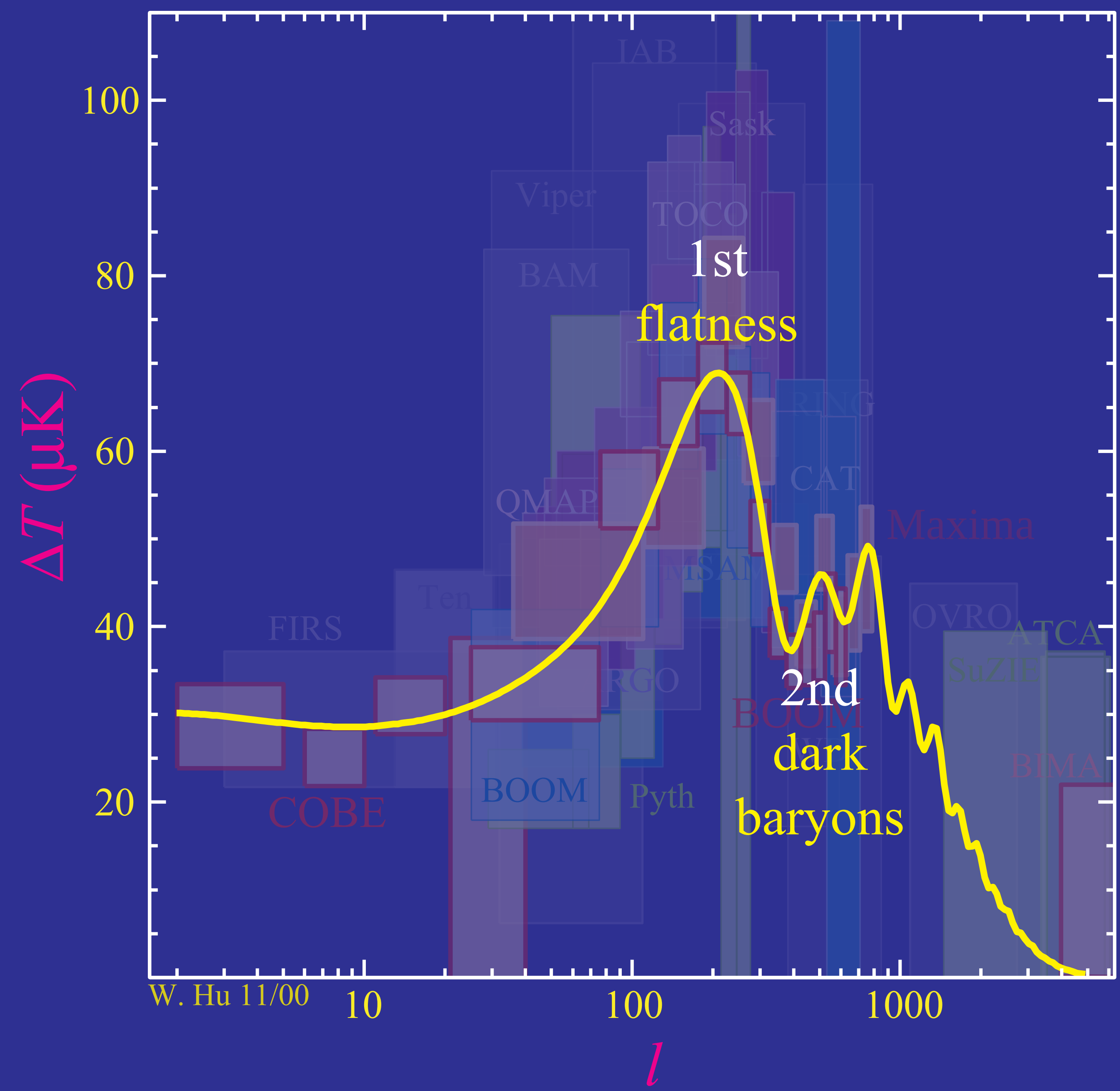
$$\frac{E}{\omega} = \frac{1}{2} m_{\rm eff} \omega A^2 = \frac{1}{2} 3 c_s^{-2} k c_s A^2 \propto A^2 (1 + R)^{1/2} = const.$$

- Amplitude of oscillation $A \propto (1 + R)^{-1/4}$ decays adiabatically as the photon-baryon ratio changes

Baryons in the Power Spectrum

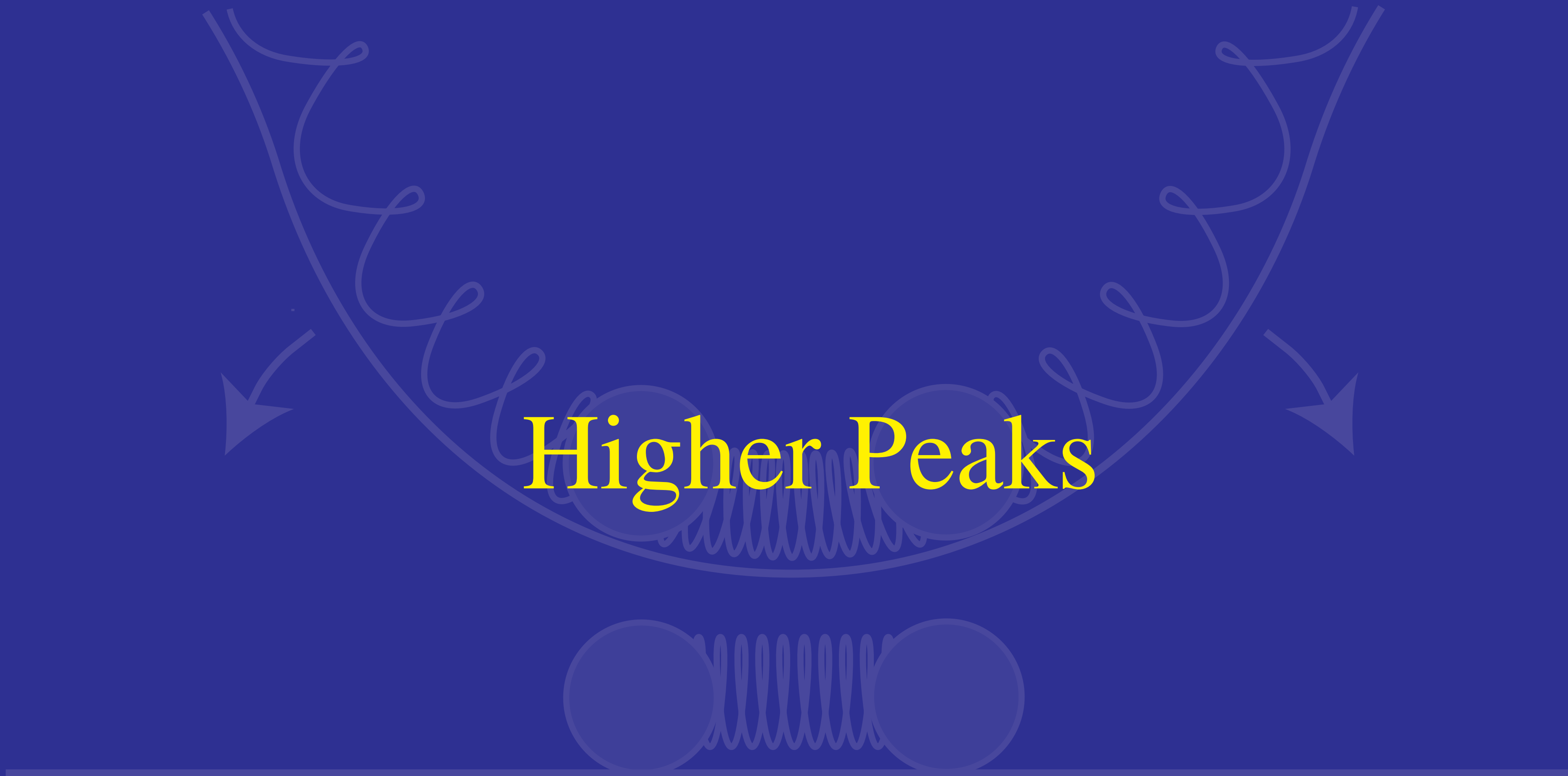


Score Card



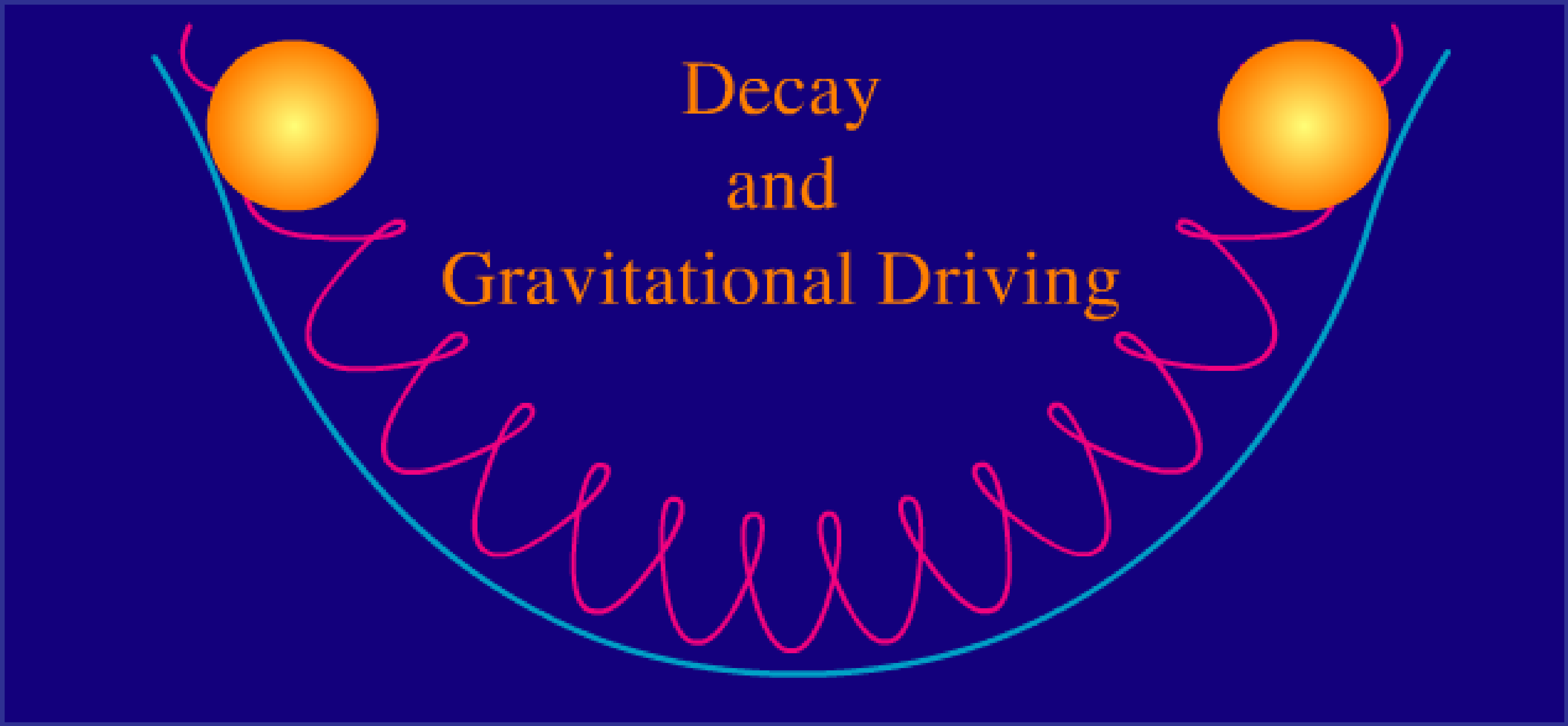
IAB
Sask
Viper
TOCO
BAM
1st
flatness
RING
CAT
QMAP
Maxima
MSAM
Ten
FIRS
OVRO
ATCA
SuZIE
RGO
2nd
BOOM
dark
baryons
BIMA
BOOM
Pyth
COBE
100
80
60
40
20
ΔT (μK)
W. Hu 11/00
10
100
1000
l

Higher Peaks



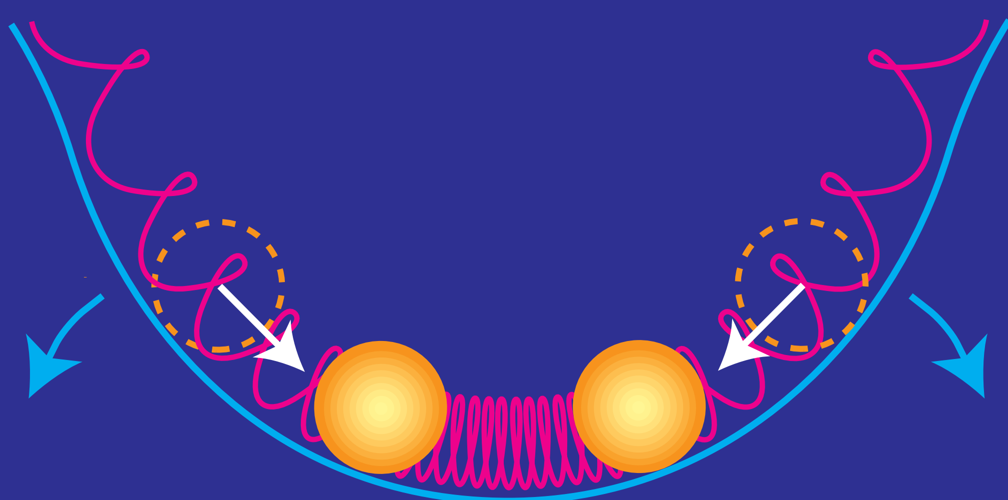
Radiation and Dark Matter

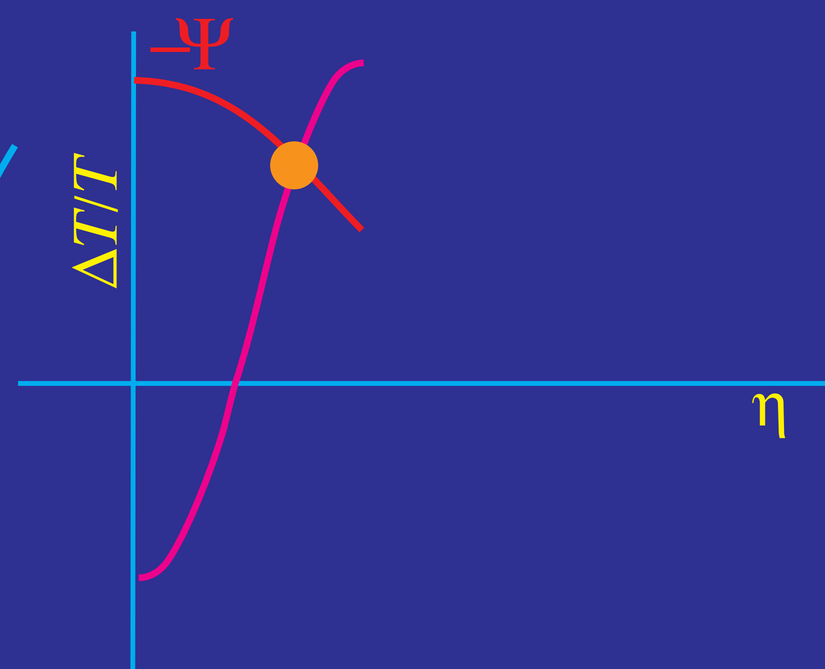
- Radiation domination:
  potential wells created by CMB itself
- Pressure support ⇒ potential decay ⇒ driving
- Heights measures when dark matter dominates



Driving Effects and Matter/Radiation

- Potential perturbation: $k^2\Psi = -4\pi G a^2 \delta\rho$ generated by radiation
- Radiation → Potential: inside sound horizon $\delta\rho/\rho$ pressure supported $\delta\rho$ hence Ψ decays with expansion





Hu & Sugiyama (1995)

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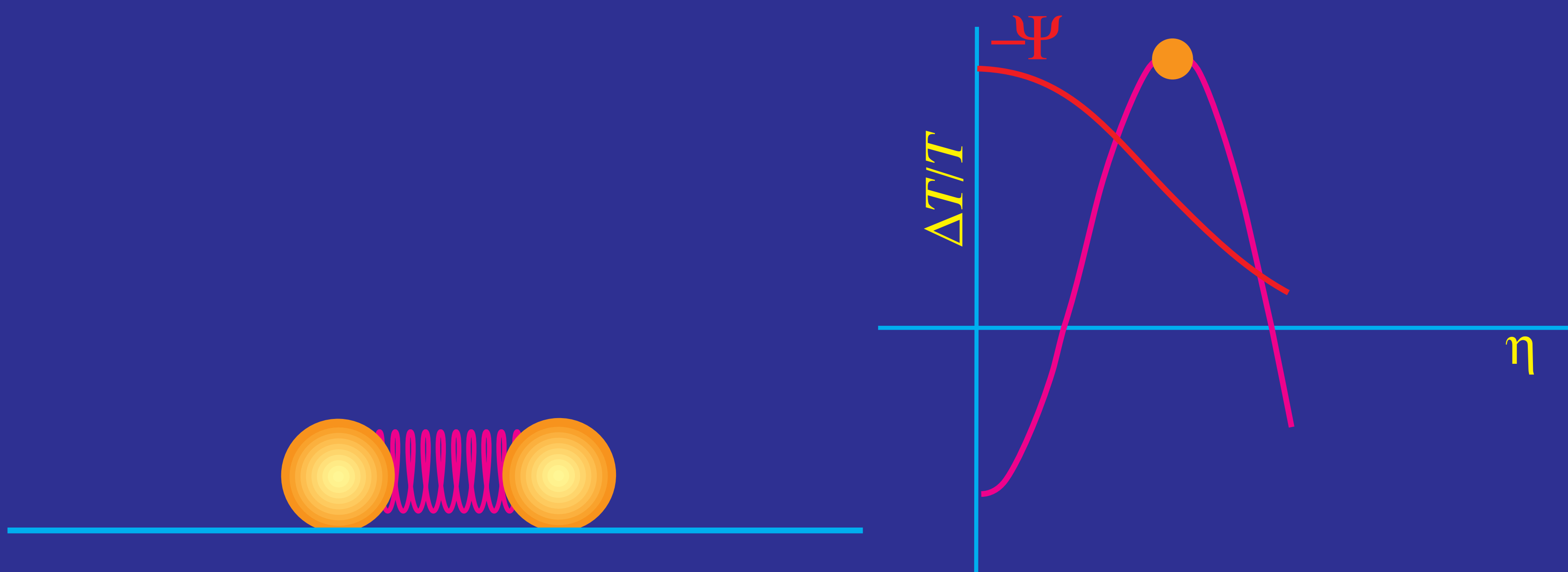
- Feedback stops at matter domination



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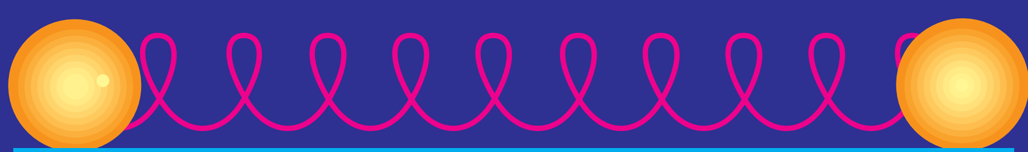
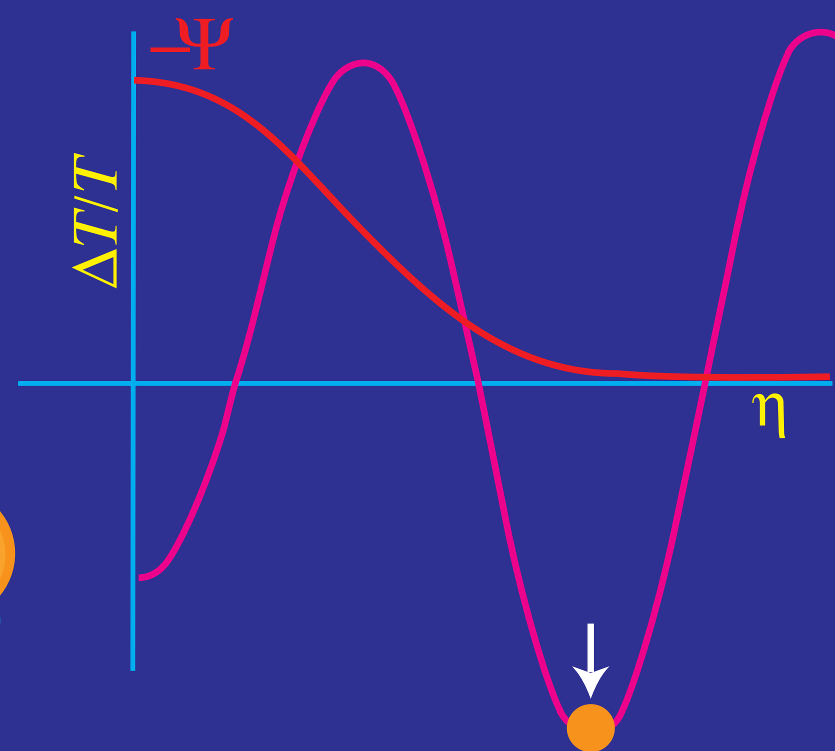




Hu & Sugiyama (1995)

Oscillator: Take Three and a Half

- The not-quite-so simple harmonic oscillator equation is a forced harmonic oscillator

$$c_s^2 \frac{d}{d\eta}(c_s^{-2}\dot{\Theta}) + c_s^2 k^2 \Theta = -\frac{k^2}{3}\Psi - c_s^2 \frac{d}{d\eta}(c_s^{-2}\Phi)$$

  changes in the gravitational potentials alter the form of the acoustic oscillations

- If the forcing term has a temporal structure that is related to the frequency of the oscillation, this becomes a driven harmonic oscillator

- Term involving Ψ is the ordinary gravitational force

- Term involving Φ involves the $\dot{\Phi}$ term in the continuity equation as a (curvature) perturbation to the scale factor

Potential Decay

- Matter-to-radiation ratio

$$\frac{\rho_m}{\rho_r} \approx 24\Omega_m h^2 \left(\frac{a}{10^{-3}}\right)$$

of order unity at recombination in a low Ω_m universe

- Radiation is not stress free and so impedes the growth of structure

$$k^2\Phi = 4\pi G a^2 \rho_r \Delta_r$$

$\Delta_r \sim 4\Theta$ oscillates around a constant value, $\rho_r \propto a^{-4}$ so the Netwonian curvature decays.

- General rule: potential decays if the dominant energy component has substantial stress fluctuations, i.e. below the generalized sound horizon or Jeans scale

Radiation Driving

- Decay is timed precisely to drive the oscillator - close to fully coherent

$$[\Theta + \Psi](\eta) = [\Theta + \Psi](0) + \Delta\Psi - \Delta\Phi$$
$$= \frac{1}{3}\Psi(0) - 2\Psi(0) = \frac{5}{3}\Psi(0)$$

- $5\times$ the amplitude of the Sachs-Wolfe effect!
- Coherent approximation is exact for a photon-baryon fluid but reality is reduced to $\sim 4\times$ because of neutrino contribution to radiation
- Actual initial conditions are $\Theta + \Psi = \Psi/2$ for radiation domination but comparison to matter dominated SW correct

External Potential Approach

- Solution to homogeneous equation

$$(1+R)^{-1/4}\cos(ks)\,, \qquad (1+R)^{-1/4}\sin(ks)$$

- Give the general solution for an external potential by propagating impulsive forces

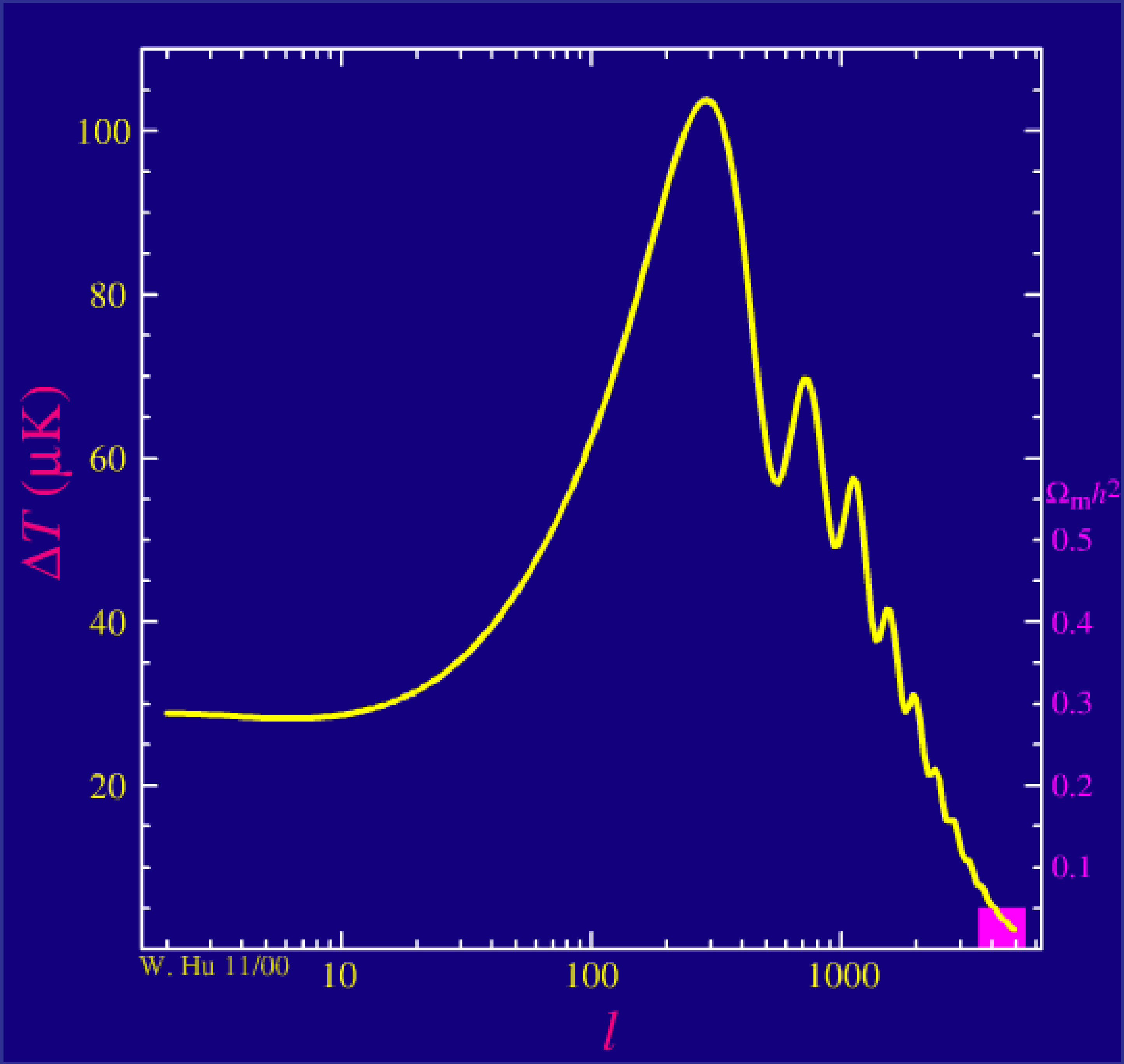
$$(1+R)^{1/4}\Theta(\eta) = \Theta(0)\cos(ks) + \frac{\sqrt{3}}{k}\left[\dot\Theta(0) + \frac{1}{4}\dot R(0)\Theta(0)\right]\sin ks$$
$$+ \frac{\sqrt{3}}{k}\int_0^\eta d\eta'(1+R')^{3/4}\sin[ks - ks']F(\eta')$$

where

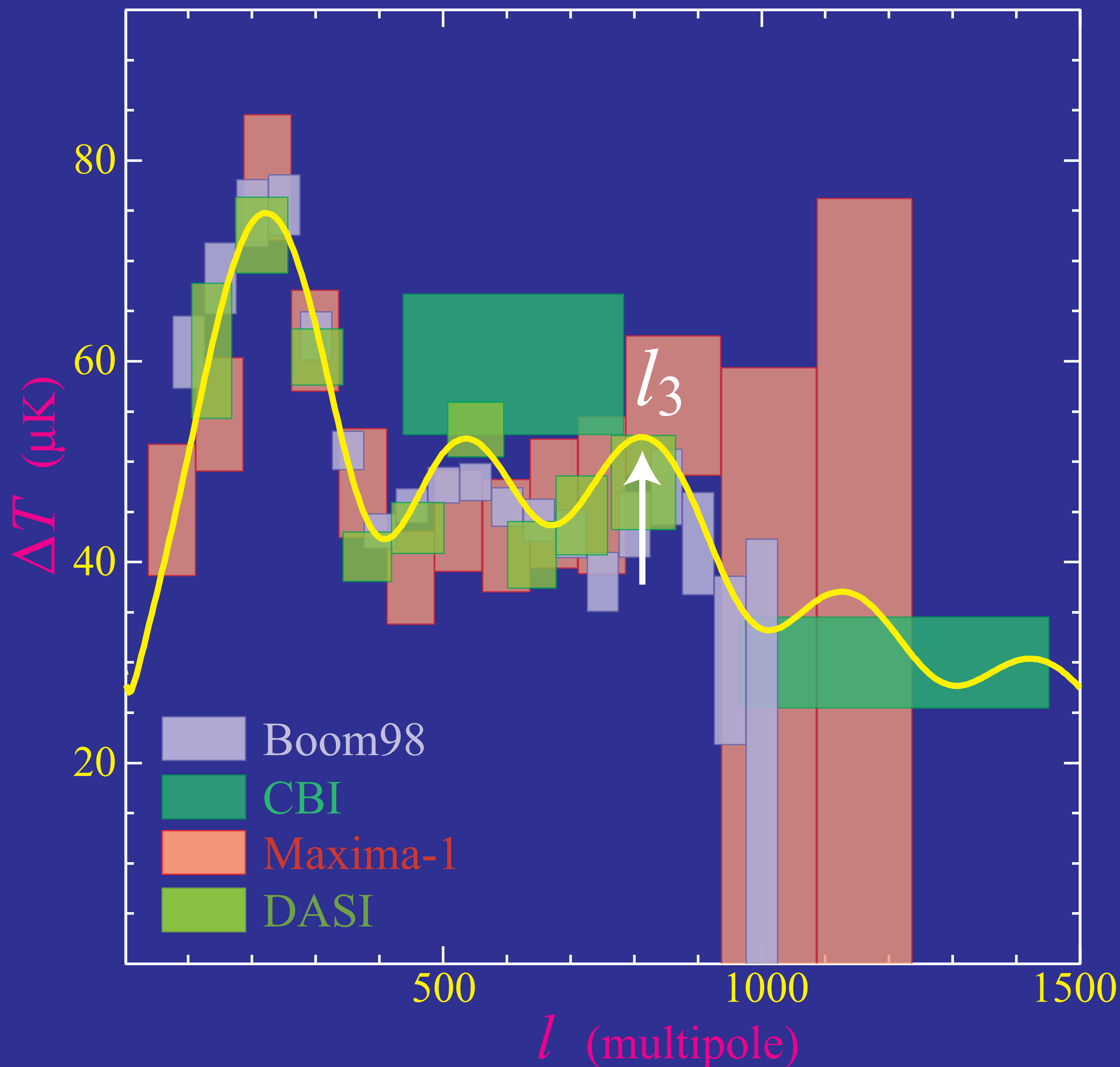
$$F = -\ddot\Phi - \frac{\dot R}{1+R}\dot\Phi - \frac{k^2}{3}\Psi$$

- Useful if general form of potential evolution is known

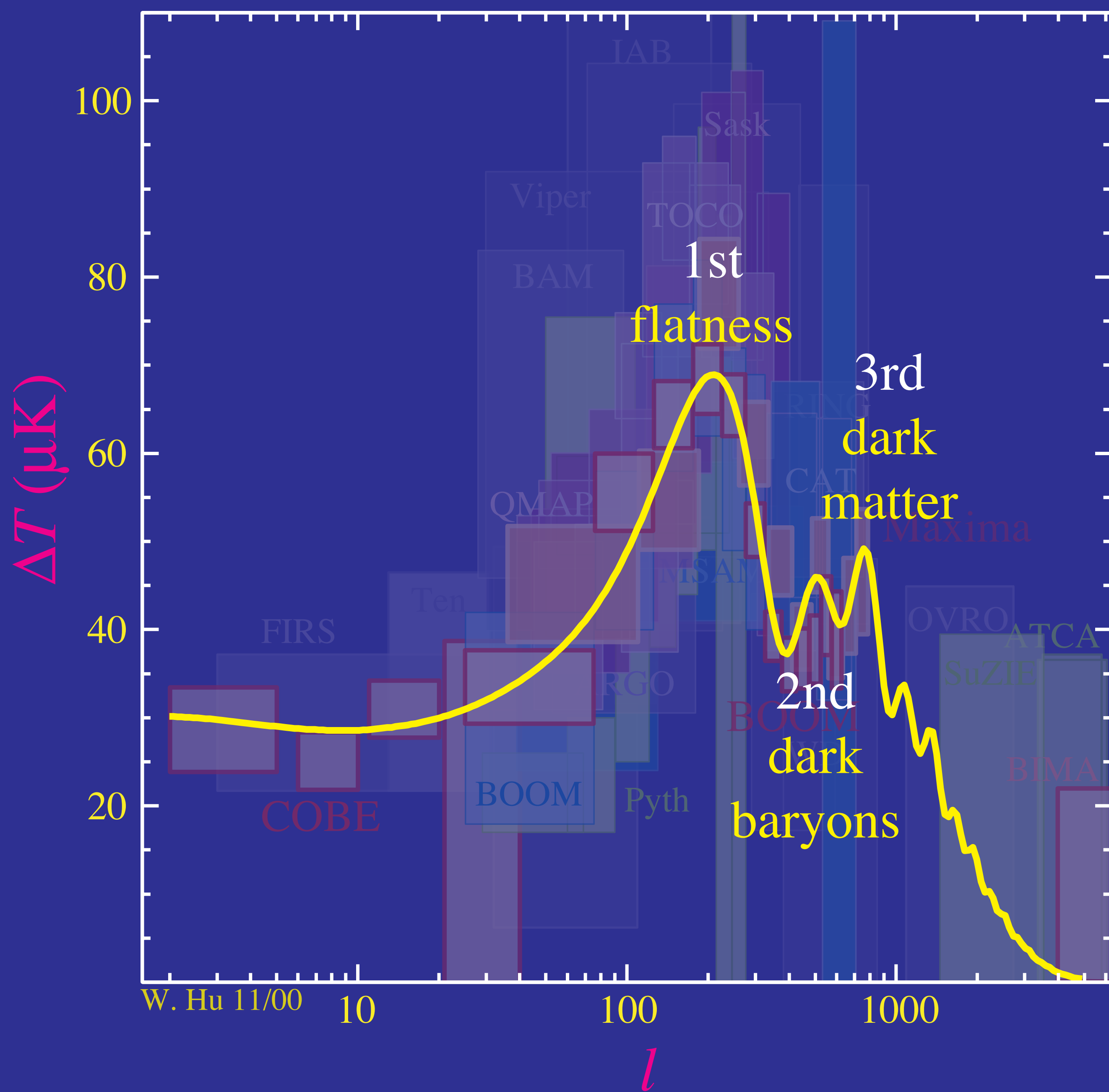
Dark Matter in the Power Spectrum



Third Peak First Measured



Score Card

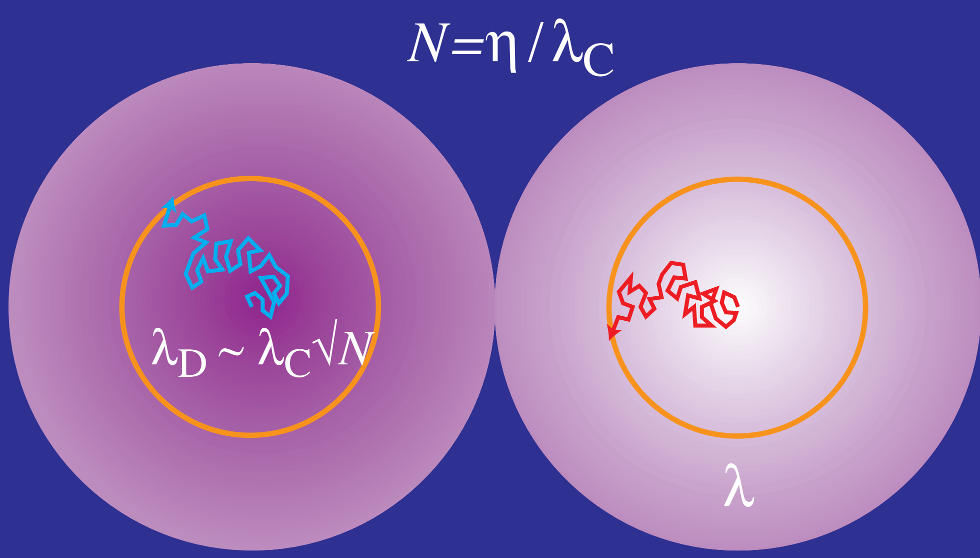


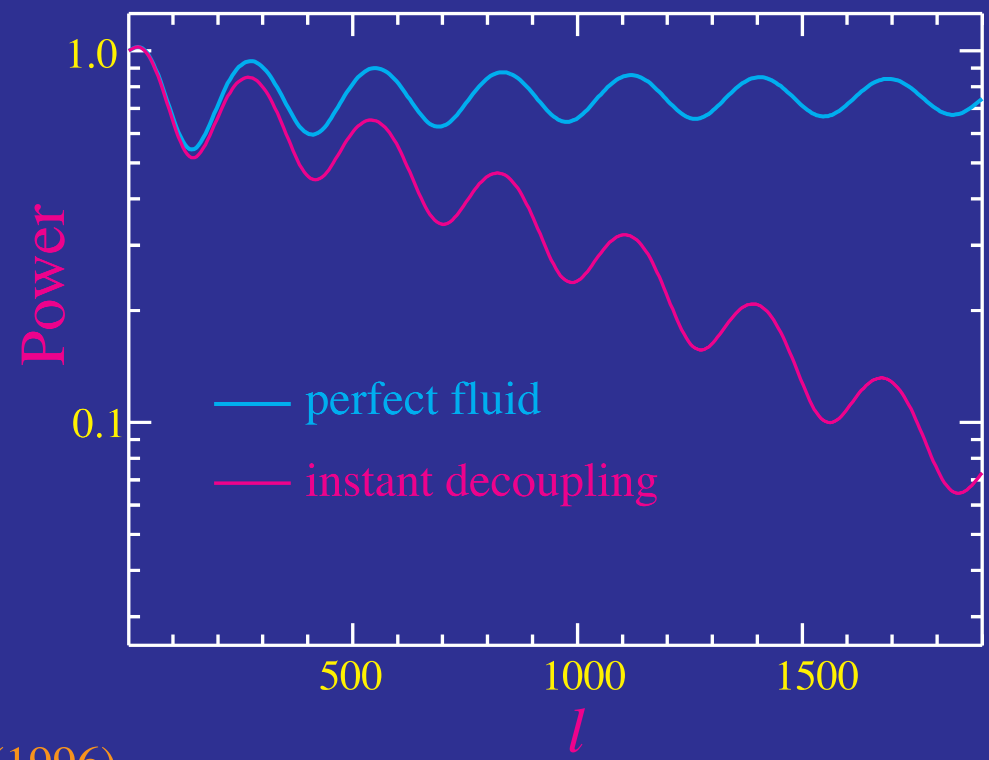
ΔT (μK)
100
80
60
40
20
10
100
1000
l
W. Hu 11/00
1st
flatness
2nd
dark
baryons
3rd
dark
matter
IAB
Sask
Viper
TOCO
BAM
QMAP
Ten
FIRS
COBE
BOOM
Pyth
RGO
CAT
Maxima
OVRO
ATCA
SuZIE
BIMA

Damping Tail

Dissipation / Diffusion Damping

- Imperfections in the coupled fluid → mean free path λ_C in the baryons
- Random walk over diffusion scale: geometric mean of mfp & horizon
  $\lambda_D \sim \lambda_C \sqrt{N} \sim \sqrt{\lambda_C \eta} \gg \lambda_C$
- Overtake wavelength: $\lambda_D \sim \lambda$; second order in λ_C/λ
- Viscous damping for R<1; heat conduction damping for R>1

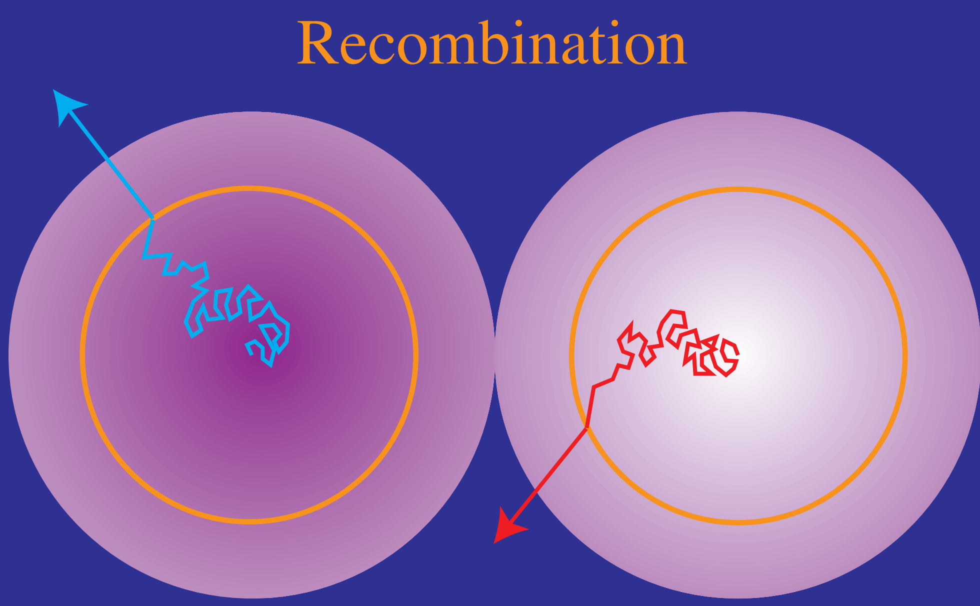


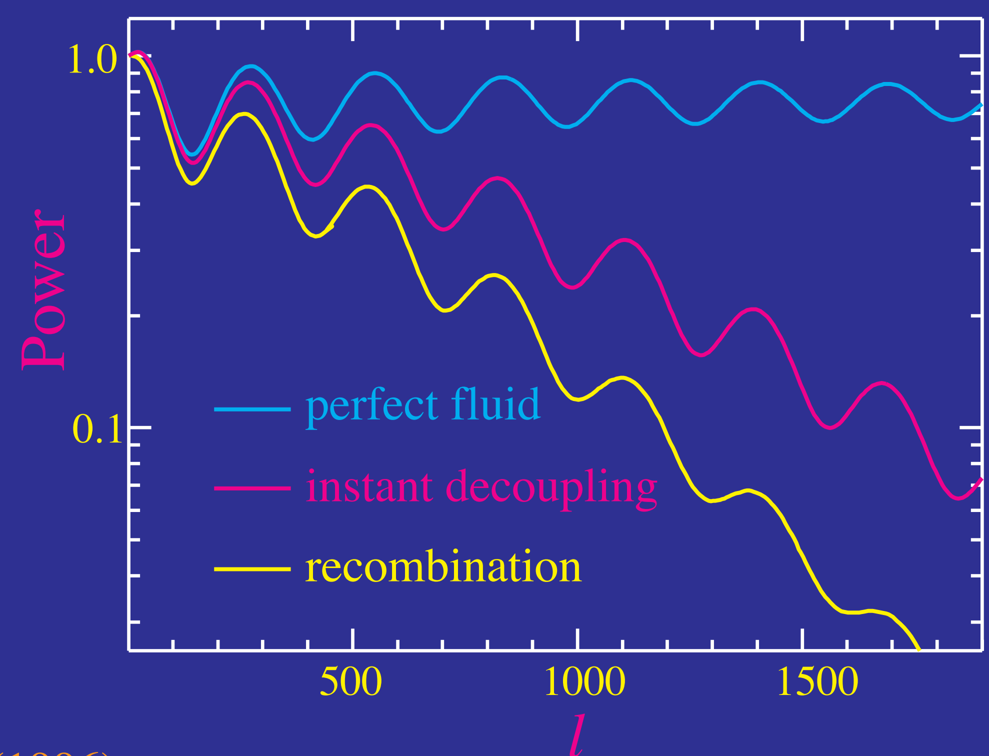


Silk (1968); Hu & Sugiyama (1995); Hu & White (1996)

Dissipation / Diffusion Damping

- Rapid increase at recombination as mfp ↑
- Independent of (robust to changes in) perturbation spectrum
- Robust physical scale for angular diameter distance test (Ω_K, Ω_Λ)





Silk (1968); Hu & Sugiyama (1995); Hu & White (1996)

Damping

- Tight coupling equations assume a perfect fluid: no viscosity, no heat conduction
- Fluid imperfections are related to the mean free path of the photons in the baryons

$$\lambda_C = \dot\tau^{-1} \quad \text{where} \quad \dot\tau = n_e \sigma_T a$$

  is the conformal opacity to Thomson scattering
- Dissipation is related to the diffusion length: random walk approximation

$$\lambda_D = \sqrt{N}\lambda_C = \sqrt{\eta/\lambda_C}\,\lambda_C = \sqrt{\eta\lambda_C}$$

  the geometric mean between the horizon and mean free path
- $\lambda_D/\eta_* \sim$ few %, so expect the peaks :>3 to be affected by dissipation

Equations of Motion

- Continuity

$$\dot{\Theta} = -\frac{k}{3} v_\gamma - \dot{\Phi}\,, \qquad \dot{\delta}_b = -k v_b - 3\dot{\Phi}$$

where the photon equation remains unchanged and the baryons follow number conservation with $\rho_b = m_b n_b$

- Euler

$$\begin{aligned} \dot{v}_\gamma &= k(\Theta + \Psi) - \frac{k}{6}\pi_\gamma - \dot{\tau}(v_\gamma - v_b) \\ \dot{v}_b &= -\frac{\dot{a}}{a} v_b + k\Psi + \dot{\tau}(v_\gamma - v_b)/R \end{aligned}$$

where the photons gain an anisotropic stress term π_γ from radiation viscosity and a momentum exchange term with the baryons and are compensated by the opposite term in the baryon Euler equation

Viscosity

- Viscosity is generated from radiation streaming from hot to cold regions
- Expect

$$\pi_\gamma \sim v_\gamma \frac{k}{\dot\tau}$$

generated by streaming, suppressed by scattering in a wavelength of the fluctuation. Radiative transfer says

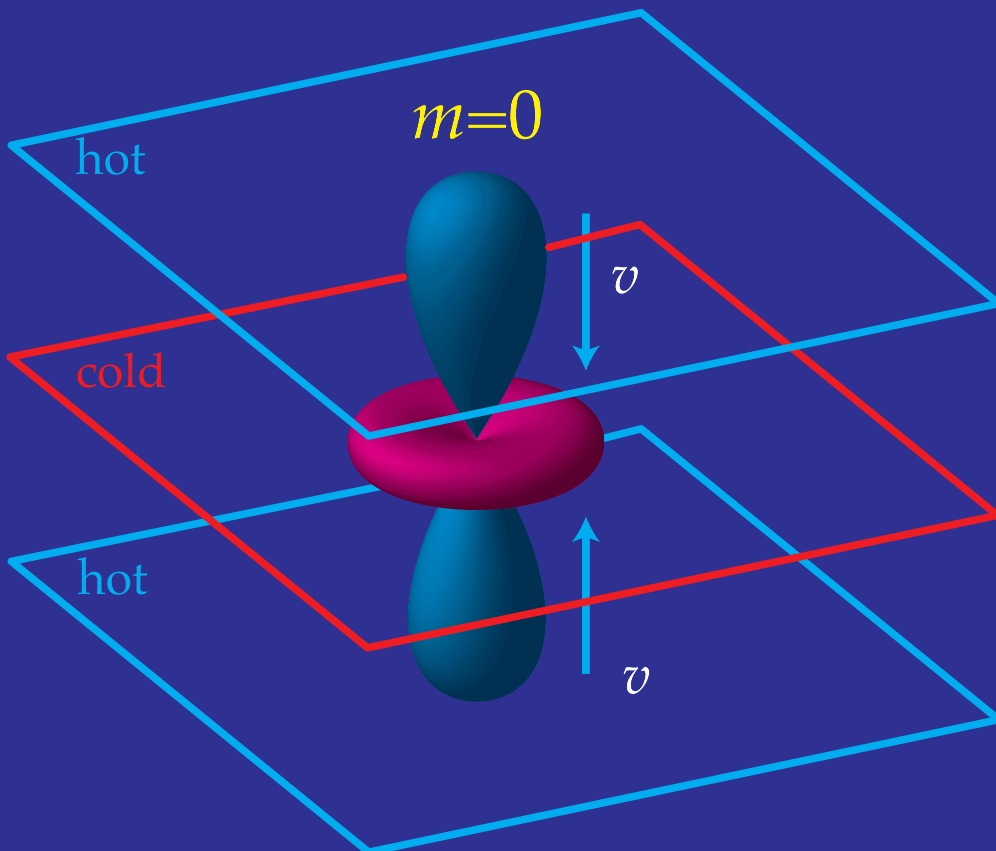
$$\pi_\gamma \approx 2 A_v v_\gamma \frac{k}{\dot\tau}$$

where $A_v = 16/15$

$$\dot v_\gamma = k(\Theta + \Psi) - \frac{k}{3} A_v \frac{k}{\dot\tau} v_\gamma$$

Viscosity & Heat Conduction

- Both fluid imperfections are related to the gradient of the velocity kv_γ by opacity $\dot\tau$: slippage of fluids $v_\gamma - v_b$.
- Viscosity is an anisotropic stress or quadrupole moment formed by radiation streaming from hot to cold regions

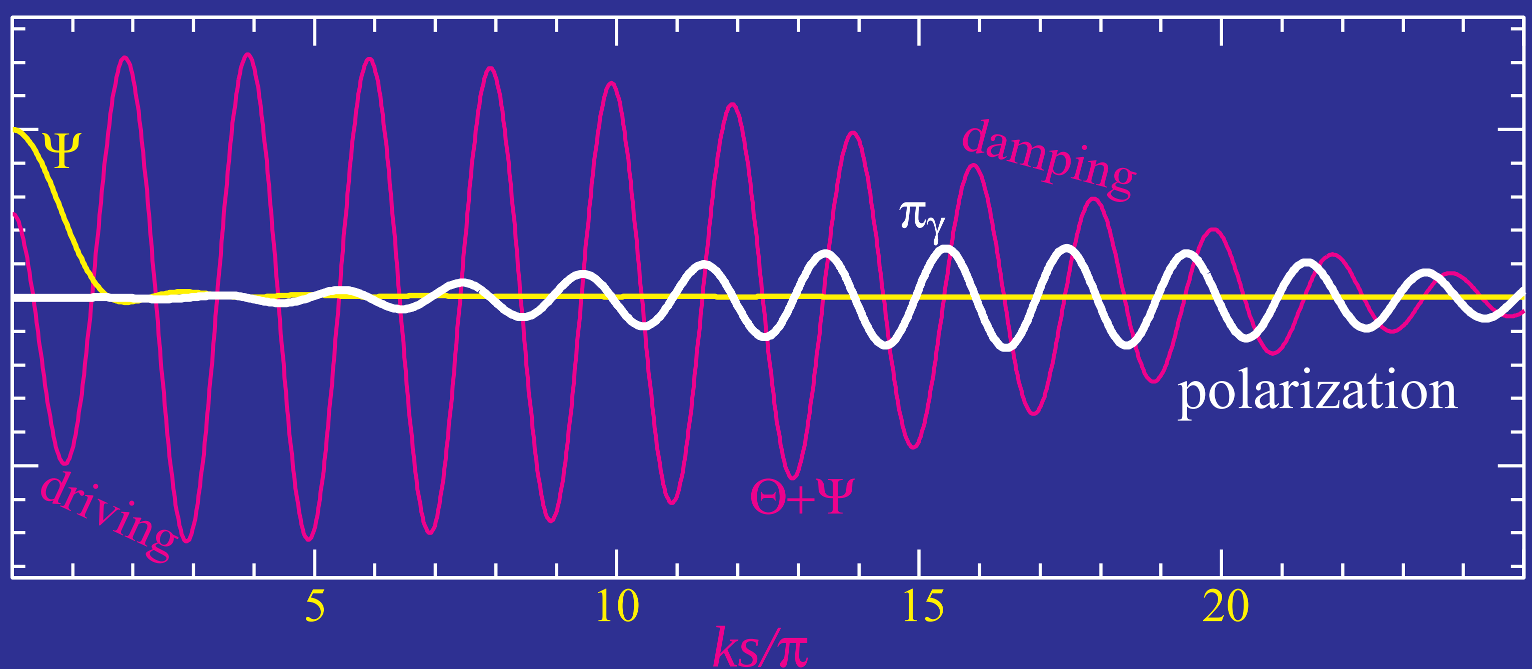


Damping & Viscosity

- Quadrupole moments:

  damp acoustic oscillations from fluid viscosity
  generates polarization from scattering (next lecture)

- Rise in polarization power coincides with fall in temperature power – $l \sim 1000$



Oscillator: Penultimate Take

- Adiabatic approximation ($\omega \gg \dot{a}/a$)

$$\dot{\Theta} \approx -\frac{k}{3} v_\gamma$$

- Oscillator equation contains a $\dot{\Theta}$ damping term

$$c_s^2 \frac{d}{d\eta}(c_s^{-2}\dot{\Theta}) + \frac{k^2 c_s^2}{\dot{\tau}} A_v \dot{\Theta} + k^2 c_s^2 \Theta = -\frac{k^2}{3}\Psi - c_s^2 \frac{d}{d\eta}(c_s^{-2}\dot{\Phi})$$

- Heat conduction term similar in that it is proportional to v_γ and is suppressed by scattering $k/\dot{\tau}$. Expansion of Euler equations to leading order in $k/\dot{\tau}$ gives

$$A_h = \frac{R^2}{1+R}$$

since the effects are only significant if the baryons are dynamically important

Oscillator: Final Take

- Final oscillator equation

$$c_s^2 \frac{d}{d\eta}(c_s^{-2}\dot{\Theta}) + \frac{k^2 c_s^2}{\dot{\tau}}[A_v + A_h]\dot{\Theta} + k^2 c_s^2 \Theta = -\frac{k^2}{3}\Psi - c_s^2 \frac{d}{d\eta}(c_s^{-2}\dot{\Phi})$$

- Solve in the adiabatic approximation

$$\Theta \propto \exp(i \int \omega d\eta)$$

$$-\omega^2 + \frac{k^2 c_s^2}{\dot{\tau}}(A_v + A_h) i\omega + k^2 c_s^2 = 0$$

Dispersion Relation

- Solve

$$\omega^2 = k^2 c_s^2 \left[1 + i\frac{\omega}{\dot\tau}(A_v + A_h)\right]$$
$$\omega = \pm k c_s \left[1 + \frac{i}{2}\frac{\omega}{\dot\tau}(A_v + A_h)\right]$$
$$= \pm k c_s \left[1 \pm \frac{i}{2}\frac{k c_s}{\dot\tau}(A_v + A_h)\right]$$

- Exponentiate

$$\exp(i\int \omega d\eta) = e^{\pm iks} \exp[-k^2 \int d\eta \frac{1}{2}\frac{c_s^2}{\dot\tau}(A_v + A_h)]$$
$$= e^{\pm iks} \exp[-(k/k_D)^2]$$

- Damping is exponential under the scale k_D

Diffusion Scale

- Diffusion wavenumber

$$k_D^{-2} = \int d\eta \frac{1}{\dot\tau} \frac{1}{6(1+R)} \left(\frac{16}{15} + \frac{R^2}{(1+R)} \right)$$

- Limiting forms

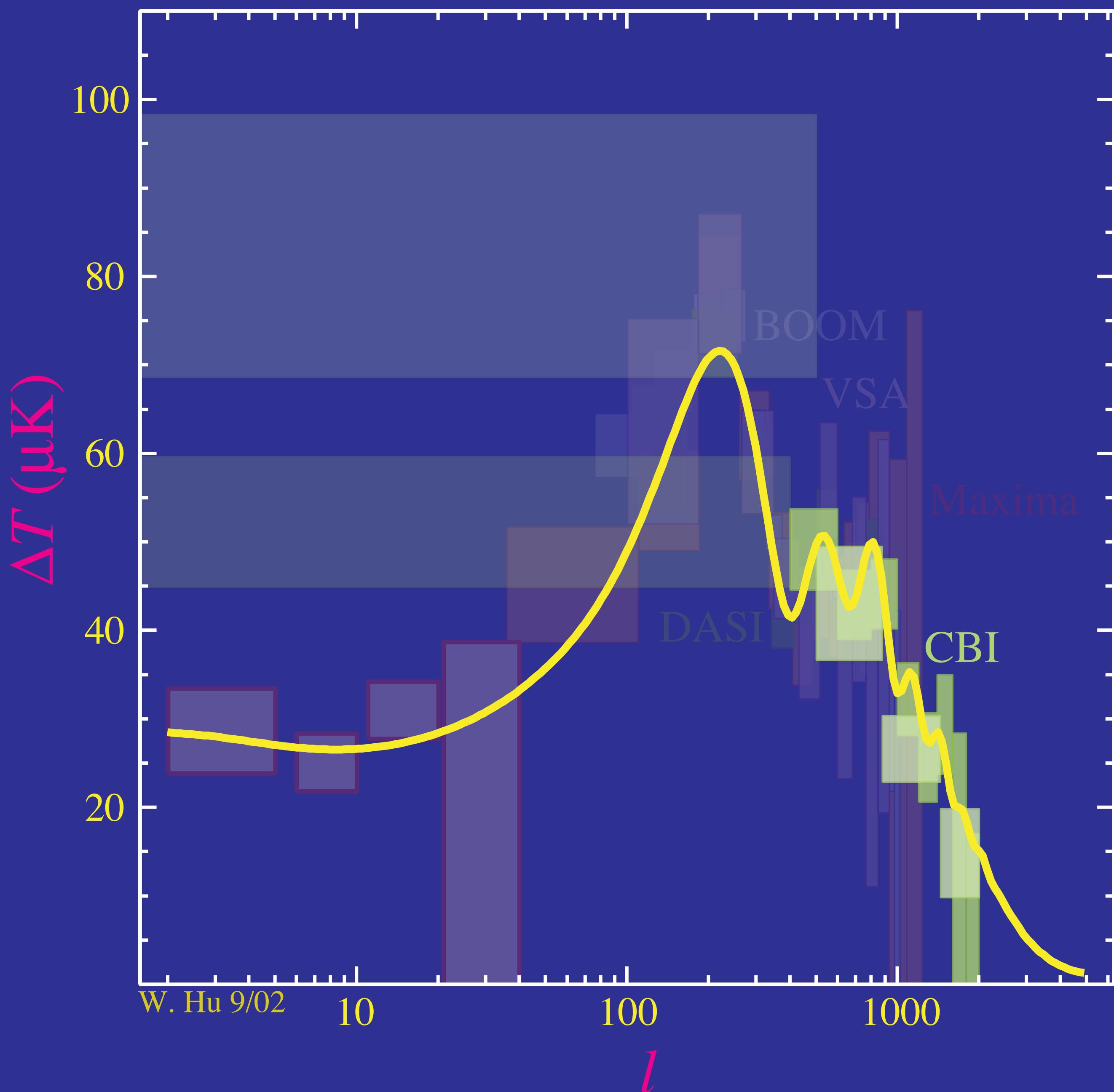
$$\lim_{R\to 0} k_D^{-2} = \frac{1}{6}\frac{16}{15} \int d\eta \frac{1}{\dot\tau}$$

$$\lim_{R\to \infty} k_D^{-2} = \frac{1}{6} \int d\eta \frac{1}{\dot\tau}$$

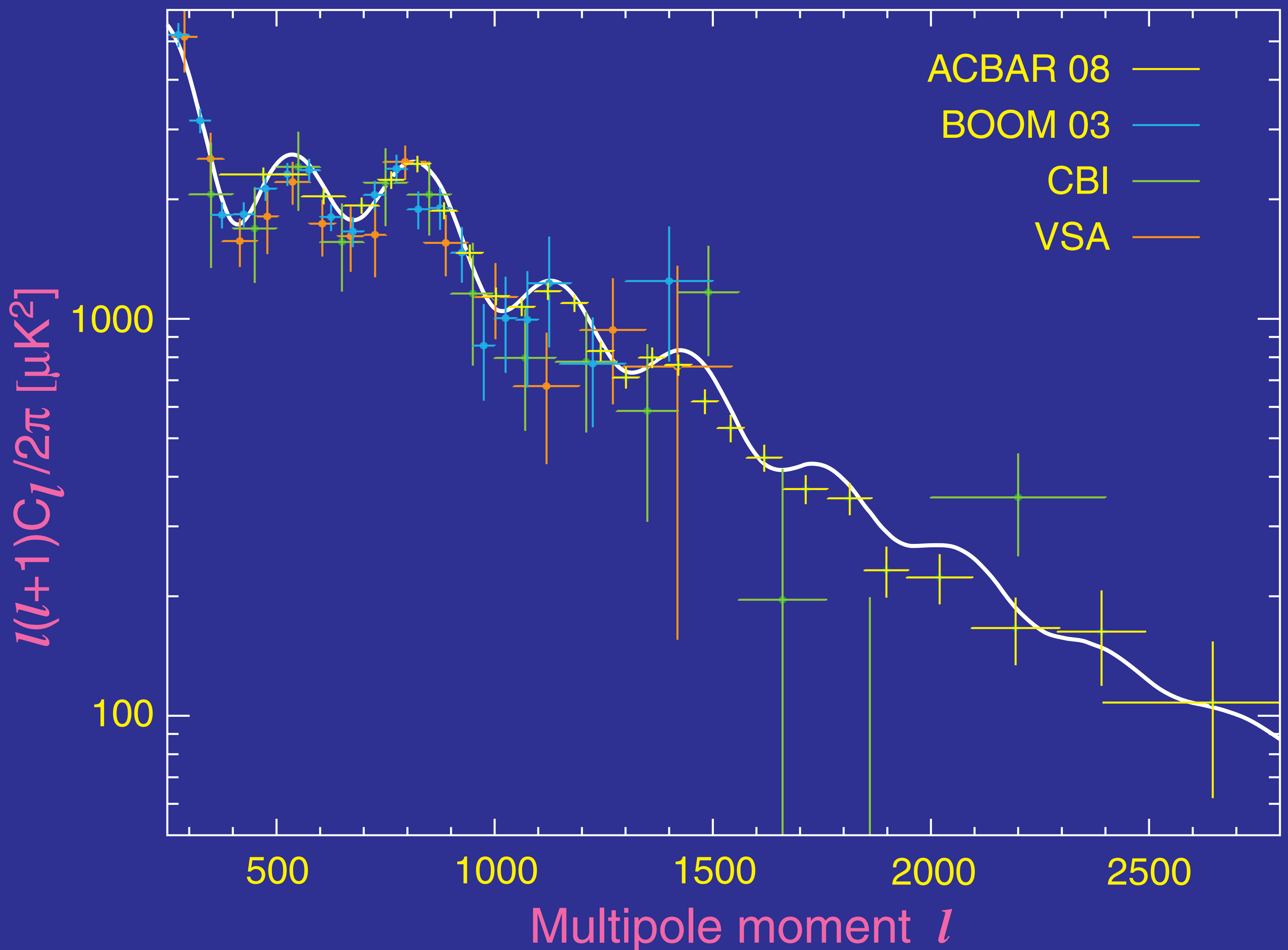
- Geometric mean between horizon and mean free path as expected from a random walk

$$\lambda_D = \frac{2\pi}{k_D} \sim \frac{2\pi}{\sqrt{6}} (\eta \dot\tau^{-1})^{1/2}$$

Damping Tail Measured

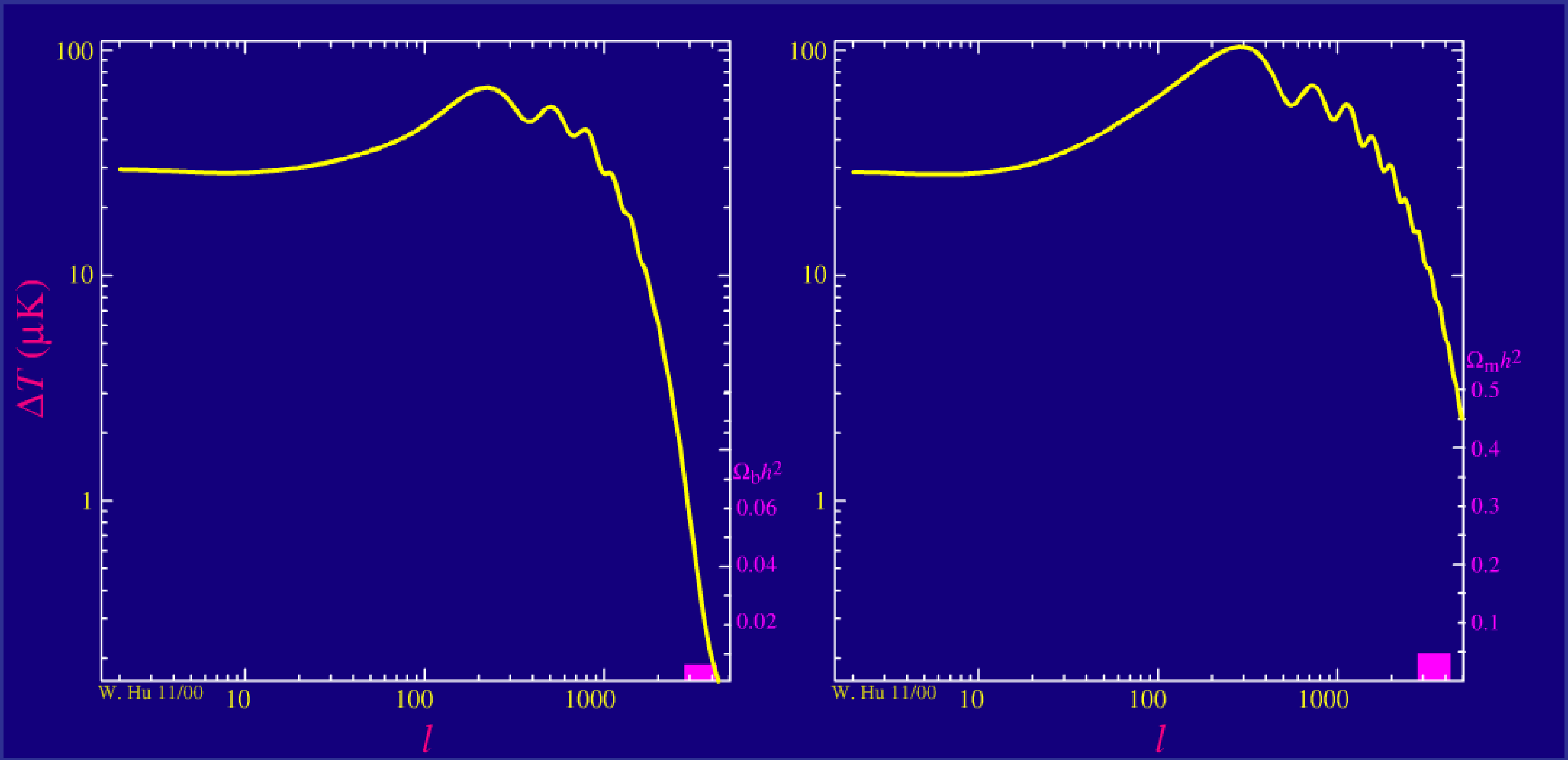


Power Spectrum Present



Standard Ruler

- Damping length is a fixed physical scale given properties at recombination
- Gemoetric mean of mean free path and horizon: depends on baryon-photon ratio and matter-radiation ratio



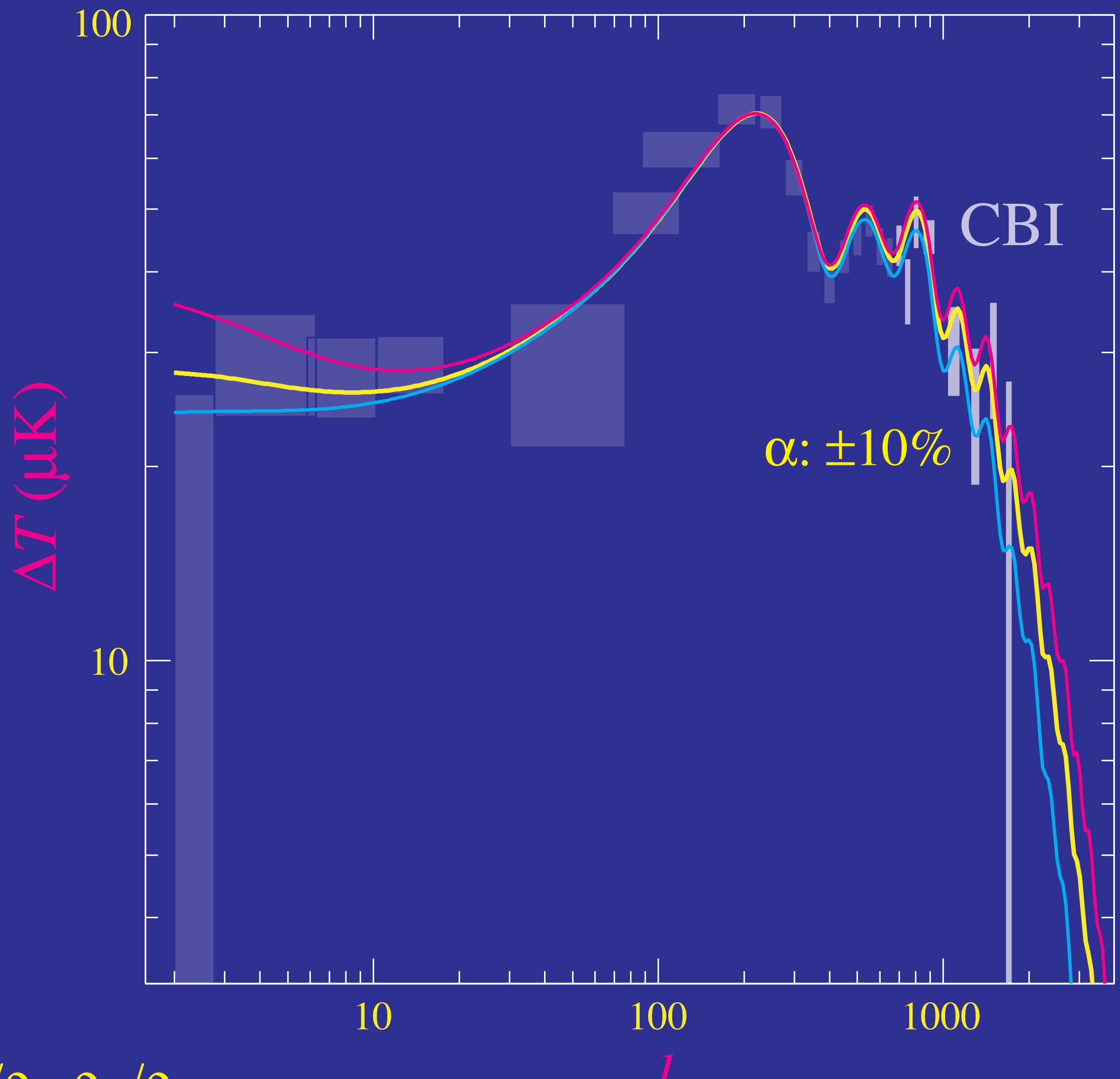
Standard Rulers

- Calibrating the Standard Rulers

- Sound Horizon

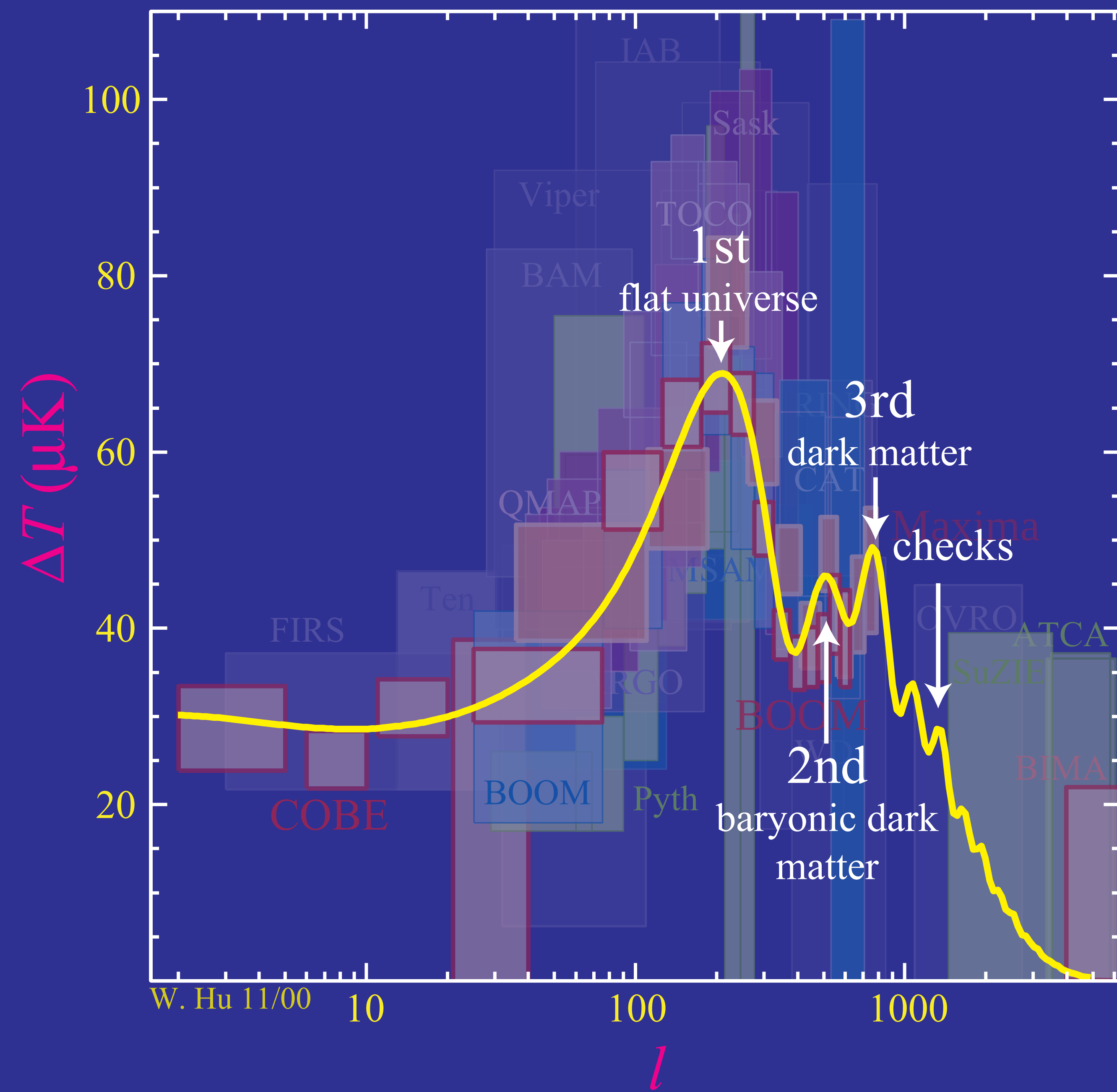
- Damping Scale

Consistency Check on Recombinaton



fixed l_A, ρ_b/ρ_γ, ρ_m/ρ_r

The Peaks



ΔT (μK)
100
80
60
40
20
10
100
1000
l
W. Hu 11/00
COBE
FIRS
Ten
BOOM
Pyth
RGO
QMAP
BAM
Viper
TOCO
Sask
IAB
CAT
MSAM
OVRO
ATCA
SuZIE
BIMA
Maxima
BOOM
1st
flat universe
2nd
baryonic dark
matter
3rd
dark matter
checks

Lecture I: Summary

- CMB photons emerge from the cosmic photosphere at $z \sim 10^3$ when the universe (re)combines
- Temperature inhomogeneity at recombination becomes anisotropy to the observer at present
- Initial temperature inhomogeneities oscillate as sound waves in the plasma
- Harmonic series of peaks based on the distance sound travels by recombination
- Distance can be calibrated if expansion history is known and baryon content known
- Angular scale measures the angular diameter distance to recombination involving the curvature and to a lesser extent the dark energy

Lecture I: Summary

- Gravitational potential redshift combines with gravitationally induced initial perturbation to form the Sachs-Wolfe effect
- Baryon loading enhances odd numbered peaks so that the ratio of first to second peak height determines the baryon density
- Decay of potentials during radiation domination drives oscillations so that the relative peak heights across the first three peaks determines the matter-radiation ratio
- Fluid imperfections due to viscosity (quadrupole stresses) and heat conduction dissipate acoustic waves in a manner predicted by baryon density and matter-radiation ratio
- Strong consistency checks for recombination physics, angular diameter distance and source of acoustic polarization