How To Exploit a Small Cryptographic Leakage

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Side Channel Attacks

- Are extremely powerful, and in many cases are the only practical way to break well designed cryptosystems
- Had been studied for more than a decade in academia, and for much longer by others
- Many types of side channel attacks are known, but each one needs different physical and mathematical techniques
- Still lacks a unifying framework

The typical Scenario Considered So Far:

- A new type of potential leakage is discovered, which provides a very small amount of very indirect information about the cryptographic key
- Specialized techniques have to be developed to extract the full key from a large number of measurements of this new source of information
- To apply it to a particular device, detailed information about the physical and logical implementation of the cryptosystem in that device is usually required
- The success of each attack is extremely sensitive to the existence of unknown countermeasures

Our Goal in This Paper:

- To develop a new type of side channel attack which can be universally applied to any device by exploiting any newly discovered source of leakage
- Applying the attack will not require detailed knowledge of the physical and logical implementation of the cryptosystem
- However, its success will not be guaranteed, and will have to be tested experimentally

Examples of Possible scenarios:

- We are given a chip, and can probe any wire in it. However, we have no idea what kind of data is passing through the wire during each cycle
- We can measure the total power consumption of the chip, but do not know how this power consumption is related to the instructions executed by the processor or to the data operated upon
- We can use a tiny antenna to measure the RF field near the surface of the chip, but do not know how this field is related to the crypto key

Leakage Attacks on Block Ciphers:

- Block ciphers are typically iterated, applying the same operations in each round to different values

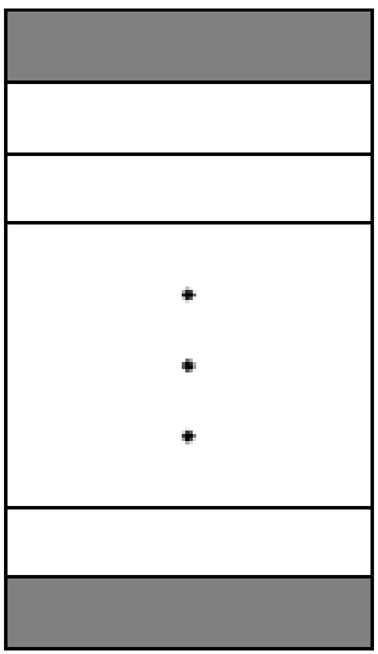
- Any type of physical leakage is likely to repeat itself in each round, and all these values will be available to the cryptanalyst

Leakage Attacks on Block Ciphers:

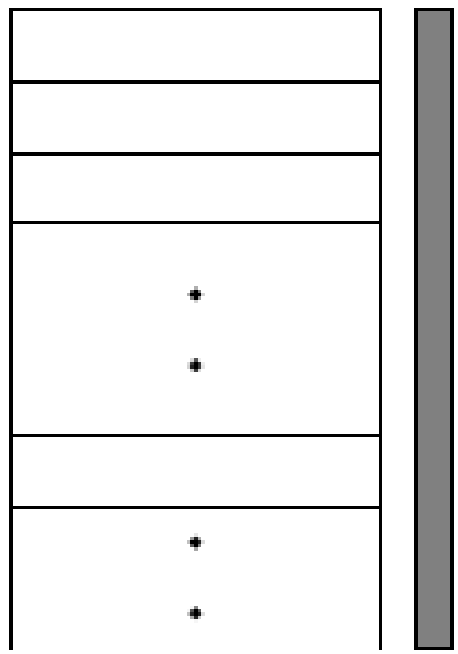
- The simplest type of leakage we consider is a single state bit, obtained e.g., by probing a single register cell or a single wire
- Another type of leakage is a single bit which is a simple function of many state bits, e.g., whether a carry occurred during an addition operation
- More complicated types of leakage can be multibit functions such as the Hamming weight of a byte written into memory

Information Available to the Attacker:

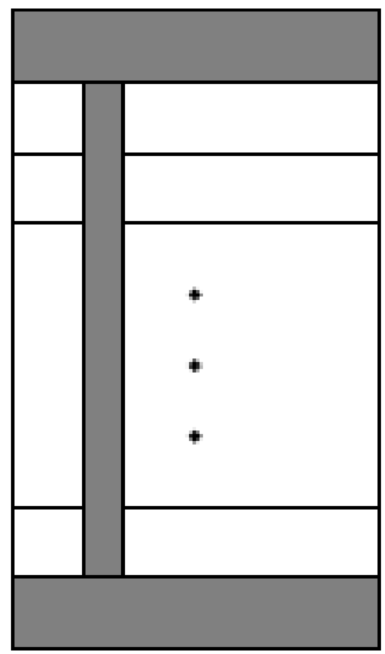
In block ciphers:



In stream ciphers:



In leakage attacks:



Which bits of information are useful?

- Single bits of information in successive rounds are difficult to relate to each other
- Our approach will be to relate a single bit of information to the fully known plaintext or ciphertext
- If the distance between them is too small, only few key bits can be typically extracted
- If the distance between them is too large, it is typically too difficult to get the key info

A Typical Example: AES-128

- In AES-128 the original 128-bit key K is expanded into eleven 128-bit subkeys K_i
- The key expansion operation is invertible, so the key can be easily derived from any subkey
- The avalanche of all the key bits into a single state bit takes a few rounds

A Typical Example: AES-128

- A single bit of state data available after the initial whitening step $P+K_0$ reveals exactly one key bit

- A single bit of state data available after the first round is a function of one bit from K_1, together with at most 32 bits from K_0

- A single bit of state data after the second round depends on all the 128 key bits

A Typical Example: AES-128

- Our attack will only use the plaintext and a single state bit leaked from the end of the second round in multiple encryptions

- It will ignore the known ciphertext (which is too far from the state bit we analyze)

- It will ignore the state bits leaked during earlier/later rounds, since they add little information/are too difficult to analyze

A Typical Example: AES-128

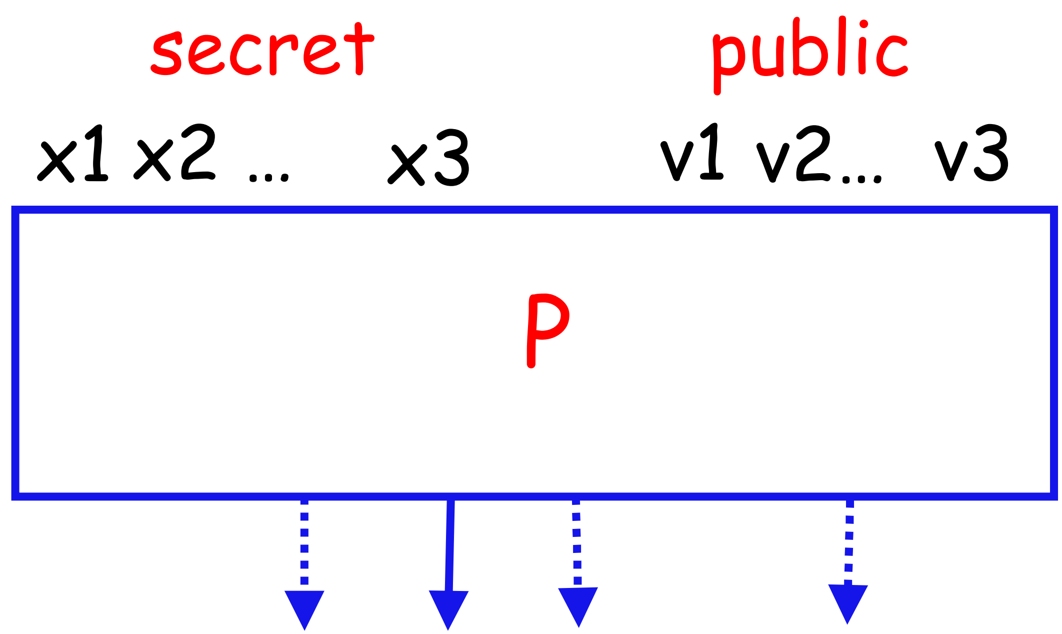
- No previous type of attack (exhaustive/statistical/differential/linear) seems to be applicable in this scenario
- The new attack is completely practical, requiring about 2^{35} time for complete key recovery
- The mathematical part of the attack was simulated successfully on a single PC in a few minutes

The new CUBE ATTACK (Dinur&Shamir):

- Is a very general key derivation algebraic attack
- Generalizes and improves some previous summation-based attacks such as Integral Attacks and Vielhaber's AIDA
- Was applied successfully to several stream ciphers (Trivium, Grain-128) but not to block ciphers
- As we show in this talk, cube attacks are ideal generic tools which can be applied to any type of leaking information in side channel attacks

Any cryptographic scheme can be described by multivariate polynomials:

- Each output bit is some multivariate polynomial $P(x_1,\ldots x_n,v_1,\ldots v_m)$ over GF(2) of secret variables x_i (key bits), and public variables v_j (plaintext bits in block ciphers/MAC's, IV bits in stream ciphers)



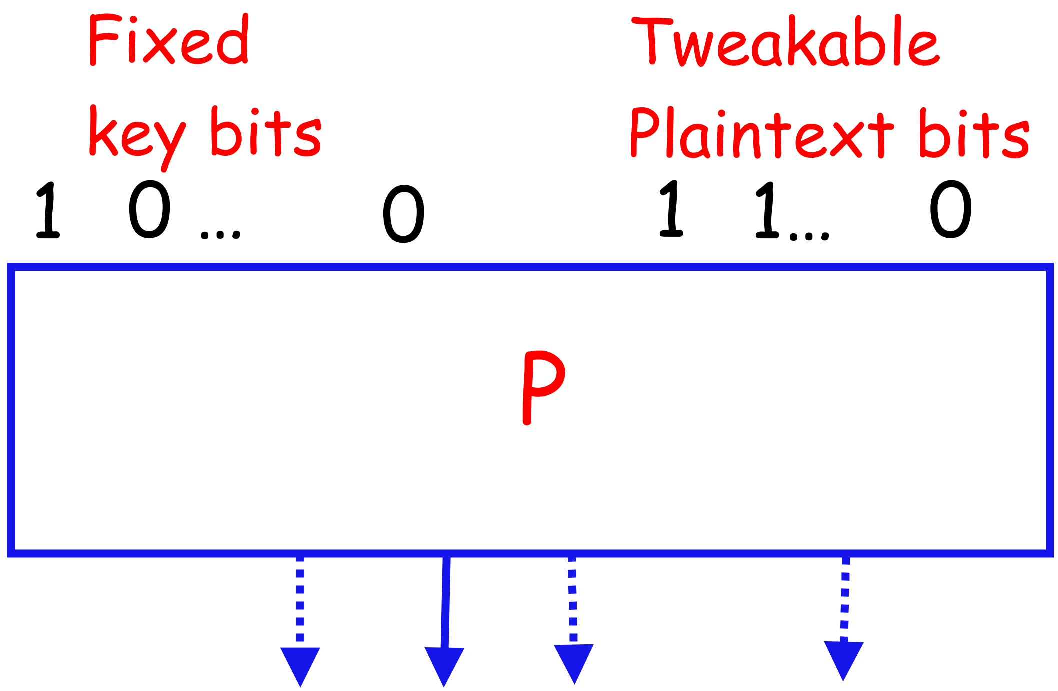
The main characteristics of cryptographically defined polynomials:

(consider the case of the AES, with 128+128 inputs)

- We consider only multivariate polynomials in fully expanded Algebraic Normal Form
- These polynomials are typically huge, and can not be explicitly defined, stored, or manipulated with a feasible complexity
- The data available to the attacker will typically be insufficient to interpolate their coefficients from their output values

Black box multivariate polynomials:

The only realistic way to deal with these polynomials is as black box polynomials, which can be evaluated on any (fully specified) set of secret and public inputs:



The typical problem of algebraic cryptanalysis:

- Solve a system of black box polynomial equations over GF(2):

$$P_1(x_1 \ldots x_n v^1_1 \ldots v^1_m)=0$$

$$P_2(x_1 \ldots x_n v^2_1 \ldots v^2_m)=1$$

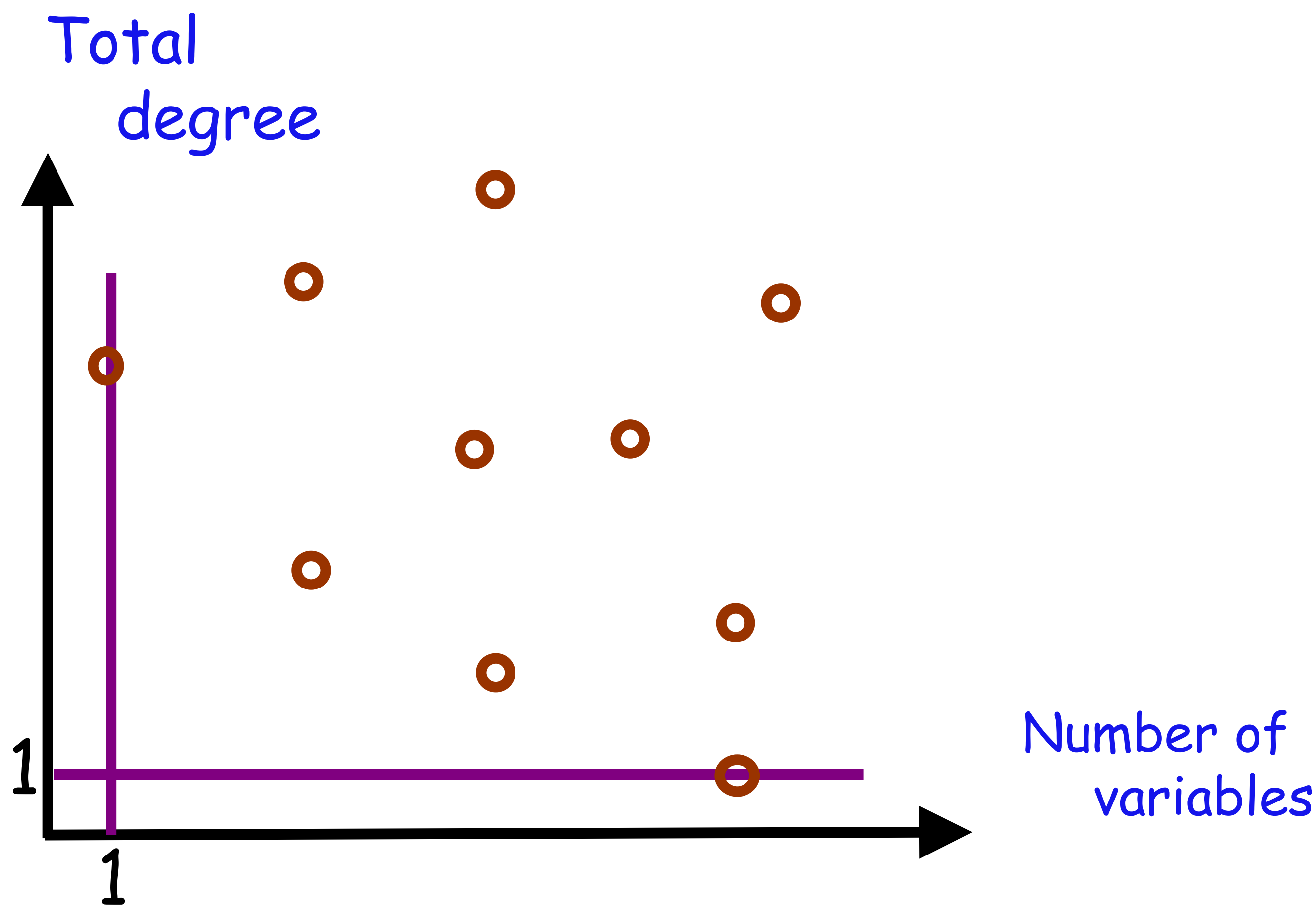
$$P_3(x_1 \ldots x_n v^3_1 \ldots v^3_m)=0$$

…

  in which the fixed key variables x_i are unknown, and the various plaintext/IV variables v^j_i are known

- The problem is NP-hard and exceedingly difficult in practice, even with explicitly given polynomials

The only easily solvable cases of simultaneous algebraic equations:



Grobner Base Techniques

- Can not be applied to black box polynomials
- Are double exponential in worst case, exponential in practice

Linearization Techniques:

- Are applicable to any explicit and sufficiently overdefined system of algebraic equations
- Assigns a new variable name to each term such as $y_{ijk} = x_i x_j x_k$, ignoring their algebraic relationships
- Solves the system of linear equations to derive the values of the singleton terms x_i

The new cube attack:

- Can be applied directly to arbitrary black box polynomials, even when they are huge
- Can be applied to unknown or partially known cryptographic schemes given as black boxes
- Can be applied automatically without careful preanalysis of the properties of the scheme
- Is provably successful when the black box polynomials are sufficiently random

Cube attacks have two phases:

- A preprocessing phase (via simulation):
  - The cryptosystem is given as a black box. The attacker can obtain one bit of output for any chosen key and plaintext.
- The online phase (via eavesdropping):
  - The stream cipher is given as a black box, with the key set to a secret fixed value. The attacker can obtain one bit of output for any chosen plaintext.

The complexity of the attack:

- For random polynomials of degree d in n input variables over GF(2), the complexity of cube attacks is $O(n2^{d-1}+n^2)$ bit operations, which is polynomial in the key size n (!)

- After two rounds of AES-128, the polynomial describing a single state bit depends on all the n=128 key bits, but its degree d is still relatively small, and it is not very random since many of the key bits do not have an opportunity to interact with each other

A typical example of a cube attack:

- To demonstrate the attack, consider the following dense master polynomial of degree $d=3$ over three secret variables x_1,x_2,x_3 and three public variables v_1,v_2,v_3:

$$P(v_1,v_2,v_3,x_1,x_2,x_3)=$$

$$v_1v_2v_3+v_1v_2x_1+v_1v_3x_1+v_2v_3x_1+v_1v_2x_3+v_1v_3x_2+v_2v_3x_2+v_1v_3x_3+v_1x_1x_3+v_3x_2x_3+x_1x_2x_3+v_1v_2+v_1x_3+v_3x_1+x_1x_2+x_2x_3+x_2+v_1+v_3+1$$

The effect of partial substitution:

- Substituting $v_1=1$ and $v_2=1$, we get a derived symbolic polynomial in the remaining variables x_1,x_2,x_3 and v_3:

$$P(v_1,v_2,v_3,x_1,x_2,x_3)=$$

$$x_1+x_2+v_3x_1+v_3x_3+x_1x_2+x_2x_3+x_1x_3+v_3x_2x_3 +x_1x_2x_3+1$$

The "miracle" created by cube attacks:

- The linearized version of the derived polynomial equations is extremely underdefined with many more columns than rows

The result of Gauss elimination:

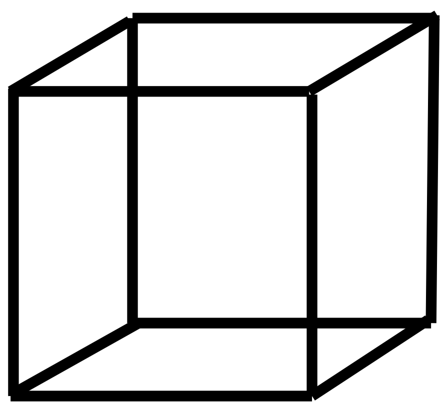
- For random unrelated polynomials in the rows, Gauss elimination can cancel only a tiny fraction of the nonlinear terms

The "miracle" created by cube attacks:

- However, polynomials derived from a single low degree master polynomial are related in a subtle way, which makes it possible to simultaneously eliminate the huge number of nonlinear terms from the relatively small number of equations by summing certain carefully selected subsets of the rows

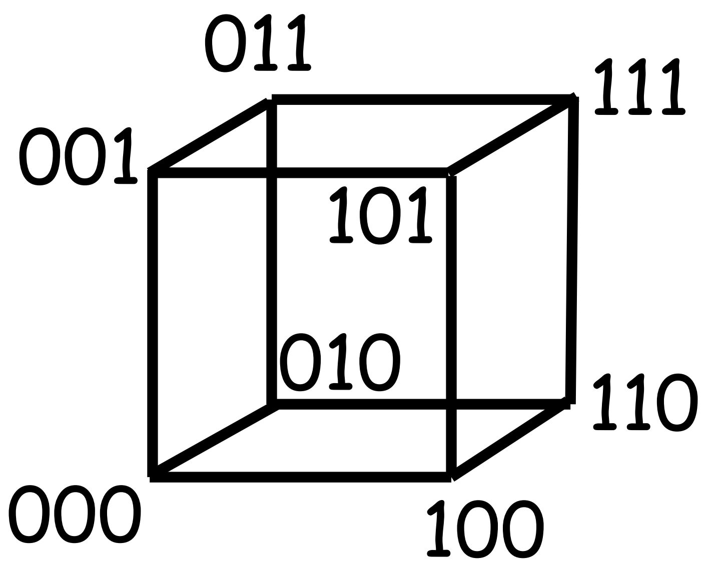
The Boolean cube:

Each corner of the Boolean cube will have 3 interpretations in cube attacks:



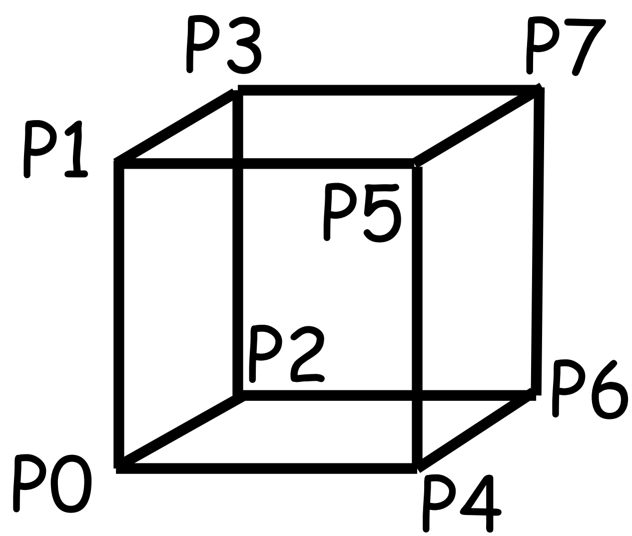
The Boolean cube:

An assignment of 0/1 values to some subset of the public v_j variables



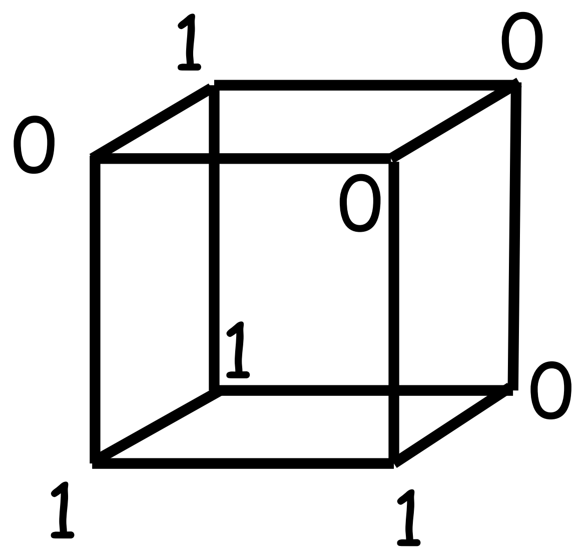
The Boolean cube:

The simplified symbolic form of the corresponding derived polynomial



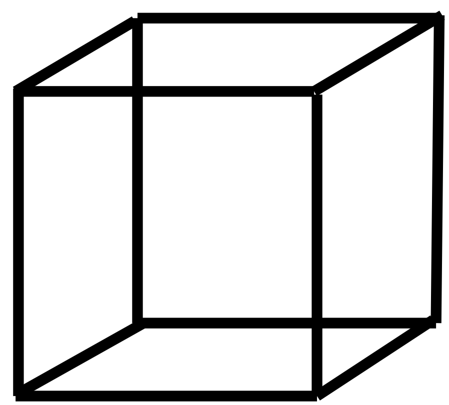
The Boolean cube:

The 0/1 value of this derived polynomial when all the other variables are set to their public and secret values

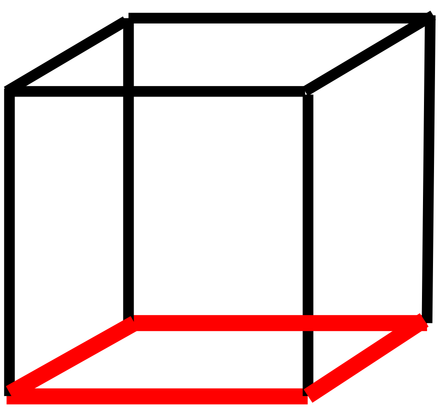


The Boolean cube:

We sum over GF(2) both the symbolic forms of the derived polynomials and their 0/1 values which occur in the vertices of various (potentially overlapping) subcubes



The summations:



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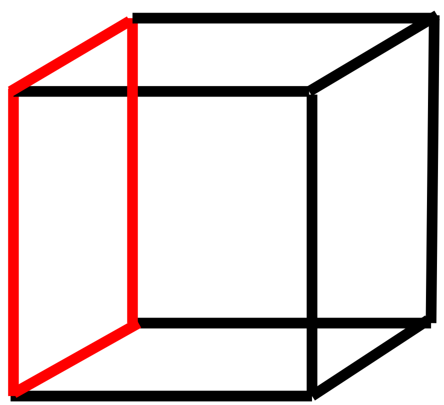

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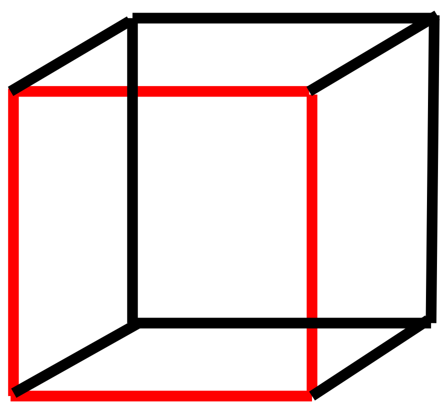


In our small example:

- Summing the 4 derived polynomials with $v_1=0$, all the nonlinear terms disappear and we get x_1+x_2; summing the 4 derived polynomials with $v_2=0$ we get $x_1+x_2+x_3$; and summing the four derived polynomials with $v_3=0$ we get x_1+x_3
- The sums of polynomials equated to their summed values give rise to three linear equations in the three secret variables x_i, which can be easily solved

Why did all the nonlinear products of secret variables disappear from the sum?

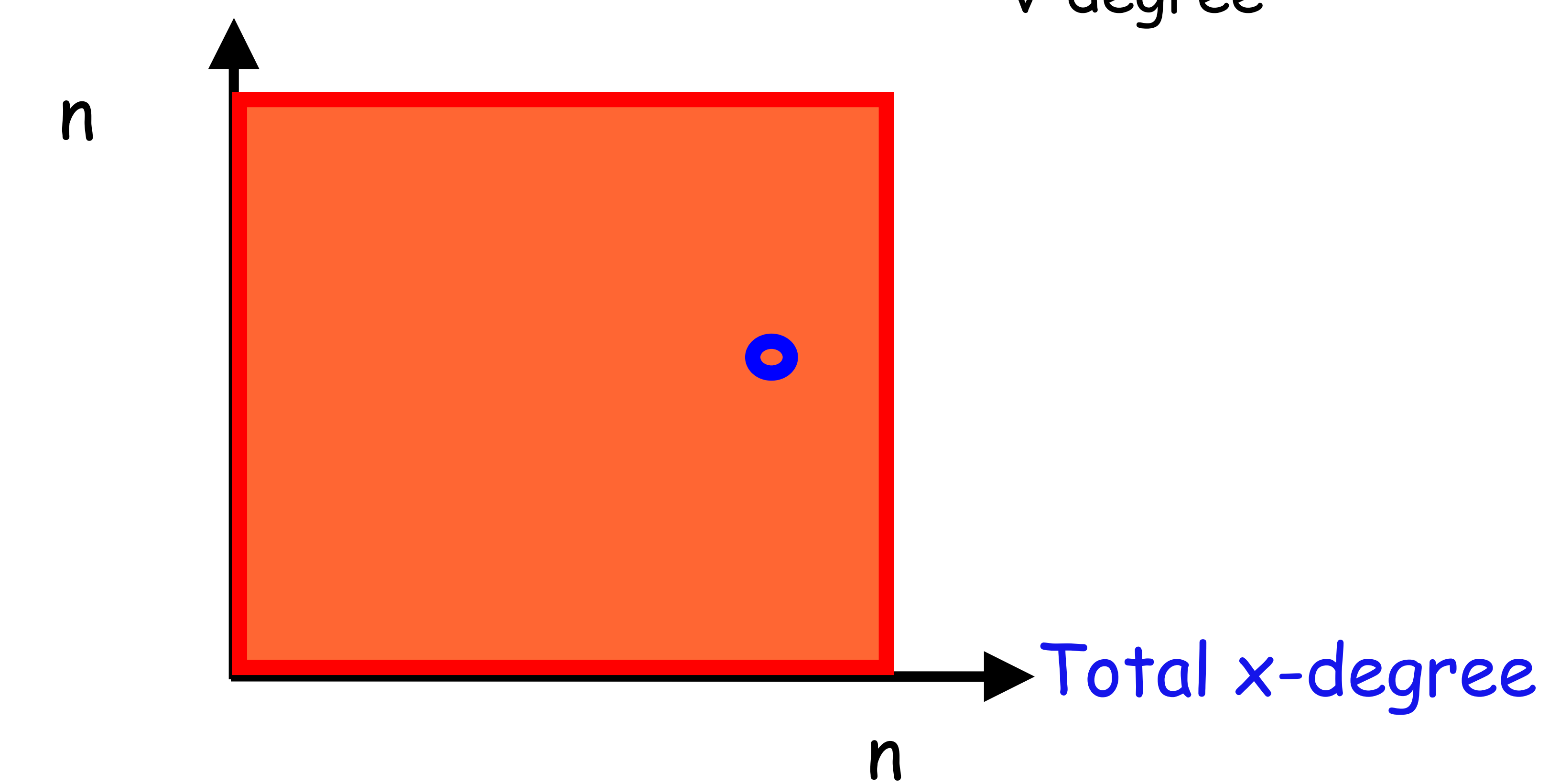
- All the terms are the products of at most 3 of the 6 x_i and v_j variables
- We sum over all the values of two v_j's
- Any term in the master polynomial P such as $x_1x_2v_1$ which contains the nonlinear product of two or more x_i in it, is missing at least one of the v_j that we sum over, and is thus added an even number of times modulo 2 to the sum

Isn't cube attack just a differentiation? No wonder that it reduces the degree...

- However, each terms has two types of variables: $v_1v_2v_4x_2x_3x_4$
- What we want: to reduce the x-degree to linear
- What we can do: to reduce the v-degree by differentiation
- Differentiating the term above wrt v_1v_2 gives $v_4x_2x_3x_4$; wrt v_1v_3 gives 0; neither has x-degree 1.

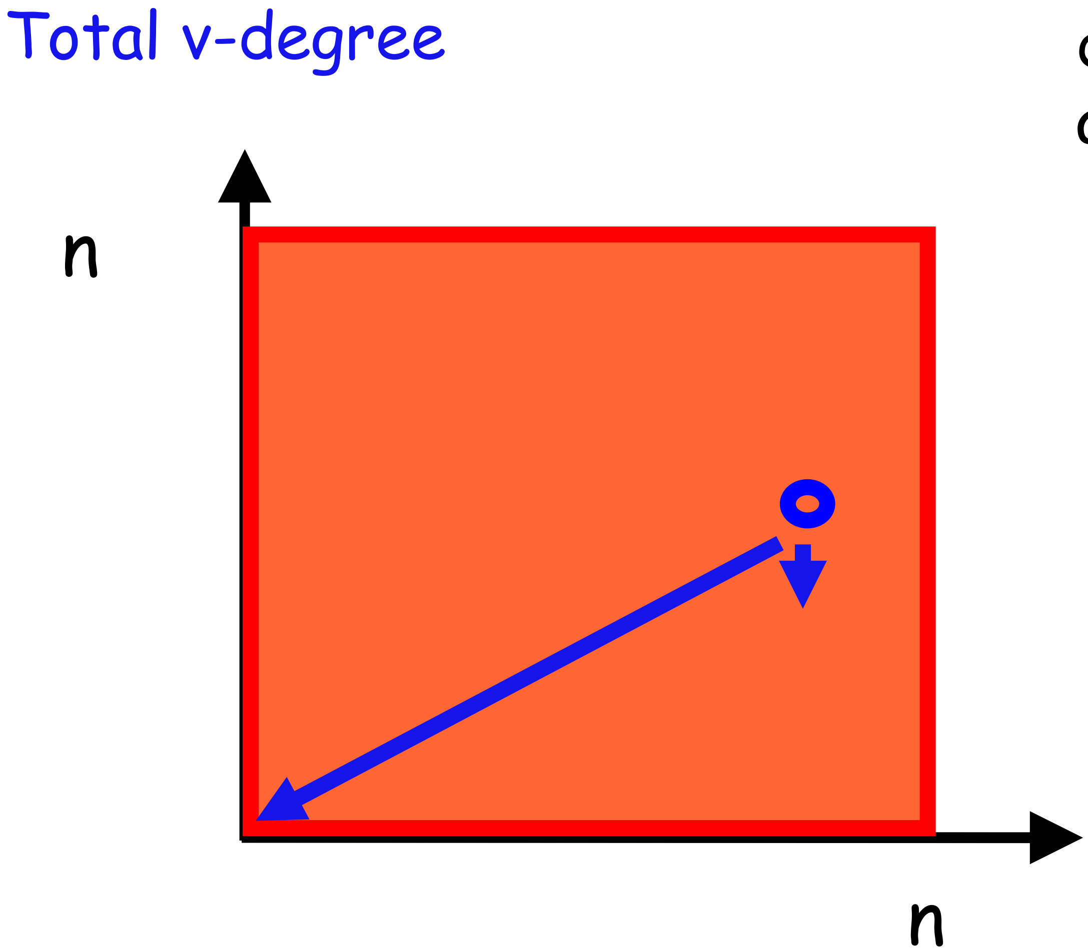
Consider a general polynomial in n secret and n public variables:

Each term has an x-degree and a v-degree



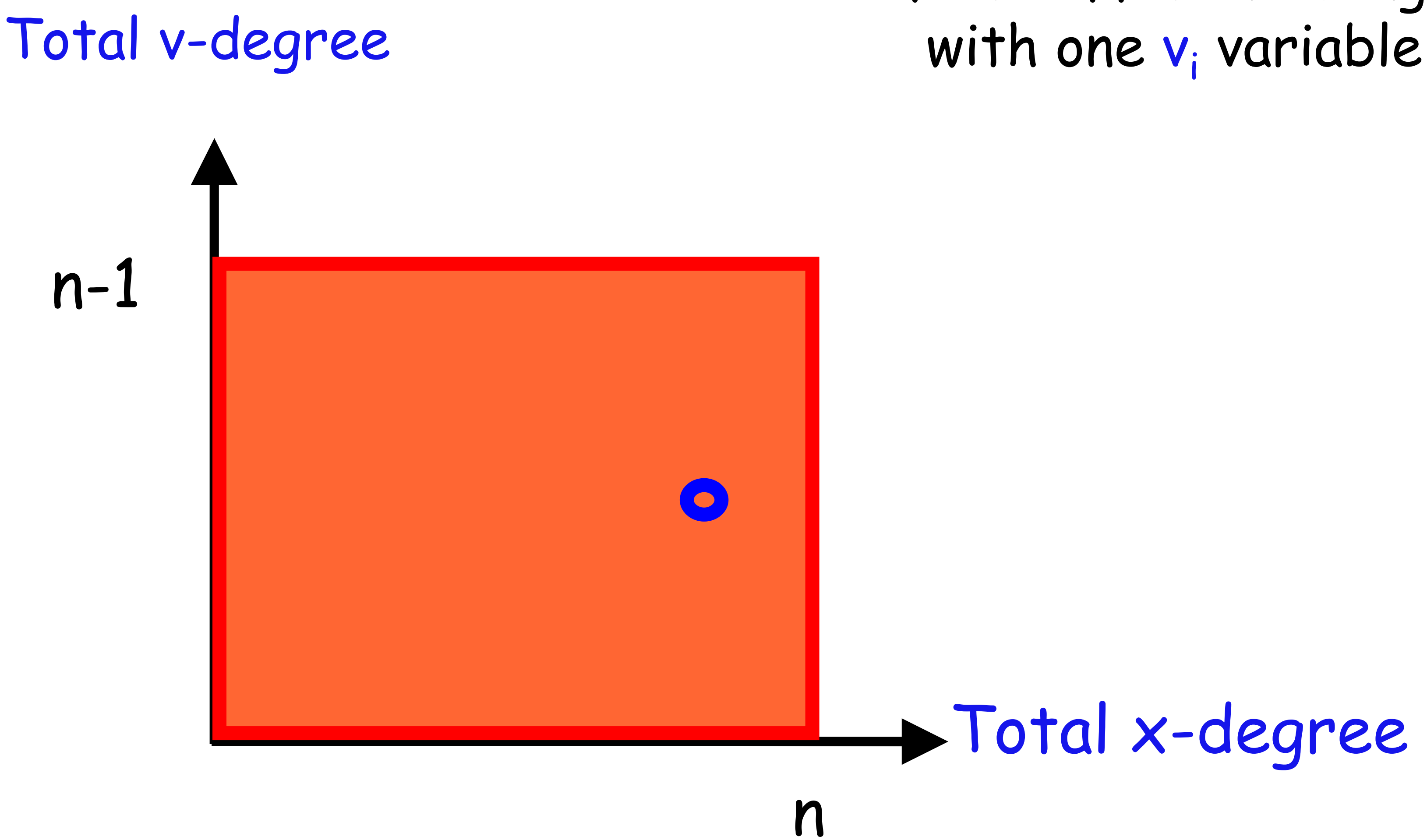
Differentiating wrt public variables reduce v-degrees

Each term moves downwards by 1 or all the way to zero



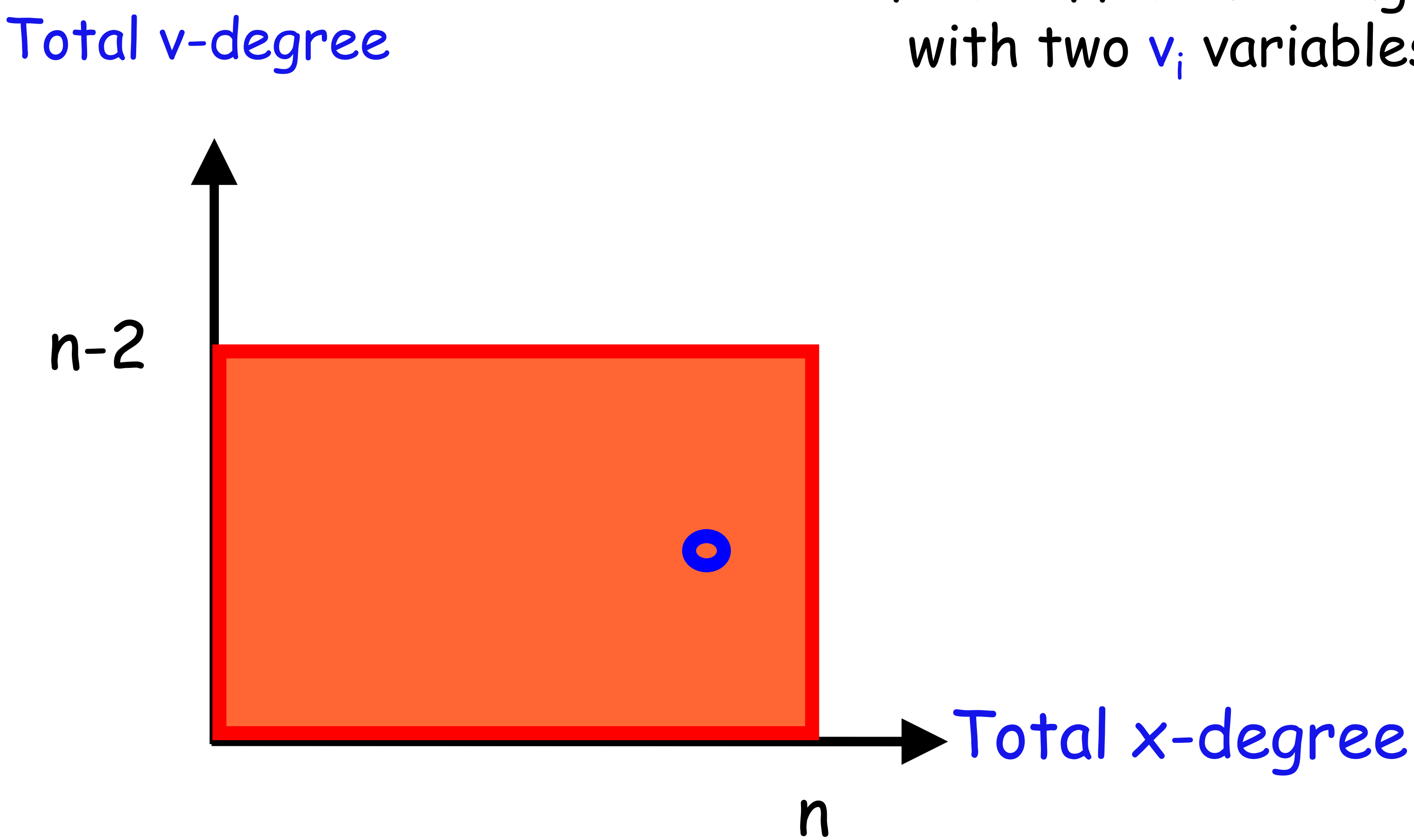
Differentiating wrt public variables reduce v-degrees

After differentiating
with one v_i variable

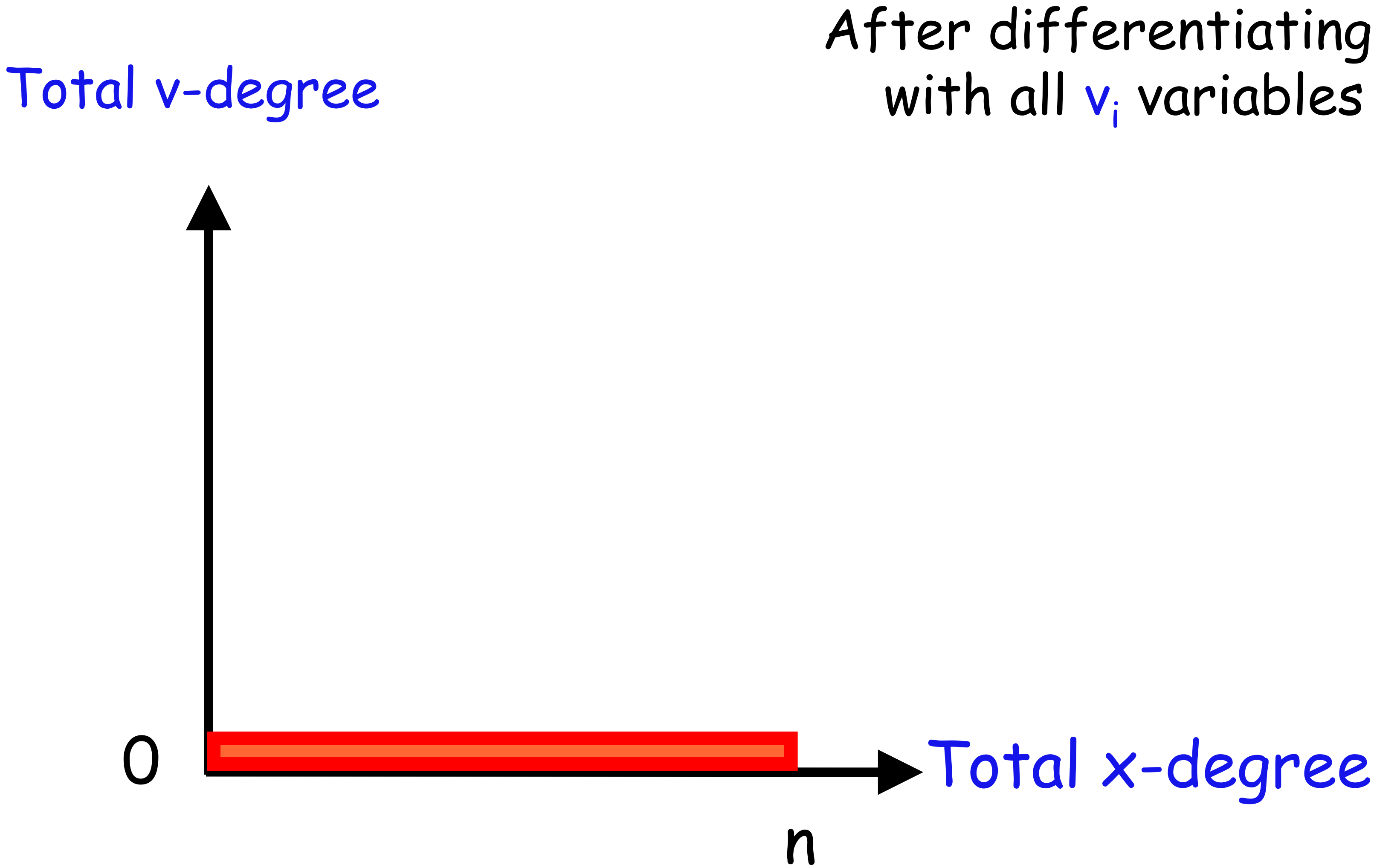


Differentiating wrt public variables reduce v-degrees

After differentiating with two v_i variables



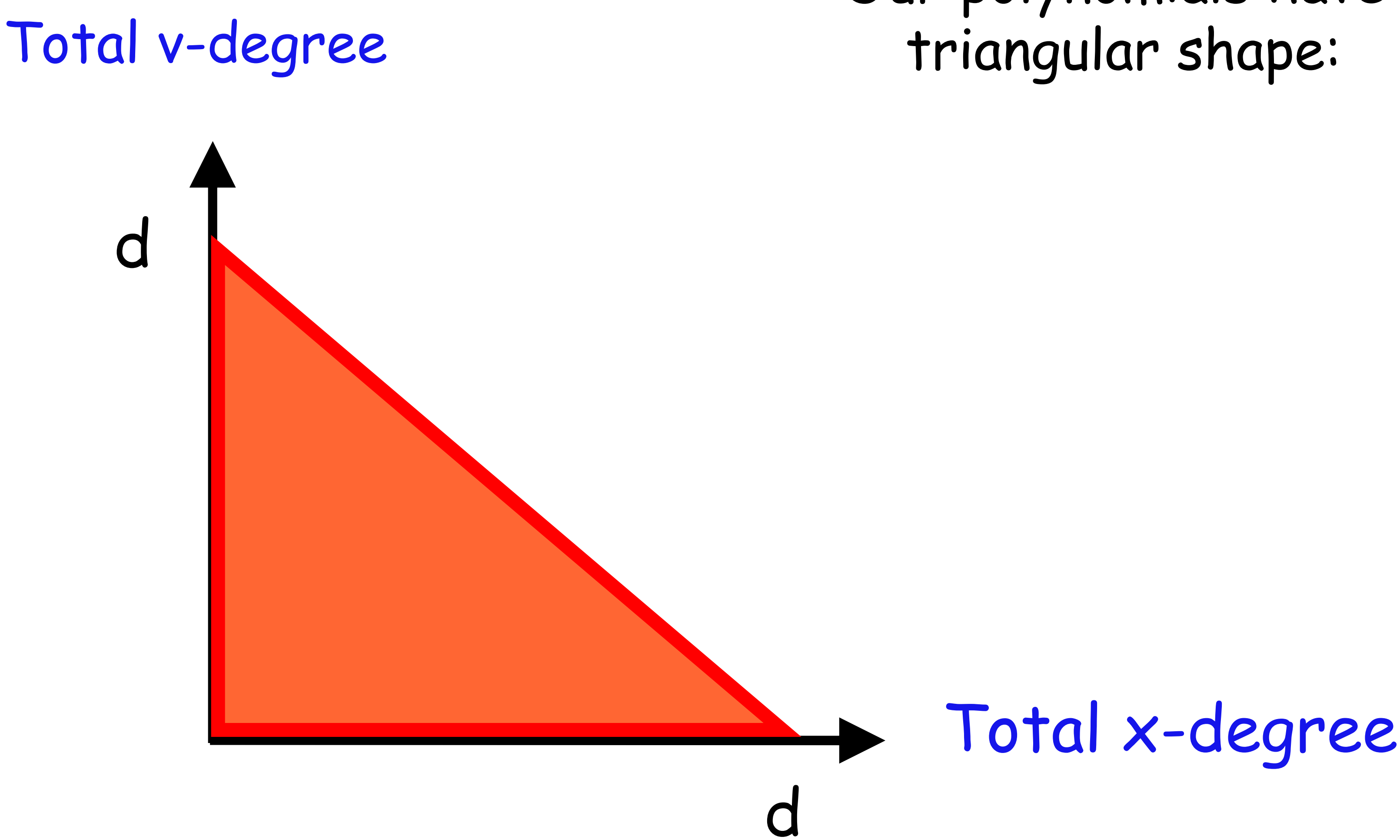
A general polynomial will still have x-degree of n even after differentiating wrt all its public variables



In cube attacks, we consider general polynomials of total degree $d<n$ in all the public and secret variables

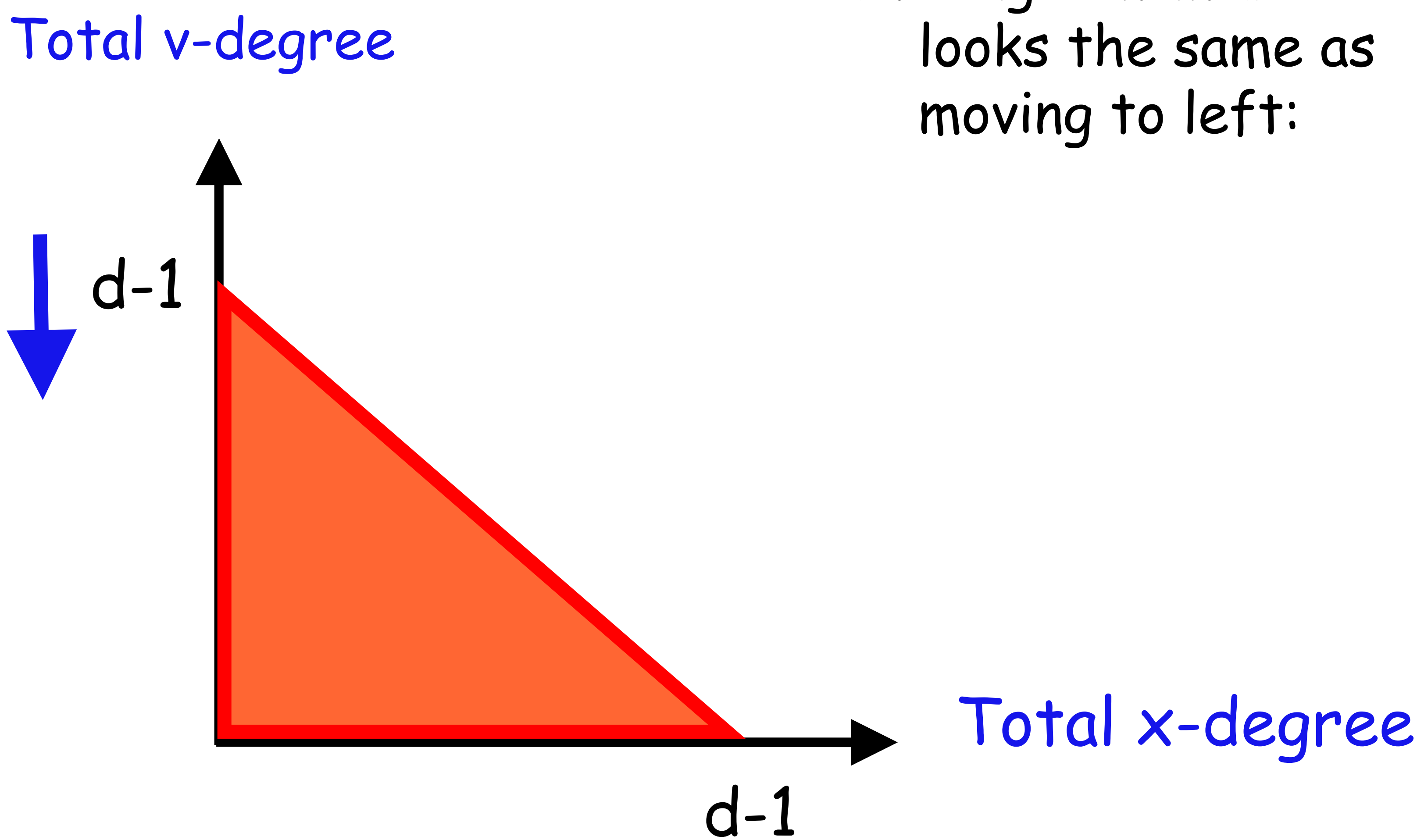
In cube attacks, we consider general polynomials of total degree d<n in all the public and secret variables

Our polynomials have triangular shape:



Differentiating with respect to one public variable:

Moving downwards looks the same as moving to left:



Differentiating with respect to i public variables:

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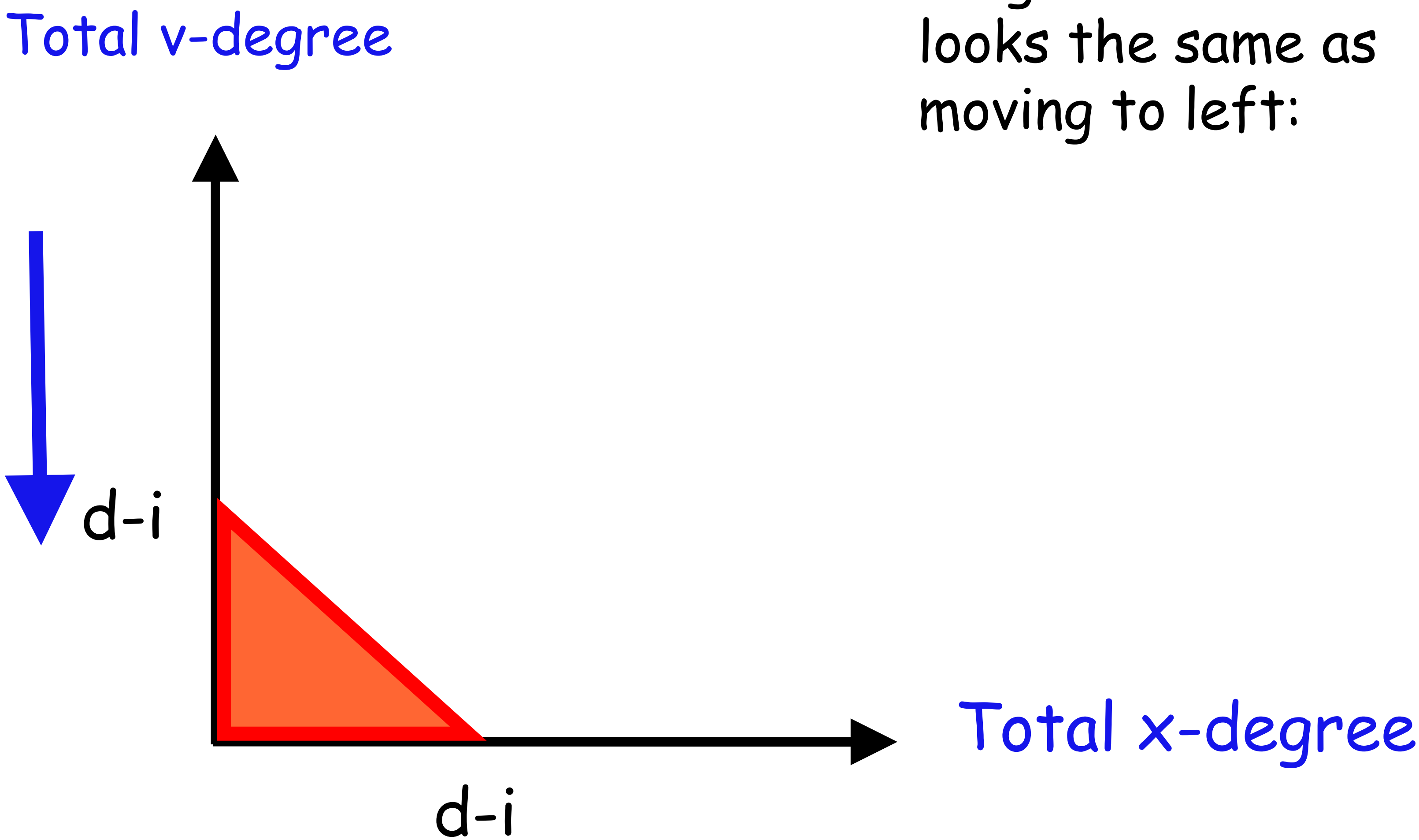
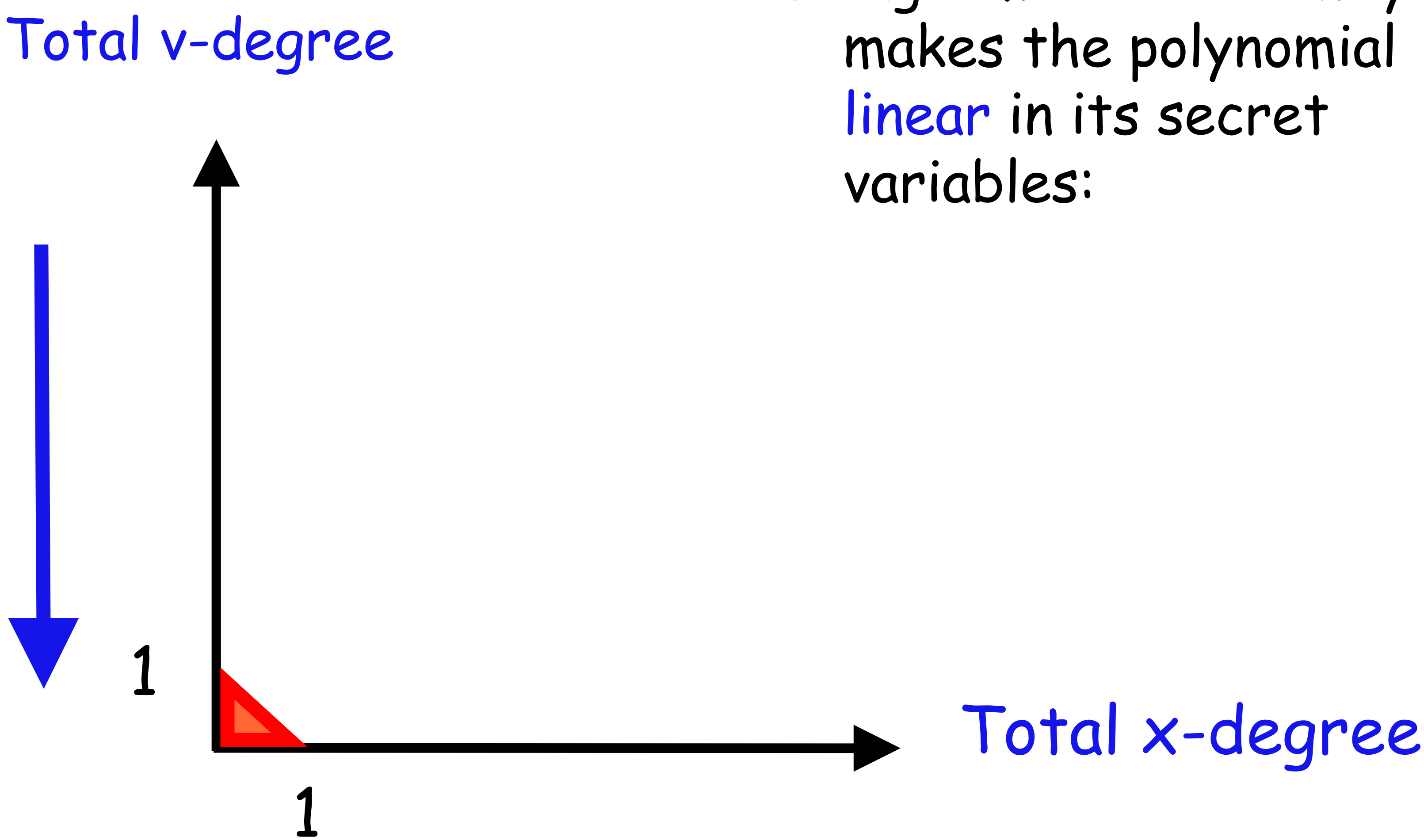


Differentiating with respect to d-1 public variables:



Going almost all the way makes the polynomial linear in its secret variables:

Remark:

- The attack is provably successful (rather than a heuristic) against any sufficiently random multivariate polynomials in which:
  - Each term occurs with probability 0.5
  - Each term of maximum degree d occurs with probability 0.5
  - Each term containing one x_i variable and d-1 v_j variables occurs with probability 0.5

- Polynomials representing cryptographic schemes are typically sufficiently random

Applying the cube leakage attack to AES:

- We found a small number of maxterms of two round AES by summing over cubes of dimension d=27, and a large number of maxterms by summing over cubes of dimension d=28
- The search was not blind, and exploited the known limitations on how the plaintext and key bits can interact with each other during the first two rounds

Applying the cube leakage attack to AES:

- The preprocessing identified a set of $n=128$ linearly independent maxterms
- During the actual attack on a particular key, we have to encrypt 2^7 sets of 2^{28} chosen plaintexts, and sum up the leaked bit in each set to determine the right hand side of each linear equation
- The total complexity of the attack is 2^{35}

Cube leakage attacks on SERPENT:

- Complete key avalanche in SERPENT occurs only at the end of the third round, due to the smaller 4-bit S-boxes and the weaker interaction between the state and key bits
- Since the degree of the polynomial grows more slowly in SERPENT than in AES, we were able to find $n=128$ linearly independent equations by summing over cubes of dimension $d=11$
- The complexity of the attack is only $2^7 \times 2^{11} = 2^{18}$

Equations for 3-round Serpent:

Table 1. Maxterms for 3-round Serpent given the first state bit. Equations are given in the working key bits that are inserted to the first Sbox layer.

Maxterm Equation	Cube Indexes	Maxterm Equation	Cube Indexes
1+x0	{3,8,21,35,46,78,85,96,99,104,117}	x16+x48	{10,25,42,57,62,80,94,106,112,121,126}
x0+x96	{7,13,32,34,45,64,66,77,98,103,109}	1+x16+x112	{6,24,25,38,48,56,57,80,102,120,121}
x32	{7,13,34,45,64,66,77,96,98,103,109}	1+x48	{3,10,13,16,35,45,80,94,99,106,109}
x64	{0,2,8,34,36,40,68,96,98,100,104}	1+x80	{10,11,16,17,48,49,74,75,106,107,113}
x1+x33	{2,3,23,34,35,65,87,97,98,99,119}	x17+x49	{3,14,22,35,54,78,81,99,110,113,118}
x1+x97	{18,19,20,33,51,52,65,82,114,115,116}	x17+x113	{0,22,32,49,54,63,81,86,95,96,127}
1+x33	{1,18,19,20,50,51,52,65,82,115,116}	x49	{0,32,54,63,81,86,95,96,113,118,127}
1+x65	{1,18,19,20,33,50,51,52,82,115,116}	1+x81	{0,17,22,32,49,54,63,86,95,96,127}
1+x2	{5,16,37,48,58,76,90,98,101,112,122}	x18+x50	{10,20,31,42,52,82,95,106,114,116,127}
x2+x34	{4,13,21,36,45,66,85,98,100,109,117}	1+x50+x114	{10,18,20,42,52,63,82,95,106,116,127}
1+x2+x98	{4,13,21,34,36,45,66,85,100,109,117}	x82	{10,18,20,31,42,52,95,106,114,116,127}
x66	{4,13,21,34,36,45,85,98,100,109,117}	1+x114	{13,18,45,53,55,77,85,87,109,117,119}
1+x3	{2,6,13,34,38,45,66,74,99,102,109}	1+x19+x115	{18,21,39,50,51,53,71,83,103,114,117}
x3+x35	{0,22,30,62,64,67,86,96,99,118,126}	x51	{3,4,24,56,67,68,83,99,100,115,120}
1+x35+x99	{0,3,22,30,62,64,67,86,96,118,126}	1+x51+x115	{18,19,21,39,50,53,71,83,103,114,117}
x67	{3,26,27,30,62,90,91,99,122,123,126}	x83	{18,21,39,50,51,53,71,103,114,115,117}
1+x4	{10,13,15,42,45,74,77,79,100,106,111}	1+x20+x116	{12,17,24,44,49,52,56,84,88,108,113}
x36	{0,16,21,32,48,53,68,80,96,100,117}	x52	{6,14,38,46,53,84,85,102,110,116,117}
1+x36+x100	{0,2,4,8,34,40,64,68,96,98,104}	1+x52+x116	{12,17,20,44,49,56,84,88,108,113,120}
1+x68	{0,2,4,8,34,36,40,64,96,98,104}	1+x84	{12,17,20,44,49,52,56,88,108,113,120}
x5+x37	{10,20,31,42,52,69,95,101,106,116,127}	1+x21+x117	{10,17,49,53,54,74,85,86,106,113,118}
1+x5+x101	{2,7,20,34,37,39,52,66,69,103,116}	1+x53	{6,10,21,27,38,59,74,85,102,106,123}

Conclusions:

- Cube attacks are ideal generic tools in leakage attacks on block ciphers

- They can be applied even to poorly understood types of leakage from unknown cryptosystems

- They do not require knowledge of the details of the implementation or the types of countermeasures employed